Tunable refraction in a two-dimensional quantum-state metamaterial

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In this paper we consider a two-dimensional quantum-state metamaterial comprising an array of qubits (two-level quantum objects). Here we propose that it should be possible to manipulate the propagation of quantum information. We show that a quantum metamaterial such as the one considered here exhibits several different modes of operation, which we have termed Aharonov-Bohm, intermediate, and quantum-Zeno. We also see interesting behavior which could be thought of as either *quantum birefringence* (where the material acts like a beam splitter) as well as the emergence of quantum correlations in the circuit's measurement statistics. Quantum-state metamaterials as proposed here may be fabricated from a variety of technologies from superconducting qubits to quantum dots and would be readily testable in existing state-of-the-art laboratories.

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I. INTRODUCTION

Quantum metamaterials, i.e., artificial quantum media, which maintain quantum coherence over the signal traversal time, hold promise of becoming a testing ground for the investigation of the quantum-classical transition, interesting new phenomena in wave propagation, and unusual technological applications [1,2]. With strong analogies existing between atomic physics, quantum optics, and superconducting systems it is natural to seek technologies that span these fields [3]. Solid state quantum metamaterials are one such class of devices where such parallels with conventional electromagnetic metamaterials can be leveraged to great utility. Indeed, emphasizing such a synergy, the implementation of a quantum metamaterial in the optical range is feasible [1,4]. In our view, the experimental realization of the concept is likely to be achieved first in the microwave range, as was the case with conventional metamaterials [5]. We believe that the best candidate system system would comprise superconducting qubits or SQUIDs [6–8] playing the role of controllable artificial atoms. This view is supported by the ability of superconducting flux qubits as quantum scatterers, a phenomenon that has been both theoretically modeled and experimentally observed [9–11]. Here, rather than using quantum circuits to construct a metamaterial for controlling electromagnetic fields, we propose an analogous device for the quantum state itself. We thus adopt the terminology *quantum-state metamaterial* to make clear the difference.

One-dimensional (1D) quantum metamaterials (in the conventional sense) can be realized readily enough. An example of this is a chain of qubits placed in a transmission line [6,7,9,12]. Unfortunately, 1D devices do not allow us to realize more interesting and useful effects such as "quantum birefringence" and other phenomena, where the change of direction of the signal by an arbitrary angle is important (e.g., Ref. [4]). Here one needs to go beyond 1D. As with any quantum circuit, we retain the essential requirement that any "proof of principle" realization of a truly quantum metamaterial must maintain global quantum coherence, and it is therefore necessary for such a system to contain as few unit elements as possible (although there may exist situations where such a constraint might be relaxed while retaining some useful quantum material properties). In this work we are therefore concerned with the minimal realization of a two-dimensional quantum-state metamaterial. The effect of changing signal transmission amplitude through a system by tuning of the material's constituents, which we consider in this paper, is related to two classes of phenomena: mesoscopic transport in quantum interferometers (for a review see, e.g., Ch. 5 of Ref. [13] or Ch. 3 of Ref. [7]) and a quantum state analogy of electromagnetically induced transparency, an effect known in optics [14–16] and recently demonstrated in an artificial atom (superconducting qubit) in the microwave range [11]. The underlying physics of either of these phenomena is the same: quantum state-dependent interference between different quantum trajectories contributing to the probability amplitude of the signal's transmission through the system.

In the case of mesoscopic transport through a quantum interferometer, the transmission amplitude is determined by the interference of electron wave components propagating through the branches of the device. Here the interference pattern occurs in real space, and it can be directly affected by the electromagnetic field acting on the electrons most spectacularly, through Aharonov-Bohm or Aharonov-Casher mechanisms [17]. Systems of coupled spin-$\frac{1}{2}$ particles have been widely studied in a range of different physical contexts. Recently there has been much interest in the potential use of spin chains or networks as buses for quantum state transfer within quantum information-processing devices [18,19]. It has been shown that the phase shift caused by an applied field (the Aharonov-Casher effect [17]) can significantly enhance the maximum attainable degree of entanglement in a spin chain, as well as improving the transfer of entanglement around a ring of spins [20]. The effects of continuously monitoring the output nodes of a spin network were investigated in Ref. [21], and it

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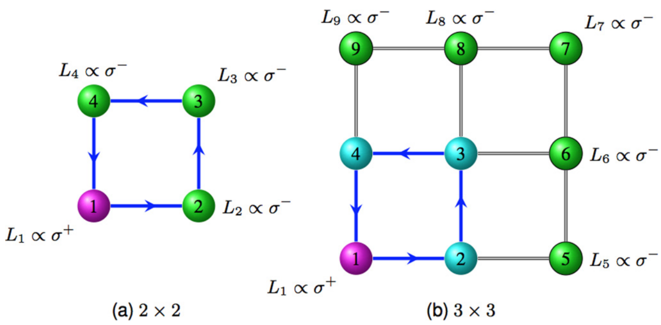

FIG. 1. (Color online) Set up, for (a) 2×2 and (b) 3×3 quantum metamaterial where each node represents a qubit and each edge the qubit-qubit coupling coupling. The directed couplings (arrowed, blue) correspond to $\Xi_{ij}(\phi) = \sigma_i^+\sigma_j^- \exp[i\phi] + \sigma_i^-\sigma_j^+ \exp[-i\phi]$ and the undirected couplings to $\Xi_{ij}(0)$. Node 1 is considered to be an input node (magenta), and the output nodes are for (a) 2–4 and (b) 5–9 (green). The Lindblad operators L_j describe the interaction with the source (L_1) and detectors (L_{2-9}).

was demonstrated that high-fidelity quantum state transfer is possible.

II. MODEL

Our simulated quantum metamaterial[1] comprises an array of interacting qubits as depicted in Fig. 1. The Hamiltonian for this system is then

$$H = \sum_i \frac{1}{2}\sigma_i^z + \sum_{(i,j)\in A} \mu_{ij}\, \Xi_{ij}(\phi_{ij}), \tag{1}$$

where σ_i^z is the Pauli matrix for the z direction and we have chosen $\mu_{ij} = \frac{1}{2}$ for the connected qubits, with each bond counted once only, as illustrated through the use of edges in Fig. 1 and zero otherwise. The coupling operator $\Xi_{ij}(\phi_{ij})$ takes the form

$$\Xi_{ij}(\phi_{ij}) = \sigma_i^+\sigma_j^- \exp[i\phi_{ij}] + \sigma_i^-\sigma_j^+ \exp[-i\phi_{ij}], \tag{2}$$

where $\sigma_i^\pm = \frac{1}{2}(\sigma_i^x \pm i\sigma_i^y)$. In order to model the effect of (for example) a magnetic field on this qubit metamaterial we have chosen the control parameter, $\phi_{ij} = \phi$, for the directed couplings, such as (1,2), (2,3), (3,4), (4,1) of Fig. 1(b), and zero otherwise as indicated in the schematic. Hence the flux enclosed by this plaquette is 4ϕ. We note that for our candidate realization of superconducting qubits the phase ϕ may be switched at high frequencies, e.g., of the order of a few GHz [23]. Consequently, rapid control of a quantum metamaterial such as the one proposed here should be possible. As it may be of utility when considering classical analogues of this system we also note that it is possible to write the Hamiltonian in the following form:

$$H = \sum_i \frac{\sigma_i^z}{2} + \sum_{\langle ij\rangle} \frac{\mu_{ij}}{2}\begin{bmatrix}\sigma_i^x & \sigma_i^y\end{bmatrix}\begin{bmatrix}\cos\phi_{ij} & \sin\phi_{ij}\\ -\sin\phi_{ij} & \cos\phi_{ij}\end{bmatrix}\begin{bmatrix}\sigma_j^x\\ \sigma_j^y\end{bmatrix},$$

which may be considered as a twisted XY model.[2]

Before we present the full model system, to gain insight into the effect of a phase and measurement on excitation dynamics let us first consider analytics for the simplified system of Fig. 1(a). The Hamiltonian can be easily diagonalized. In particular, the eigenstates in the single spin-flip sector,

$$|n\rangle = \frac{1}{2}\sum_{j=1}^{4} \exp[n\pi i(j-1)/2]\sigma_j^+ |{\downarrow\downarrow\downarrow\downarrow}\rangle$$

($n = 0,1,2,3$), have energy

$$E_n(\phi) = -1 + 2\mu\cos(\phi - n\pi/2).$$

For simplicity we henceforth shift the levels by the constant term 1. The amplitude for a spin flip created on site j at time 0 to be detected at site i at time t is

$$\begin{aligned}G_{ij}(t) = G_{ji}^*(t) &= \langle{\downarrow\downarrow\downarrow\downarrow}|\sigma_i^- \exp(-iHt)\sigma_j^+|{\downarrow\downarrow\downarrow\downarrow}\rangle\\ &= \sum_k \langle{\downarrow\downarrow\downarrow\downarrow}|\sigma_i^-|k\rangle\langle k|\sigma_j^+|{\downarrow\downarrow\downarrow\downarrow}\rangle \exp[-iE_k(\phi)t],\end{aligned}$$

giving

$$G_{11}(t) = \frac{\cos(2\mu t\cos\phi) + \cos(2\mu t\sin\phi)}{2},$$

$$G_{12}(t) = \frac{-i\sin(2\mu t\cos\phi) - \sin(2\mu t\sin\phi)}{2},$$

$$G_{13}(t) = \frac{\cos(2\mu t\cos\phi) - \cos(2\mu t\sin\phi)}{2},$$

and cyclically. We see immediately from Eq. (3) that when ϕ is an odd multiple of $\frac{\pi}{4}$ destructive interference suppresses propagation across the diagonal of the square.

However, the metamaterial that is the subject of this paper will require "inputs" and "outputs" and as such is an open system: as well as coherent evolution there is a measurement process. We can model this by a relaxation rate α and evaluate the transmission

$$P_{ij} \equiv \alpha\int_0^\infty e^{-\alpha t}|G_{ij}(t)|^2 dt. \tag{3}$$

There are three cases (two limiting):

[1]This idea may be connected with more conventional approaches to metamaterials if we consider our model as a reduced one for qubits coupled to electromagnetic modes where the field degrees of freedom have been eliminated producing effective couplings between the qubits similar to Ref. [22].

[2]Our motivation for this particular quantum metamaterial is a quantum circuit realisation fabricated using an array of superconducting qubits. This justifies our chosen Hamiltonian as it is of the same form as those currently used to model such circuits. It is, however, interesting to note that it would be possible to construct an equivalent system of interacting fermions (this can be achieved via a Jordan-Wigner or similar transform [24]). The results presented here in terms of the states of a qubit can, therefore, also be viewed in terms of the motion of fermions. Such a process might, for example, take the form of electrons propagating in a lattice of quantum dots in the presence of a magnetic flux, which is precisely the case of mesoscopic transport mentioned previously.

(1) If $\alpha \ll \mu$, then this corresponds to weak measurement of the metamaterial, and interference between multiple pathways is important. It therefore seems appropriate to term this limit the *Aharonov-Bohm regime*. Here we see sharp notches in the transmission across the diagonal when $|\phi - (2n+1)\pi/4| \lesssim \alpha/\mu$:

$$\lim_{\alpha\to 0} P_{12} = \lim_{\alpha\to 0} P_{14} = \begin{cases} \frac{1}{8}, & \phi = n\frac{\pi}{2} \\ \frac{1}{4}, & \text{otherwise} \end{cases},$$

$$\lim_{\alpha\to 0} P_{13} = \begin{cases} 0, & \phi = (2n+1)\frac{\pi}{4} \\ \frac{3}{8} & \phi = n\frac{\pi}{2} \\ \frac{1}{4}, & \text{otherwise} \end{cases}.$$

Here we expect to see a uniform distribution of counts but with enhanced transmission of along the diagonal and suppression of counts on the two corners at $\phi = n\frac{\pi}{2}$. We also expect to see a suppression of counts on the diagonal for $\phi = (2n+1)\frac{\pi}{4}$.

(2) The *intermediate regime* $\alpha \approx \mu$. Here the transmission has a $\cos 4\phi$ component with weaker higher-order harmonics. In this modulation the counts on the diagonal will have a maximum at $\phi = n\pi/2$ having strong suppression of counts at its minimum. The counts on detectors 2 and 4 are out of phase and higher on average than those on detector 3.

(3) When $\alpha \gg \mu$ we view the metamaterial as being strongly measured, and it seems natural to refer to this limit as the *quantum-Zeno regime*. Here measurements are taken much faster than the evolution time of the system, suppressing both hopping to second order and interference (which requires two hops) to fourth order, leaving a weak modulation:

$$P_{12} = O(\mu^2/\alpha^2) + O(\mu^4/\alpha^4)(3 + \cos 4\phi),$$
$$P_{13} = O(\mu^4/\alpha^4)(1 + \cos 4\phi);$$

that is, we expect a low number of counts in this case with the diagonal having far fewer counts than the corner detectors (as the excitation is more likely to be measured at sites 2 or 4 before it can get to 3).

III. QUANTUM JUMPS MODEL

In order to make analytic predictions in our above discussion we assumed that the outcome of measuring the outside edge of the metamatrial could be approximated by a relaxation rate. As the size of the material grows such an analysis becomes impractical. Furthermore, it is desirable that we introduce a more realistic model of a measurement process that could be readily associated with a real-world experiment. In order to characterize the properties of our proposed quantum metamaterial we need to generate some kind of excitation flow of the quantum state through the material. Hence, we will need an input, which we arbitrarily have coupled to qubit 1. Moreover, we also need to measure the output of this flow on the outer edges of the material, that is, qubits 2–4 of Fig. 1(a) and qubits 5–9 of Fig. 1(b). We have chosen to introduce both the input and the outputs through the introduction of a quantum jumps model of the measurement process. Here the outcome of a measurement is clear, i.e., either a jump is measured or it is not, providing a measurement record comprising a time series of "clicks" which we can then analyze (see, e.g., Ref. [25]). In effect, we measure in and create excitations on node 1 in Fig. 1 and measure out and destroy excitations on the opposing edges. Between measurements the circuit evolves unitarily, following quantum coherent Schrödinger evolution. Quantum jumps and quantum trajectories in superconducting qubits were successfully detected in experiments [26,27]. Specifically, the quantum jumps model is an unraveling of the master equation corresponding to the irreversible emission or absorption of absorbed or emitted excitations over very short time scales. The equation for this unravelling takes the form of stochastic Itô increment equation for the state vector according to

$$|d\psi\rangle = -\frac{i}{\hbar}H|\psi\rangle dt - \frac{1}{2}\sum_{j\in B}[L_j^\dagger L_j - \langle L_j^\dagger L_j\rangle]|\psi\rangle dt + \frac{1}{2}\sum_{j\in B}\left[\frac{L_j}{\langle L_j^\dagger L_j\rangle} - 1\right]|\psi\rangle dN_j, \qquad (4)$$

where B is the set of all qubits for Fig. 1(a) and {1,5,6,7,9,8} for Fig. 1(b). Here dN_j is a Poissonian noise process satisfying $dN_j\, dN_k = \delta_{jk}\, dN_j$, $dN_j\, dt = 0$ and $\overline{dN_j} = \langle L_j^\dagger L_j\rangle\, dt$; that is, jumps (or "clicks" of the detector) occur randomly at a rate that is determined by $\langle L_j^\dagger L_j\rangle$. Here the Lindblad associated with the input node is $L_1 = \gamma_{\rm in}\sigma_1^+$ and with the output nodes is, as depicted in Fig. 1, $L_{\rm i} = \gamma_{\rm out}\sigma_i^-$ for $i \in B$ and zero otherwise. Here γ is related (but not necessarily numerically equal) to the relaxation rate α introduced in Eq. (3).

In order to take account of the statistical nature of unravelings of the master equation we have, in the results that follow, summed the counts measured over 10 trajectories. For each trajectory we integrated the counts measured over a reasonable duration adjusted to allow for comparison between results for different values of γ (specifically the integration time was $10\,000/\gamma_{\rm in}$). These two factors together are sufficient to average out most of the statistical fluctuations that arise when using quantum trajectories methods (we are confident that the slight asymmetries that remain in our results are due to the remaining fluctuations).

We first consider the case of the 2×2 metamaterial of Fig. 1(a) where we have already identified three distinct regimes of operation. The results of our simulations are presented in Figs. 2(a), 2(b), and 2(c) and comprise simply the sum total of counts registered at each detector as a function of qubit and applied phase. In each case we have chosen circuit parameters $\gamma_{\rm in} = \gamma_{\rm out} = 0.1$ [Fig. 2(a)], $\gamma_{\rm in} = \gamma_{\rm out} = 1$ [Fig. 2(b)], and $\gamma_{\rm in} = \gamma_{\rm out} = 5$ [Fig. 2(c)] corresponding to the Aharonov-Bohm, intermediate, and quantum-Zeno regimes, respectively. Here we see remarkably good agreement between our theory and numerics with each of the three domains of operation being clearly resolved.

In Figs. 2(d), 2(e), and 2(f) we extend our calculations, using the same respective circuit parameters, to the 3×3 metamaterial of Fig. 1(b). In increasing the size of the quantum metamaterial the number of degrees of freedom of the system has increased substantially. Nevertheless, each set of results is still clearly identifiable with the same regime of operation retaining all the salient features of the previous case. This result suggests that a tuneable quantum-state metamaterial based

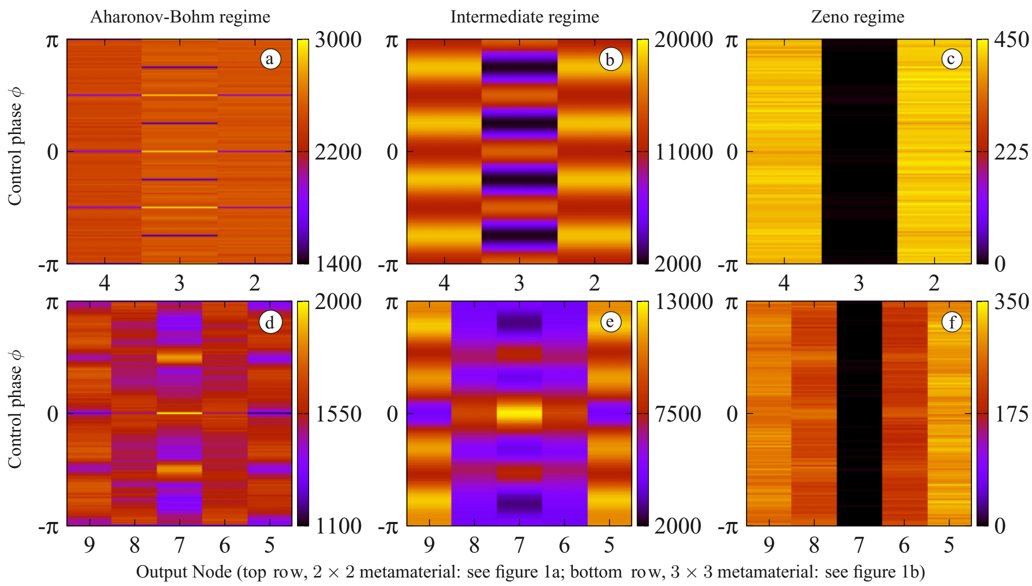

FIG. 2. (Color online) Plots of detector counts as a function of control phase ϕ and decay constant for the 2×2 array, top row (a, b, c), and the 3×3 array, bottom row (d, e, f). In the left-hand panes $\gamma_{\text{in}} = \gamma_{\text{out}} = 0.1$ (a, d), while $\gamma_{\text{in}} = \gamma_{\text{out}} = 1$ in the middle panes (b, e), and then $\gamma_{\text{in}} = \gamma_{\text{out}} = 5$ in the right-hand panes (c, f). In each case the time over which the statistics for each trajectory were collected was adjusted by a factor of $1/\gamma_{\text{in}}$ to allow for comparison.

on such an architecture might form the basis of a scalable technology.

We now describe, as an example of the possible rich behavior that quantum-state metamaterials can exhibit, the specific example of a 3×3 metamaterial in the intermediate regime whose sum total counts are shown in Fig. 2(e). For certain values of the controlling parameter (phase) ϕ we see behavior that is consistent with that of a quantum birefringent material acting as a beam splitter. We see this from $\phi = \pm\pi$ to around $\pm\frac{3}{4}\pi$ (where the effect is strongest) where we see that the vast majority of counts are measured on qubits 9 and 5. This pattern is again repeated around $\phi = \pm\frac{1}{4}\pi$. Although not conclusive, the possibility that the material is acting as a quantum birefringent beam splitter is supported by the calculation of the second order correlation coefficient

$$g^{(2)} = 1 + \frac{\langle \Delta N_9 \Delta N_5 \rangle}{\langle N_9 \rangle \langle N_5 \rangle}$$

between the qubit 9 and 5 at $\phi = 3\pi/4$, which we have determined to be approximately 0.92, which, by analogy with Ref. [28], is indicative of nonclassical scattering by the material (here N_i is the record, as a time series, of photons counted at each detector). In contrast, at $\phi = 0$ we see that counts are measured in a bell shape centered around qubit 7. Here it could be argued that this behavior corresponds to propagation of a signal as a beam across the diagonal of the quantum metamaterial. Finally we note that at $\phi = \pm\frac{1}{2}\pi$ and approximately $\phi = \pm\frac{1}{8}\pi$ the distribution of counts is approximately flat over all the output qubits. At these points the material is somewhat equally opaque across all the detectors with a quantum analog of electromagnetically induced transparency occurring for the other possible values of ϕ.

IV. CONCLUDING REMARKS

We have presented a model of a two-dimensional quantum-state metamaterial whose behavior is tuneable. Depending on circuit parameter the material can operate in three distinct regimes of operation: Aharonov-Bohm, intermediate, and quantum-Zeno. For different values of the control phase we see behavior consistent with (i) a quantum-state birefringent beam splitter, (ii) reduced quantum-state transmission plain wave propagation, and (iii) a quantum-state-beam directed along the leading diagonal. This system contains many degrees of freedom, and the analysis we have presented here only just begins to scratch the surface of what we believe quantum metamaterials will be capable of. Some example applications, that are beyond the scope of this work, might be demonstrating possible violations of Bell's inequalities for the material in its "birefringent" state or further modifying the materials' properties by applying the control flux in a manner that breaks the symmetry we assume in this paper. Due to the fact that there is no need to undergo a quantum state preparation or initialization stage, it is our view that quantum-state metamaterials will be more readily realizable than other quantum technologies such as quantum computing. Quantum-state metamaterials as proposed here may be fabricated in existing state-of-the-art laboratories and look set to form the basis for a new class of scalable quantum technologies. Given the phase dependence of our toy-model quantum-state metamaterial, immediate utility may be found in metrology, perhaps enabling the realization of solid state sensors whose operation is, in some way, analogous with quantum illumination and interferometer protocols, which could in turn lead to the determination of an unknown phase from an analysis of the quantum statistics of photon counts received at each of the sensors from a random source.

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