One-photon wave packet interacting with two separated atoms in a one-dimensional waveguide: Influence of virtual photons

S. Derouault and M. A. Bouchene*

Laboratoire de Collisions Agrégats Réactivité, CNRS UMR 5589, IRSAMC, Université Paul Sabatier, 118 Route de Narbonne, 31062 Toulouse Cedex 4, France

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We present a theoretical study of a one-photon wave packet scattered by two atoms in a one-dimensional waveguide. We investigate the role of terms beyond the rotating-wave approximation to correctly take into account the effects of the virtual photons that are exchanged between the atoms. These terms are shown to drastically influence the reflected and the transmitted fields, imposing strict constraints on their temporal envelopes.

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I. INTRODUCTION

The control of the interaction between light and matter is a research area undergoing continuous evolution because of the appearance of ever new challenges. A recent issue is the realization of all-optical quantum devices in a one-dimensional (1D) waveguide for quantum information purposes [1–3]. Recent experimental progress in the designing of these systems [4–14] and the possibility to reach the strong interaction regime between photons and atoms (or artificial atoms) open new perspectives, allowing the controllable transport of the flying qubits (photons) and the realization of fundamental quantum information operations [1–3,15–19]. Besides these challenges, the interaction of light and a collection of atoms in such systems represents on its own an interesting new theoretical problem. The photon scattering by a single atom in a 1D waveguide has been studied by Domokos *et al.* in a two-level system [20] using the Heisenberg approach, whereas spectral studies involving different experimental configurations have also been realized in Refs. [2,15], and the case for three-level atoms has been studied by Witthaut and Sørensen [21]. The extension of these studies to systems with two artificial atoms and an array of N artificial atoms has also been investigated [22–27]. However, all these studies were restricted to the regime where the rotating-wave approximation (RWA) is performed. Introduction of frequencies cutoff and the extension to negative frequencies are some procedures that are generally invoked to justify the neglect of far resonance frequencies or to recover finite coupling. In the case of two atoms in a dispersionless waveguide, non-RWA contributions cannot be neglected and are essential for the correct treatment of the problem and a deep understanding of virtual photon effects on the system dynamics. These features are well known in the field of super-radiance since the exhaustive papers of Friedberg *et al.* [28], Manassah [29], Milloni and Knight [30], and others [31,32] following the pioneering paper of Dicke [33]. The influence of virtual photons on the collective spontaneous emission of a photon wave packet by a [three-dimensional (3D)] cloud of dense atoms has recently received a great deal of attention [34–36]. This problem is particularly rich in new striking quantum effects, such as collective Lamb shift, collective encoding, entanglement, and directive photon reemission [37–40]. This interaction is associated with atomic shifts that modify the dynamics even for large samples.

Here, we present a detailed study of the scattering of a one-photon wave packet by a system of two atoms in a lossless and dispersionless 1D waveguide, taking into account the effects of the virtual photons. RWA is not performed, and we are interested in both the temporal and the spectral behaviors of the scattered field. We show that both the atomic and the field dynamics depend strongly on the nature (real or virtual) of photons exchanged by the atoms, and we clarify the role of each. Moreover, we establish the expression of the effective coupling between atoms, and we show that it results from a subtle interference effect between parts of virtual photons. We discuss the consequences of using RWA and introducing artificial frequencies cutoff. We demonstrate the important result that the central wave-packet frequency is always reflected only if non-RWA terms are taken into account. An additional feature in our approach is the development of a "time-dependent" point of view for the interaction. We show that the total reflection of the resonant frequency is related to the specific behavior of the temporal envelopes of the reflected and transmitted fields. Moreover, the transmitted wave packet obeys a strong constraint that forces the electric field to distort so that its pulse area (e.g., integral of the electric-field envelope) vanishes, whereas for the reflected field the pulse area is opposite to the incident one. This feature was already pointed in our previous study of photon scattering by a *single* atom in a 1D waveguide [41] and turned out to be fruitful to straightforwardly understand some temporal shaping effects. The time-dependent approach is only little addressed in quantum optics in contrast with semiclassical optics where intensive studies have been carried out leading to fascinating experiments for optical control and manipulation of quantum systems [42].

II. THE THEORETICAL MODEL

We consider two identical atoms that interact *resonantly* with a one-photon wave packet propagating in the $+z$ direction of an infinite lossless waveguide (Fig. 1). The transverse dimension of the waveguide d is assumed to be much smaller than λ_0 (the resonant wavelength) and the interatomic distance l (e.g., $d \ll \lambda_0,l$). An important consequence is that the

*aziz@irsamc.ups-tlse.fr

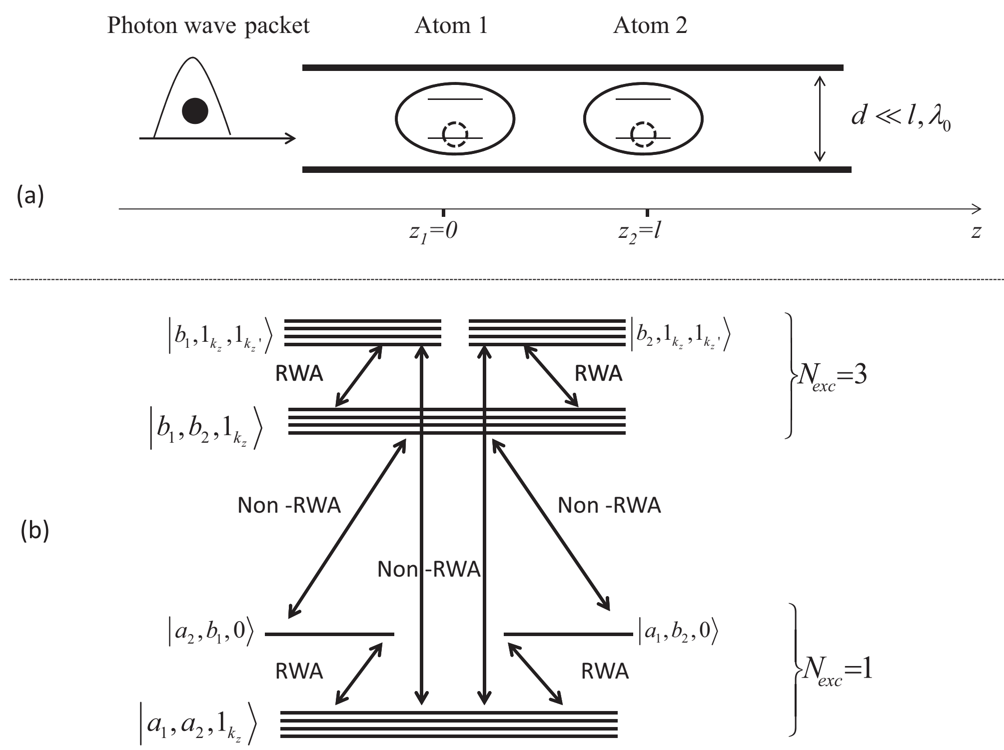

FIG. 1. (a) Configuration of the atoms and the initial photon wave packet in the waveguide. The dimension of the waveguide transverse section is d. The atoms are in the ground level and are separated by a distance l. The resonant wavelength is λ_0. (b) Atoms + field states involved in the interaction process with RWA or non-RWA nature of the coupling.

electrostatic dipole-dipole interaction between the atoms is strongly inhibited in the waveguide and will be neglected through the paper [43,44]. Moreover, the atoms no longer radiate outside the z direction, and the field remains uniform in the longitudinal direction of propagation [20,21]. The confinement of light in this waveguide also ensures that the strong interaction regime between the atoms and the photons can be realized.

The identical atoms are labeled $j = 1,2$ and are each modelized by a two-level system (ground states $|a_j\rangle$ and excited states $|b_j\rangle$ with eigenfrequencies 0 and ω_0, respectively). In our formalism, the Hamiltonian of the system $\hat{H}$ can be separated into three terms $\hat{H} = \hat{H}_{\text{atomic}} + \hat{H}_{\text{field}} + \hat{H}_{\text{inter}}$. In this notation, $\hat{H}_{\text{atomic}} = \sum_{j=1}^{2} \hbar\omega_0 |b_j\rangle\langle b_j|$ is the Hamiltonian of the free atoms, $\hat{H}_{\text{field}} = \int_{-\infty}^{+\infty} (\hbar\omega_k)\hat{a}_{k_z}^{\dagger}\hat{a}_{k_z} dk_z$ is the Hamiltonian of the free field with $\omega_k = c\,|k_z|$, and $\hat{a}_{k_z}$ is the photon annihilation operator that follows the usual bosonic commutation rules $[\hat{a}_{k_z},\hat{a}_{k_z'}^{\dagger}] = \delta(k_z - k_z')$. $\hat{H}_{\text{inter}} = \sum_{j=1}^{2}\int_{-\infty}^{+\infty}(\hbar g_k)(\hat{a}_{k_z}^{\dagger}e^{-ik_z z_j} + \hat{a}_{k_z}e^{ik_z z_j})(\hat{\sigma}_j + \hat{\sigma}_j^{\dagger})dk_z$ is the interaction Hamiltonian written in the Coulomb gauge with z_j as the position of atom j (with $z_2 - z_1 = l$), $g_k = \frac{(\omega_0 d_{ab})}{[4\pi\varepsilon_0(\hbar\omega_k)A]^{1/2}}$ is the coupling constant (A is the effective transverse guide section, and d_{ab} is the dipole moment), and $\hat{\sigma}_j = |a_j\rangle\langle b_j|$ is the lowering operator. Note that since the coupling g_k diverges in the infrared and decreases only slowly in the UV domain, we cannot neglect the contributions of any frequency and RWA cannot be performed [28–32,34–39,45].

With the atoms initially in the ground state and for the second order in the interaction Hamiltonian, the wave function $|\psi\rangle(t)$ of the whole system (atoms + field) can be formally expanded as

$$\begin{aligned}
|\psi\rangle(t) = &\int_{-\infty}^{+\infty} \alpha_{k_z}(t)e^{-i\omega_k t}|a_1,a_2,1_{k_z}\rangle dk_z \\
&+ \sum_{j=1}^{2} \beta_j(t)e^{-i\omega_0 t}|a_{j\neq j},b_j,0\rangle \\
&+ \int_{-\infty}^{+\infty} \gamma_{k_z}(t)e^{-i(2\omega_0+\omega_k)t}|b_1,b_2,1_{k_z}\rangle dk_z \\
&+ \sum_{j=1}^{2}\int_{-\infty}^{+\infty} dk_z \int_{-\infty}^{+\infty} dk_z' \eta_{j,k_z,k_z'}(t) \\
&\times e^{-i(\omega_k+\omega_{k'}+\omega_0)t}|b_j,1_{k_z},1_{k_z'}\rangle.
\end{aligned} \tag{1}$$

The two first terms correspond to states with an excitation number equal to 1. In the first term, we have states with one photon in the field and both atoms in the ground level, whereas in the second term, we have states with only one atom (j) in the excited state and no photons in the field. The last two terms correspond to an excitation number of 3. The third term describes the situation where both atoms are excited and there is one photon in the field, whereas the last term corresponds to the situation with one excited atom (j) and two photons in the field. These states are necessary for the correct treatment of the virtual photon and the collective Lamb-shift effects [34,35,39].

The evolution of the system is determined by the Schrödinger equation $i\hbar\frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle$ with the initial conditions $\beta_j(t\to-\infty)=\eta_{j,k_z,k'_z}(t\to-\infty)=\gamma_{j,j',k_z}(t\to-\infty)=0$ and $\alpha_{k_z}(t\to-\infty)=\sqrt{\frac{c}{\Delta}\sqrt{\frac{1}{2\pi}}}e^{-(\omega_k-\omega_0/\Delta)^2}$ (Δ is the spectrum bandwidth). Using Eq. (1), we obtain the following set of equations for the amplitudes:

$$i\dot{\alpha}_{k_z}(t)=\sum_{j=1,2}\left\{g_k[\beta_j(t)e^{-i(\omega_0-\omega_k)t}e^{-ik_zz_j}]+\left[2\int_{-\infty}^{+\infty}g_{k'}\eta_{j,k_z,k'_z}(t)e^{-i(\omega_0+\omega_{k'})t}e^{ik'_zz_j}dk'_z\right]\right\},\tag{2a}$$

$$i\dot{\beta}_j(t)=\int_{-\infty}^{+\infty}g_k\{[\alpha_{k_z}(t)e^{i(\omega_0-\omega_k)t}e^{ik_zz_j}]+[\gamma_{k_z}(t)e^{-i(\omega_0+\omega_k)t}e^{ik_zz_{j'}}]\}dk_z\quad(j'\neq j),\tag{2b}$$

$$i\dot{\gamma}_{k_z}(t)=g_k\sum_{\substack{j=1,2\\ j\neq j'}}\beta_j(t)e^{-ik_zz_{j'}}e^{i(\omega_0+\omega_k)t}+2\sum_{\substack{j=1,2\\ j\neq j'}}\int_{-\infty}^{+\infty}g_{k'}\eta_{j,k_z,k'_z}(t)e^{i(\omega_0-\omega_{k'})t}e^{ik'_zz_{j'}}dk'_z,\tag{2c}$$

$$i\dot{\eta}_{j,k_z,k'_z}(t)=\frac{1}{2}\{g_{k'}[\alpha_{k_z}(t)e^{i(\omega_0+\omega_{k'})t}e^{-ik'_zz_j}]+g_{k'}[\gamma_{k_z}(t)e^{-i(\omega_0-\omega_{k'})t}e^{-ik'_zz_{j'}}]+(k_z\leftrightarrow k'_z)\}\quad(j'\neq j).\tag{2d}$$

These equations show that states with excitation numbers equal to 1 (e.g., $|a_1,a_2,1_{k_z}\rangle$ and $|a_{j'\neq j},b_j,0\rangle$) are coupled through RWA coupling terms (operators $\hat{a}^\dagger_{k_z}\hat{\sigma}_j$ and $\hat{a}_{k_z}\hat{\sigma}^\dagger_j$), whereas states with excitation numbers of 3 (e.g., $|b_1,b_2,1_{k_z}\rangle$ and $|b_j,1_{k_z},1_{k'_z}\rangle$), respectively, are coupled to $|a_{j'\neq j},b_j,0\rangle$ and $|a_1,a_2,1_{k_z}\rangle$ because of non-RWA coupling terms (operators $\hat{a}_{k_z}\hat{\sigma}_j$ and $\hat{a}^\dagger_{k'_z}\hat{\sigma}^\dagger_j$). Finally, RWA coupling between highly excited states $|b_1,b_2,1_{k_z}\rangle$ and $|b_j,1_{k_z},1_{k'_z}\rangle$ also appears in (2c) and (2d).

A. Atomic coupling

The system of Eqs. (2) can be considerably simplified because of the presence of a continuum of modes. In Appendix A, we show that when $\omega_0,c/l\gg\Gamma,\Delta$, a Markovian approximation can be used leading to the fundamental equation for the amplitudes $\beta_j(t)(j=1,2)$,

$$\dot{\beta}_j(t)=S_{0,i}(t)-\Gamma\beta_j-M\beta_{j'\neq j}(t),\tag{3}$$

where $\Gamma=\frac{2\pi}{c}\frac{g_k^2\omega_k}{\omega_0}$ is a relaxation constant term (independent of frequency ω_k) and $S_{0,j}(t)=-i\sqrt{\frac{\Gamma}{2\pi}}\int_{-\infty}^{+\infty}\sqrt{c\frac{\omega_0}{\omega_k}}\alpha_{k_z}(t\to-\infty)e^{i(\omega_0-\omega_k)t}e^{ik_zz_j}dk_z$ is a source term due to the presence of an initial incident photon. Equation (3) also exhibits a third term that results from the coupling of the two atoms through the field and that involves a coupling parameter M that is the sum of four contributions $M=\sum_{i=1}^4 M_i$ corresponding to different quantum paths as will be explained further,

$$M_1=\frac{\Gamma\omega_0}{2\pi}\int_0^\infty\int_0^\infty\frac{e^{i(\omega_0-\omega)\tau}e^{ik_zl}}{\omega}d\omega\,d\tau,\tag{4a}$$

$$M_2=\frac{\Gamma\omega_0}{2\pi}\int_0^\infty\int_0^\infty\frac{e^{-i(\omega_0+\omega)\tau}e^{ik_zl}}{\omega}d\omega\,d\tau,\tag{4b}$$

$$M_3=M_1(l\leftrightarrow-l)=\frac{\Gamma\omega_0}{2\pi}\int_0^\infty\int_0^\infty\frac{e^{i(\omega_0-\omega)\tau}e^{-ik_zl}}{\omega}d\omega\,d\tau,\tag{4c}$$

$$M_4=M_2(l\leftrightarrow-l)=\frac{\Gamma\omega_0}{2\pi}\int_0^\infty\int_0^\infty\frac{e^{-i(\omega_0+\omega)\tau}e^{-ik_zl}}{\omega}d\omega\,d\tau.\tag{4d}$$

Using the mathematical relations,

$$\int_0^{+\infty}e^{i(\omega-\omega_0)T}dt=\pi\delta(\omega-\omega_0)+i\mathrm{P}\left(\frac{1}{\omega-\omega_0}\right),\tag{5a}$$

$$\int_0^{+\infty}e^{i(\omega+\omega_0)T}dT=i\mathrm{P}\left(\frac{1}{\omega+\omega_0}\right),\tag{5b}$$

where P designs the Cauchy principal part of the integral, we obtain

$$M_1=\frac{\Gamma e^{ik_0l}}{2}+\frac{i\Gamma\omega_0}{2\pi}\mathrm{P}\left(\int_0^\infty\frac{e^{i\omega l/c}}{\omega_0-\omega}\frac{d\omega}{\omega}\right),\tag{6a}$$

$$M_2=-\frac{i\Gamma\omega_0}{2\pi}\int_0^\infty\frac{e^{i\omega l/c}}{\omega_0+\omega}\frac{d\omega}{\omega},\tag{6b}$$

$$M_3=\frac{\Gamma e^{-ik_0l}}{2}+\frac{i\Gamma\omega_0}{2\pi}\mathrm{P}\left(\int_0^\infty\frac{e^{-i\omega l/c}}{\omega_0-\omega}\frac{d\omega}{\omega}\right),\tag{6c}$$

$$M_4=-\frac{i\Gamma\omega_0}{2\pi}\int_0^\infty\frac{e^{-i\omega l/c}}{\omega_0+\omega}\frac{d\omega}{\omega}.\tag{6d}$$

The integrals appearing in (6) can be evaluated with the introduction of the sine (Si) and cosine (Ci) integral functions defined by $\mathrm{Ci}(x)=-\int_x^\infty\frac{\cos t}{t}dt;\mathrm{Si}(x)=\int_0^x\frac{\sin t}{t}dt$ [46]. For real arguments, these functions are even and odd, respectively. The asymptotic values are $\mathrm{Ci}(|x|\gg1)=0,\mathrm{Si}(|x|\gg1)=\pi/2$, and we have $\mathrm{Ci}(0)=\infty$. We obtain the following relations for the coupling elements:

$$M_1=\frac{\Gamma}{2}e^{i(\omega_0l/c)}+\frac{\Gamma}{2\pi}\left[e^{i(\omega_0l/c)}\left(\mathrm{Si}(\omega_0l/c)+\frac{\pi}{2}+i\,\mathrm{Ci}(\omega_0l/c)\right)-G_+\right],\tag{7a}$$

$$M_2=\frac{\Gamma}{2\pi}\left[e^{-i(\omega_0l/c)}\left(\mathrm{Si}(\omega_0l/c)-\frac{\pi}{2}-i\,\mathrm{Ci}(\omega_0l/c)\right)+G_+\right],\tag{7b}$$

$$M_3=\frac{\Gamma}{2}e^{-i(\omega_0l/c)}+\frac{\Gamma}{2\pi}\left[-e^{-i(\omega_0l/c)}\times\left(\mathrm{Si}(\omega_0l/c)+\frac{\pi}{2}-i\,\mathrm{Ci}(\omega_0l/c)\right)-G_-\right],\tag{7c}$$

$$M_4=\frac{\Gamma}{2\pi}\left[-e^{i(\omega_0l/c)}\left(\mathrm{Si}(\omega_0l/c)-\frac{\pi}{2}+i\,\mathrm{Ci}(\omega_0l/c)\right)+G_-\right].\tag{7d}$$

$G_\pm$ is a constant given by $G_\pm = \pm\frac{\pi}{2} + i\ \mathrm{Ci}(\varepsilon l/c \to 0)$, and its imaginary part diverges (ε is an artificial infrared frequency cutoff). This is not surprising since the atom-photon coupling is $g_k \propto \omega_k^{-1/2}$ and diverges in the infrared. However, only the sum of these integrals is involved in the integral in (3), and the final coupling term $M = \sum_{i=1}^{4} M_i$ is convergent and is given by

$$M = \Gamma e^{ik_0 l}. \tag{8}$$

The dependence of the coupling coefficient with the interatomic distance appears through the dephasing term $e^{ik_0 l}$. Thus, the coupling term does not decrease with the atomic separation in contrast with the free space situation. This is because in our situation (1D waveguide with $l \gg d$), the propagating photons are confined along the interatomic axis making the energy flux unchanged between atoms. This is in contrast with the free space where the emission of the photon with wave vectors out of the interatomic axis is allowed, reducing the photon exchange probability by $1/l$ decreasing term for isotropic emission, $1/l^2$ and $1/l^3$ decreasing terms for anisotropic emission [28–30,34]. Note that the infrared divergence is particular to the 1D case where the state density is constant with the frequency ω_k. In the 3D situation (free space), the state density ($\propto \omega_k^2$) compensates for the g_k^2 contribution ($\frac{1}{\omega_k}$), and one therefore deals with an ultraviolet divergence of the amplitudes [34–39,47].

B. Quantum paths

The field and the atomic dynamics can also be understood from Eq. (3) in terms of photon exchange between atoms. Moreover, we represent in Fig. 2 the paths corresponding

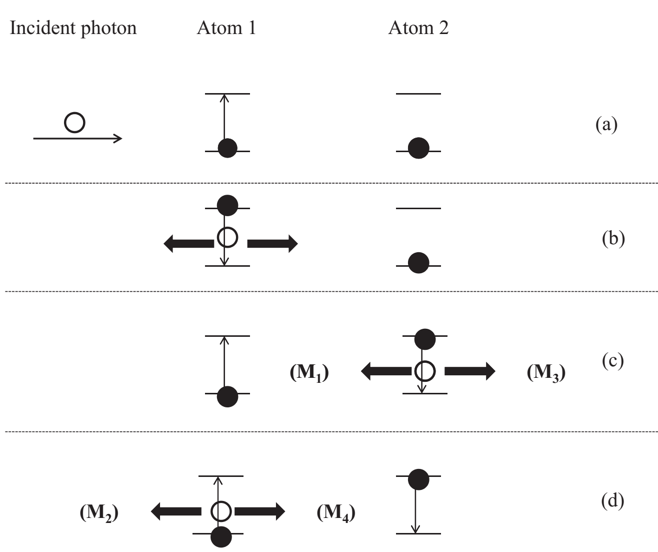

FIG. 2. Quantum paths leading to the modification of β_1, the excited-state amplitude of atom 1. Paths are associated with (a) absorption of the initial photon, (b) relaxation of atom 1 with emission of a photon in the forward or backward direction, (c) relaxation of atom 2 with emission of a photon in the backward (M_1 amplitude) or forward (M_3 amplitude) direction and that interacts further with atom 1 (RWA terms), and (d) excitation of atom 1 with emission of a photon in the backward (M_2 amplitude) or forward (M_4 amplitude) direction and that interacts further with atom 2 (non-RWA terms). Similar photon diagrams exist for the modification of β_2.

to all terms of Eq. (3) stressing on the photon absorption and emission processes. We have considered the evolution of the excited state of the first atom for simplicity. The first term (source term) is represented schematically in (a) and corresponds to the situation where the incident photon is absorbed by the atom in the ground state leading to an increase in the excited-state amplitude. The second term corresponds to (b) and represents the excited-state relaxation because of the coupling to a continuum of photons. The photons are emitted in both reflected and transmitted directions. Note that these two schemes also hold when only a single atom is present. The other remaining terms correspond to the interaction between atoms. Case (c), representing the case where atom 2 relaxes and emits photons corresponds to the presence of RWA contributions. The photon emitted in the backward direction interacts with atom 1, leading to a modification of the excited-state amplitude. This situation is associated with the M_1 contribution to the coupling term. The case where the photon is emitted in the forward direction is associated with the M_3 contribution to the coupling term. Case (d) represents the situation corresponding to the presence of non-RWA terms. The path corresponding to the M_4 contribution represents the case where atom 1 emits a photon and transits to the excited states. The forward photon is then absorbed by atom 2 that relaxes to the ground state. The M_2 contribution is the same process as the M_4 contribution but with the absorption of the backward photon by atom 2. It is worthy here to notice that, although the photon emitted in the forward direction flies away from atom 1 (M_3 contribution) or 2 (M_2 contribution), it can interact with these atoms because the coupling diverges for long wavelengths explaining nonvanishing contributions of these terms.

C. Real and virtual photon interplay

The coupling term and paths associated with M_i contributions can also be related to the virtual and real characters of the photons involved in the process. According to the common signification of these expressions, the real photons are those created in resonant physical processes that conserve the bare energy (e.g., energy without atom-radiation coupling), whereas virtual photons are the ones that are created in nonresonant processes that do not conserve this energy [48]. In our situation, a real photon corresponds to the resonant photon ($\omega = \omega_0$) involved in RWA contributions (M_1 and M_3). These observe bare-energy conservation during the whole exchange process. Virtual photons are the other nonresonant photons ($\omega \neq \omega_0$) involved in RWA processes (M_1 and M_3) and all photons involved in non-RWA processes (M_2 and M_4) that obviously violate bare-energy conservation in intermediate states of the system. Both real and virtual photons can modify the dynamics of the quantum system but in a different manner as we see next. We refer back to formulas (6) to understand the photon contribution. It is worthy to notice that real photon contribution originates from the Dirac function part in M_1(term $\frac{\Gamma e^{ik_0 l}}{2}$) and M_3(term $\frac{\Gamma e^{-ik_0 l}}{2}$). Their sum originates in the presence of the real part $\Gamma\cos(k_0 l)$ of the coupling term in (8). Virtual photons involve the remaining terms and contribute to the imaginary part of the coupling $i\Gamma\sin(k_0 l)$. Thus, the real part expresses population modifications for the atoms (e.g., transitions), whereas the imaginary part is associated with a frequency shift in the atomic resonances.

D. Importance of non-RWA contribution

The expression of the M_i (7) show clearly that non-RWA contributions can be as important as RWA ones, although the inequality $\Gamma \ll \omega_0$ is assumed to use the adiabatic approximation. To strengthen the importance of non-RWA terms and to find situations where it would be possible to restrict the calculations to RWA, we consider many instructive situations.

If RWA is performed, the coupling term between atoms would be $M^{(\mathrm{RWA})} = M_1 + M_3$ and is given by

$$M^{(\mathrm{RWA})} = \Gamma\cos(\omega_0 l/c) + i\frac{\Gamma}{\pi}\bigg[\sin(\omega_0 l/c)\bigg(\mathrm{Si}(\omega_0 l/c) + \frac{\pi}{2}\bigg) + \cos(\omega_0 l/c)\mathrm{Ci}(\omega_0 l/c) - \mathrm{Ci}(\varepsilon l/c \to 0)\bigg]. \quad (9)$$

Only the real part is correct with respect to the true value of M. The imaginary part diverges. The non-RWA contributions are thus necessary for convergence of the coupling parameter (*without the need for any frequency cutoff*). However, it is worthy to notice that this is not the only role these terms play. Indeed, the addition of non-RWA parts M_2, M_4 *also introduces an additional partial shift* $\frac{\Gamma}{\pi}[\sin(\omega_0 l/c)(-\mathrm{Si}(\omega_0 l/c) + \frac{\pi}{2}) - \cos(\omega_0 l/c)\mathrm{Ci}(\omega_0 l/c)]$ *giving the true shift* $\Gamma\sin(\omega_0 l/c)$ *in* M.

Another interesting situation is the case where $\omega_0 l/c \gg 1$. In this case, $\mathrm{Ci}(\omega_0 l/c) \simeq 0; \mathrm{Si}(\omega_0 l/c) \simeq \pi/2$. The RWA gives the right result only if an infrared frequency cutoff ε is introduced, such as $\varepsilon \gg c/l$. In this case $\mathrm{Ci}(\varepsilon l/c) \simeq 0$ and $M^{(\mathrm{RWA})} \simeq M = \Gamma e^{i(\omega_0 l/c)}$.

The correct model does not need any frequency cutoff as said before, and a physical interpretation involving virtual photons can be given when $\omega_0 l/c \gg 1$. Indeed, in this case we can separate the nonresonant contribution (e.g., the integral) in (6a) and (6c) into two parts, one corresponding to photons nearly resonant with frequencies ω located in a domain $\delta \geqslant c/l$ around ω_0 (with $\omega_0 \gg \delta$) and another part with the remaining photons. In this situation, considering the M_1 contribution (M_3, respectively), we have $\frac{i\Gamma\omega_0}{2\pi}\mathrm{P}(\int_{\omega_0-\delta/2}^{\omega_0+\delta/2}\frac{e^{i\omega l/c}}{\omega_0-\omega}\frac{d\omega}{\omega}) \simeq \frac{\Gamma e^{ik_0 l}}{2}$ [$\frac{i}{2\pi}\mathrm{P}(\int_{\omega_0-\delta/2}^{\omega_0+\delta/2}\frac{e^{-i\omega l/c}}{\omega_0-\omega}\frac{d\omega}{\omega}) \simeq -\frac{\Gamma e^{-ik_0 l}}{2}$, respectively]. The sum of these contributions gives rise to the imaginary part of the coupling $i\Gamma\sin(k_0 l)$. In other words, only nearly resonant photons contribute to the atomic coupling. The role of the remaining part (highly nonresonant photons in M_1 and M_3) is to annihilate the non-RWA photon contributions (M_2 and M_4). This result is in line with the (undesired) noncausal character of the interaction associated with these photons. Indeed, due to time-energy incertitude energy, non-RWA photons are present for a time $|\omega + \omega_0|^{-1} \leqslant \omega_0^{-1}$. So, they should not exceed a travel distance of about $\lambda_0 = c/\omega_0$ in accordance with causality (finite c). However, the divergence of the interaction parameter in the infrared domain [responsible for the $1/\omega$ term in the integrals (6) and (7)] leads to an efficient interaction between atoms, even if $l > \lambda_0$, thus violating causality. These contributions necessarily have to be compensated in the expression of any measurable physical quantity to fulfill the causality principle.

A frequent situation also considered is the case where the RWA is used and the frequency variation in the coupling g_ω is neglected [e.g., $g_\omega(\omega_k) = g_\omega(\omega_0)$]. In this case, using our notations, the same calculations lead to substitution of $1/\omega$ by $1/\omega_0$ in the integrals of Eqs. (4a), (4c), (6a), and (6c) giving a coupling $M^{(\mathrm{RWA})}_{g_\omega cte}$,

$$M^{(\mathrm{RWA})}_{g_\omega cte} = \Gamma\cos(\omega_0 l/c) + i\frac{\Gamma}{\pi}\bigg[\sin(\omega_0 l/c)\bigg(\mathrm{Si}(\omega_0 l/c) + \frac{\pi}{2}\bigg) + \cos(\omega_0 l/c)\mathrm{Ci}(\omega_0 l/c)\bigg]. \quad (10)$$

This result corresponds to $M^{(\mathrm{RWA})}$ with the removal of the diverging term. However, except for the situation where $\omega_0 l/c \gg 1$, this model is not suitable to recover the true coupling parameter $M = \Gamma e^{i(\omega_0 l/c)}$.

Finally, another procedure used in some papers [49] is to perform the RWA with the extension of integration in the coupling parameter to negative frequencies [e.g., in (6a) and (6c) the integration is performed from $-\infty$ to $+\infty$]. This gives a coupling parameter $M^{(\mathrm{RWA})}_{-\infty,\infty} = \Gamma e^{i(\omega_0 l/c)}$ that exactly matches with the true coupling M. This procedure was introduced in previous papers to preserve causality in photodetection processes [50] and found its justification here in our case.

III. FIELD BEHAVIOR

A. Photoelectric signal

The field behavior is modified by the interaction with the atoms. An important feature already mentioned in the one-atom case [2,15,20,21] is the reflection of the resonant frequency of the field. This property leads to a transmission of an electric field that distorts temporally, such as its algebraic area vanishes (pulse-area theorem [41]). This important feature is the key point for understanding the field dynamics. Here, we show that the presence of non-RWA terms is necessary to preserve this feature and is the consequence of the compensation between parts of virtual photon contributions.

We consider the mean-field intensity at a photodetector located at a distance z from atom 1. We assume that the photodetector is fast enough to resolve the temporal variation in the entering field. In the Glauber theory of photodetection (RWA performed in the detector), the photodetector signal—in the Coulomb gauge—is then given by $I(t,z) = s\langle\psi(t)|\hat{A}^{(-)}(z)\hat{A}^{+}(z)|\psi(t)\rangle$ (s a constant set equal to 1 for simplicity). In this notation, $\hat{A}^{(\pm)} = \int_{-\infty}^{+\infty} B(\omega_k)dk_z\frac{\varepsilon_k}{\omega_k}\hat{a}_{k_z}e^{\pm ik_z z}$ is the positive (respectively, negative) frequency part of the potential vector field operator, ε_k is the vacuum electric field, and $|\psi\rangle$ is the wave function. We introduce in these expressions $B(\omega_k)$ as the spectral acceptance of the detector defined as $B(\omega_k) = 1$ for $\omega_1 < \omega_k < \omega_2$ and $B(\omega_k) = 0$ elsewhere, $\omega_{2,1} = \omega_0 \pm \frac{\Delta_0}{2}$, $\Delta_0 = \omega_2 - \omega_1$ is the detector spectral bandwidth satisfying $\Delta_0 \gg \Delta,\Gamma$ to ensure the spectral collection of all emitted photons. We also assume that $|z - z_j| \gg c\Gamma^{-1}, c\Delta^{-1}$ to ensure that the field emission is complete before its entry in the photodetector. Using the above definitions and expression (1) of the wave function, we find the following expression for the mean-field intensity $I(t,z) = I_1 + I_2 + I_3$ with $I_1(t,z) = |A_{\mathrm{eff}}(t,z)|^2$, A_{eff} given by:

$$A_{\mathrm{eff}}(t,z) = \int_{-\infty}^{+\infty} \left(\frac{\varepsilon_k}{\omega_k}\right) B(\omega_k)\alpha_{k_z}(t) e^{-i\omega_k[t-\mathrm{sgn}(k_z)z/c]} dk_z, \tag{11}$$

and

$$I_2(t,z) = \left| \int_{-\infty}^{+\infty} \left(\frac{\varepsilon_k}{\omega_k}\right) B(\omega_k)\gamma_{k_z}(t) e^{-i\omega_k[t-\mathrm{sgn}(k_z)z/c]} dk_z \right|^2, \tag{12a}$$

$$I_3(t,z) = 2\sum_{j=1}^{2} \int_{-\infty}^{+\infty} dk_z \left| \int_{-\infty}^{+\infty} dk'_z \left(\frac{\varepsilon_{k'}}{\omega_{k'}}\right) B(\omega_{k'}) \eta_{j,k_z,k'_z}(t) \right. \\ \left. e^{-i\omega_{k'}[t-\mathrm{sgn}(k'_z)z/c]} \right|^2. \tag{12b}$$

I_1 represents the intensity due to the incident field and the field radiated through RWA processes. The intensity expression is similar to that obtained in the classical regime with an effective potential vector field A_{eff}. I_2 and I_3 are associated with fields radiated through non-RWA processes and are thus *exclusively due* to virtual photons. In Appendix B, we show that $I_2, I_3 \simeq 0$ as long as $|z| \gg c/\omega_1$ and $z < 0$ or $|z - l| \gg c/\omega_1$ and $z > 0$ (conditions that are automatically fulfilled in our situation with $|z - z_j| \gg c\Gamma^{-1}, c\Delta^{-1}$). The vanishing of I_2 and I_3 can be understood from the fact that these intensities are associated with (non-RWA) virtual photons that are located within a wavelength from the atoms. As c/ω_1 corresponds to the maximum wavelength accepted by the detector, none of these virtual photons influence the photodetection process when the detector is located at a longer distance from the atoms. If the detector is in the near-field regime ($|z - z_j| \leqslant \lambda_1 = c/\omega_1$), a non-RWA treatment of the whole interaction (atoms + photodetector) is needed [50,51]. The effective field A_{eff} in relation (11) can be related to the population amplitude β_j of the excited states. In Appendix C, we show that in the limit of Markovian approximation, the effective field can be decomposed in three propagating parts as follows:

$$\begin{aligned} A_{\mathrm{eff}}(t,z) = {} & h(-z)A_{\mathrm{inc}}(t - z/c)e^{-i\omega_0(t-z/c)} \\ & + h(-z)A_{\mathrm{refl}}(t + z/c)e^{-i\omega_0(t+z/c)} \\ & + h(z - l)A_{\mathrm{trans}}(t - z/c)e^{-i\omega_0(t-z/c)}, \end{aligned} \tag{13}$$

with

$$A_{\mathrm{inc}}(t-z/c) = \int_0^{\infty} B(\omega_k)\frac{\varepsilon_k}{\omega_k}[\alpha_{k_z}(-\infty)e^{-i(\omega_k-\omega_0)(t-z/c)}]dk_z, \tag{14a}$$

$$A_{\mathrm{trans}}(t-z/c) = A_{\mathrm{incid}}(t-z/c) \\ - i\frac{g_k\varepsilon_k}{c}\frac{2\pi}{\omega_0}\sum_{j=1}^{2} e^{-i(\omega_0/c)z_j}\beta_j(t - (z - z_j)/c), \tag{14b}$$

$$A_{\mathrm{refl}}(t + z/c) = -i\frac{g_k\varepsilon_k}{c}\frac{2\pi}{\omega_0}\sum_{j=1}^{2} e^{i(\omega_0/c)z_j}\beta_j(t + (z - z_j)/c). \tag{14c}$$

$A_{\mathrm{inc}}(t,z)$, $A_{\mathrm{refl}}(t,z)$, and $A_{\mathrm{trans}}(t,z)$, respectively, are the incident, reflected, and transmitted electric wave packets [$h(z)$ is the Heaviside function]. Finally, another interesting quantity used in our investigations is the spectral distribution of the field $\tilde{I}_\alpha(\omega) = |\tilde{A}_\alpha(\omega)|^2$ that gives the energy distribution of the corresponding photons [α stands for incident, transmitted, and reflected, and $\tilde{A}_\alpha(\omega) = \int_{-\infty}^{+\infty} A_\alpha(\tau)e^{i(\omega-\omega_0)\tau}d\tau^2$].

B. Transmitted and reflected wave packets: Pulse-area theorem

We establish in this section that the algebraic pulse areas of the transmitted and reflected pulses obey strict conditions. Indeed, the transmitted potential field is given by (14b) and for $z = z_j (j = 1,2)$, we have $A_{\mathrm{trans}}(t - z_j/c) = A_{\mathrm{incid}}(t - z_j/c) - i\frac{g_k\varepsilon_k}{c}\frac{2\pi}{\omega_0}\sum_{j'=1}^{2} e^{-i(\omega_0/c)z_{j'}}\beta_{j'}(t-(z_j-z_{j'})/c)$ and $A_{\mathrm{refl}}(t + z_j/c) = -i\frac{g_k\varepsilon_k}{c}\frac{2\pi}{\omega_0}\sum_{j=1}^{2} e^{i(\omega_0/c)z_{j'}}\beta_{j'}(t + (z_j - z_{j'})/c)$. Introducing the constant $G_{0j} = \frac{g_k\omega_k}{\varepsilon_k}e^{ik_0z_j}$ (independent of ω_k) and remembering that we are working within the Markovian approximation $\beta_j(t \pm l/c) \simeq \beta_j(t)$ [and $A_{\mathrm{incid}}(t - l/c) \simeq A_{\mathrm{incid}}(t)$], we found that the variation $\dot{\beta}_j$ of the excited-state population given by (3) is directly related to the contribution of propagating fields through the following relations:

$$i\dot{\beta}_1(t) = G_{01}[A_{\mathrm{inc}}(t) + A_{\mathrm{refl}}(t)], \tag{15a}$$

$$i\dot{\beta}_2(t) = G_{02}A_{\mathrm{trans}}(t). \tag{15b}$$

This an important property: Although virtual photons are taken into account, only the propagating causal fields (incident, reflected, and transmitted) evaluated at the atomic position modify the dynamics of the corresponding population. This is the consequence of the interference of the contribution of virtual photons due to two-photon terms (non-RWA terms) with the contribution that originates from virtual photons with energies located outside a narrow bandwidth around the resonant frequency (RWA contribution) as discussed in Sec. II C. We define the pulse area as $S_i = \int_{-\infty}^{+\infty} A_i(\tau = t - z/c)d\tau$ (i = inc,trans,refl). Integration of Eq. (12b) turns into $S_{\mathrm{trans}} = iG_{02}[\beta_2(t \to +\infty) - \beta_2(t \to -\infty)]$. The asymptotic behavior of β_j (j = 1,2) can be obtained by deduction from Eq. (3). Moreover, because of the presence of the relaxation term, we have $\beta_j(t \to +\infty) = 0$, meaning that the atoms come back to the initial ground states after the end of the photon-scattering process. Because initially $\beta_j(t \to -\infty) = 0$ the transmitted pulse area vanishes, e.g., $S_{\mathrm{trans}} = \int_{-\infty}^{+\infty} A_{\mathrm{trans}}(\tau = t - z/c)d\tau = 0$. Similarly, we have $\int_{-\infty}^{+\infty}(A_{\mathrm{inc}} + A_{\mathrm{refl}})(\tau = t - z/c)d\tau = 0$. We finally obtain the following important results:

$$S_{\mathrm{trans}} = \int_{-\infty}^{+\infty} A_{\mathrm{trans}}(\tau = t - z/c)d\tau = 0, \tag{16a}$$

$$\begin{aligned} S_{\mathrm{refl}} &= \int_{-\infty}^{+\infty} A_{\mathrm{refl}}(\tau = t - z/c)d\tau \\ &= -\int_{-\infty}^{+\infty} A_{\mathrm{inc}}(\tau = t - z/c)d\tau = -S_{\mathrm{inc}}. \end{aligned} \tag{16b}$$

Note that this result is valid whatever the coupling Γ, the pulse width Δ, and the distance l are between the atoms. The pulse area can be identified in the spectral domain with the spectrum

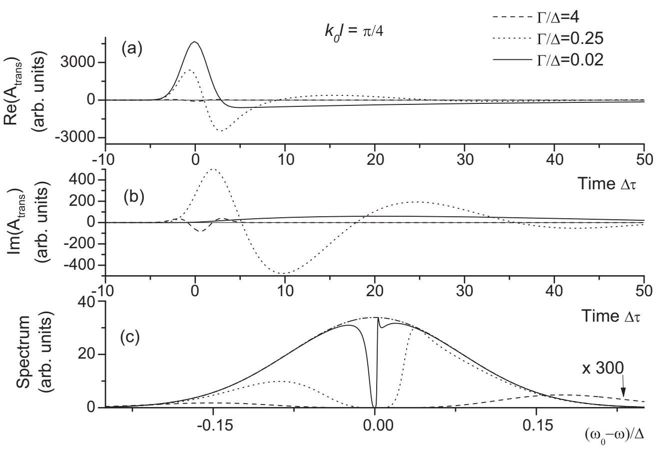

FIG. 3. Pulse-area theorem. Temporal behavior of the transmitted field with (a) real part and (b) imaginary part. We represent the curves for several ratios of Γ/Δ. The interatomic distance is $k_0 l = \frac{\pi}{4}$. The temporal area vanishes in all cases. In (c) the corresponding spectra are represented, and the incident spectrum is represented by the dashed-dotted line. The resonance frequency is not transmitted.

at resonance. Thus, Eqs. (16) mean that the central frequency is always *totally* reflected. Moreover, the atoms radiate in both backward and forward directions, but for the resonance frequency the interference between radiated fields is always destructive (constructive) in the forward (backward) direction.

An important remark has to be made at this level. The pulse-area theorem holds only when Eqs. (15) are valid. As noticed above, this is the consequence of the interference between the contributions of virtual photons due to non-RWA terms and those of nonresonant RWA terms in the expression of the fields. Thus only RWA 1D models that recover the correct shift can recover this feature (see Sec. II C).

These results are illustrated in Fig. 3 where the temporal and spectral profiles of the transmitted pulse are plotted for a fixed value of the distance l, such as both virtual and real photons are involved ($k_0 l = \pi/4$) and for three increasing values of the ratio Γ/Δ. In all cases, both the (a) real and the (b) imaginary parts of the field exhibit both positive and negative parts to ensure the vanishing of the pulse area. The distortion of the pulse also increases with the coupling parameter, and oscillations appear with a characteristic time that becomes shorter than the initial pulse duration for large values of the coupling. This distortion is also accompanied with a significant decrease in the total amplitude. This is in line with Eq. (15). When Γ increases, G_{02} decreases, and $A_{\rm trans}(t - l/c) = -iG_{02}^{-1}\dot{\beta}_2(t)$ vanishes as a consequence. In the same manner, we obtain from (15) that $A_{\rm refl}(t) \simeq -A_{\rm inc}(t)$ for large coupling parameter Γ/Δ. In Fig. 3(c), we represent the spectrum of the transmitted field for corresponding values of the coupling parameter. We see that the central frequency is never transmitted and is hence totally reflected. The spectrum also exhibits a profound dip whose width increases with the coupling parameter.

IV. CONCLUSION

A detailed study of the scattering of an incident photon wave packet by two atoms in a one-dimensional waveguide has been presented. We clarify the role and importance of non-RWA terms to correctly account for virtual photon contributions. Moreover, we have shown that a subtle interplay between parts of virtual photon contributions leads to strong constraints on the pulse area of temporal envelopes. This study show that virtual photons can lead to substantial—quantitative—modification of both atomic and radiated fields in the one-dimensional waveguide, in line with the 3D case. Extension of this paper to an array of N atoms and for the non-Markovian case is a natural perspective. Moreover, the interpretation in terms of temporal behavior for the fields developed here turns out to be a useful concept for understanding shaping effects. This constitutes a step in the manipulation of photon wave-packet characteristics thus adding a new control parameter—the shape—for the transport of flying qubits.

APPENDIX A: FUNDAMENTAL EQUATION FOR THE POPULATIONS

The set of Eq. (2) can be simplified so a simple equation can be obtained for the excited-state populations $\beta_j(t)$. First, the relevant parts of the amplitudes of highly excited states involved in (2a) and (2b) can be obtained by formally integrating Eqs. (2c) and (2d) using the approximation $\int_{-\infty}^{t} dt' f(t') e^{i(\omega_0+\omega_k)(t'-t)} \simeq f(t)\int_{-\infty}^{t} dt' e^{i(\omega_0+\omega_k)(t'-t)} \simeq \frac{f(t)}{i(\omega_0+\omega_k)}$ [$f(t) = \alpha_{k_z}(t)$ or $\beta_j(t)$]. We obtain

$$\gamma_{k_z}(t) \simeq -\frac{g_k}{\omega_0+\omega_k}\sum_{\substack{j=1,2\\ j\neq j'}} \beta_j(t) e^{-ik_z z_{j'}} e^{i(\omega_0+\omega_k)t}, \quad \text{(A1a)}$$

$$\eta_{j,k_z,k_z'}(t) \simeq -\frac{g_k}{\omega_0+\omega_{k'}}[\alpha_{k_z}(t) e^{i(\omega_0+\omega_{k'})t} e^{-ik_z' z_j}]. \quad \text{(A1b)}$$

Injecting Eq. (A1b) into (2a), we obtain the following equation for $\alpha_{k_z}(t)$:

$$i\dot{\alpha}_{k_z}(t) \simeq \left\{ g_k \sum_{j=1,2} [\beta_j(t) e^{-i(\omega_0-\omega_k)t} e^{-ik_z z_j}] \right\} - \left(\alpha_{k_z}(t)\int \frac{2g_{k'}^2}{\omega_0+\omega_{k'}} dk_z'\right). \quad \text{(A2)}$$

The inclusion of two-photon states $|\beta_j, 1_{k_z}, 1_{k_z'}\rangle$ in the dynamics of the system leads to a shift in the ground levels [factor 2 in (A2) appears because of the summation over j]. This is the usual Lamb shift due to the vacuum because of the emission-absorption cycles of virtual photons by atoms in the ground states. We next rewrite γ_{k_z} as $\gamma_{k_z}(t) \simeq -\frac{g_k}{\omega_0+\omega_k}\beta_j(t) e^{-ik_z z_{j'}} e^{i(\omega_0+\omega_k)t} - i g_k \int_{-\infty}^{t} \beta_{j'}(t') e^{-ik_z z_j} e^{i(\omega_0+\omega_k)t'}$ (e.g., we take the part of γ_{k_z} in β_j [from (A1a)] and maintain the integral form $\beta_{j'}$ [from (2c)]). Injecting this expression in (2b), we obtain

$$i\dot{\beta}_j(t) = \int dk_z \left\{ g_k[\alpha_{k_z}(t) e^{i(\omega_0-\omega_k)t} e^{ik_z z_j}] + \left(-i g_k^2 \int_{-\infty}^{t} \beta_{j'}(t') e^{ik_z(z_{j'}-z_j)} e^{(\omega_0+\omega_k)(t'-t)} dt'\right)\right\} - \left(\beta_j(t)\int \frac{g_k^2}{\omega_0+\omega_k} dk_z\right), \quad j' \neq j, \quad \text{(A3)}$$

The excited state also exhibits a shift that is half that of the ground state. Shifting the total energy of the system by the amount of the ground-state shift, formally integrating (A2), we obtain

$$\alpha_{k_z}(t) = \alpha_{k_z}(t \to -\infty) - i\left\{ g_k e^{-ik_z z_j} \sum_{j=1,2} \int_{-\infty}^{t} dt'[\beta_j(t')e^{-i(\omega_0-\omega_k)t'}] \right\}. \tag{A4}$$

Using this expression in the equation of evolution (A3) and the adiabatic elimination of the continuum [52] for the first term of the integral $\int g_k^2 dk_z [\int_{-\infty}^{t} \beta_j(t') e^{-i(\omega_0-\omega_k)(t'-t)} dt'] \simeq \beta_j(t)[\int_{-\infty}^{t} (\int g_k^2 dk_z e^{-i(\omega_0-\omega_k)(t'-t)}) dt']$, we obtain the following fundamental equation for $\beta_j(t)$,

$$\dot{\beta}_j(t) = S_{0,j}(t) - (\Gamma - i\delta_0)\beta_j(t) - \Gamma\omega_0\left(\int_{-\infty}^{t} \beta_{j'}(t')\bar{M}(t'-t)dt'\right), \tag{A5}$$

with $\Gamma = 2\pi g_0^2$ $(g_0 = \frac{g_k}{\sqrt{c}}\sqrt{\omega_k/\omega_0})$ and $S_{0,j}(t) = -i\sqrt{\frac{\Gamma}{2\pi}} \int_{-\infty}^{+\infty} \sqrt{c\frac{\omega_0}{\omega_k}} \alpha_{k_z}(t \to -\infty) e^{i(\omega_0-\omega_k)t} e^{ik_z z_j} dk_z$. $\bar{M} = \sum_{i=1}^{4} \bar{M}_i$ is the memory function, and the $\bar{M}_i$ are defined by the relations,

$$\bar{M}_1(t-t') = \frac{1}{2\pi}\int_0^\infty \frac{e^{i(\omega_0-\omega_k)(t-t')}e^{ik_z l}}{\omega_k} d\omega_k, \tag{A6a}$$

$$\bar{M}_2(t-t') = \frac{1}{2\pi}\int_0^\infty \frac{e^{-i(\omega_0+\omega_k)(t-t')}e^{ik_z l}}{\omega_k} d\omega_k, \tag{A6b}$$

$$\bar{M}_3(t-t') = M_1(t-t', l \leftrightarrow -l) = \frac{1}{2\pi}\int_0^\infty \frac{e^{i(\omega_0-\omega_k)(t-t')}e^{-ik_z l}}{\omega_k} d\omega_k, \tag{A6c}$$

$$\bar{M}_4(t-t') = M_2(t-t', l \leftrightarrow -l) = \frac{1}{2\pi}\int_0^\infty \frac{e^{-i(\omega_0+\omega_k)(t-t')}e^{-ik_z l}}{\omega_k} d\omega_k. \tag{A6d}$$

$\delta_0 = \int \frac{g_k^2}{\omega_0+\omega_k} dk_z + \mathrm{P}(\int \frac{g_k^2}{\omega_k-\omega_0} dk_z)$ is the resultant shift in the excited state and can be incorporated in the definition of the transition frequency ω_0. An important case is the Markovian situation where the atoms are close enough so that the interaction (exchange of photons) can be considered as instantaneous compared to the atomic dynamics [53]. This is the case when the photon time of flight l/c and the resonant period $(\frac{2\pi}{\omega_0})$ are smaller than the time characteristics of population amplitudes β_j that are Γ^{-1} and Δ^{-1}. This is obtained for $l,\lambda_0 \ll c\Gamma^{-1}, c\Delta^{-1}$ (but $l < \lambda_0$ or $l > \lambda_0$ is allowed). In this case, we can set $\beta_j(t') \simeq \beta_j(t)$ in the integral appearing in (A5). We then obtain the equation,

$$\dot{\beta}_j(t) = S_{0,i}(t) - \Gamma\beta_j - M\beta_{j'\neq j}(t), \tag{A7}$$

with $M = \int_{-\infty}^{t} \bar{M}(t-t')dt'$; $M_i = \int_{-\infty}^{t} \bar{M}_i(t-t')dt'$.

APPENDIX B: CONTRIBUTION OF NON-RWA PHOTONS TO THE PHOTOELECTRIC SIGNAL

We consider $I_2(t,z) = |\int_{-\infty}^{+\infty} (\frac{\varepsilon_k}{\omega_k}) B(\omega_k)\gamma_{k_z}(t) e^{-i\omega_k[t-\mathrm{sgn}(k_z)z/c]} dk_z|^2$. We formally integrate Eq. (2c) and insert it in the expression of $I_2(t,z)$. Using the adiabatic elimination of the continuum technique we obtain

$$I_2(t,z) = \left| 2\sum_{j=1}^{2} \beta_j(t) \int_0^\infty d(\omega_k/c) B(\omega_k) \times \left(\frac{\varepsilon_k g_k}{\omega_k(\omega_0+\omega_k)}\right) \cos(\omega_k|z-z_j|/c) \right|^2. \tag{B1}$$

The integration over ω_k can be performed analytically since $\varepsilon_k g_k$ is constant. We have

$$\int_0^\infty d(\omega_k/c) B(\omega_k) \left(\frac{\cos(\omega_k|z-z_j|/c)}{\omega_k(\omega_0+\omega_k)}\right) = f(\omega_2,\omega_0,a) - f(\omega_1,\omega_0,a), \tag{B2}$$

where

$$f(\omega,\omega_0,a) = \frac{1}{c\omega_0}\{-\cos(\omega_0 a)\mathrm{Ci}[(\omega+\omega_0)a] + \mathrm{Ci}(\omega a) - \sin(\omega_0 a)\mathrm{Si}[(\omega+\omega_0)a]\}, \tag{B3}$$

with $a = |z - z_j|/c$. Ci and Si are the cosine and sine integral functions, respectively [46]. For real arguments, these functions are even and odd, respectively, and the asymptotic values are $\mathrm{Ci}(|x| \gg 1) = 0, \mathrm{Si}(|x| \gg 1) = \pi/2$. From these properties it follows that $f(\omega_2,\omega_0,a) - f(\omega_1,\omega_0,a)$ vanishes as long as $\omega_1|z - z_{j'}|/c \gg 1$ (and thus $\omega_2|z - z_{j'}|/c \gg 1$). In this case, the intensity $I_2(t,z)$ vanishes as a result.

We consider now $I_3(t,z) = \sum_{j=1}^{2} \int_{-\infty}^{+\infty} dk_z |\int_{-\infty}^{+\infty} dk'_z (\frac{\varepsilon_{k'}}{\omega_{k'}}) B(\omega_{k'}) \eta_{j,k_z,k'_z}(t) e^{-i\omega_{k'}[t-\mathrm{sgn}(k'_z)z/c]}|^2$. We calculate this expression by using Eq. (A1b) and inject it in the above expression of I_3. We obtain $I_3(t,z) = \sum_{j=1}^{2} \int_{-\infty}^{+\infty} dk_z |D_{k_z,j}|^2$ with

$$D_{k_z} = -\sqrt{2}\alpha_{k_z}(t) \int_{\omega_1}^{\omega_2} d\omega_{k'} \left(\frac{g_{k'}\varepsilon_{k'}}{c\omega_{k'}(\omega_0+\omega_{k'})}\right) \times B(\omega_{k'}) \cos[\omega_{k'}(z-z_j)] - i\frac{\sqrt{2} g_k e^{i(\omega_0+\omega_k)t}}{2(\omega_0+\omega_k)} \times e^{-ik_z z_j} \int_{-\infty}^{+\infty} dk'_z \left(\frac{\varepsilon_{k'}}{\omega_{k'}}\right) B(\omega_{k'}) \alpha_{k_{z'}}(t) e^{i(k_{z'}z-\omega_{k'}t)}. \tag{B4}$$

If $\omega_1|z - z_j|/c \gg 1$ the cosine term in the first integral in (B4) strongly oscillates, and the corresponding integral vanishes. This can also be explicitly demonstrated using relations (B2) and (B3). In the second term in (B5), we recognize the effective field $A_{\mathrm{eff}}(t,z)$ given in (11). We then obtain the following expression for I_3,

$$I_3 \simeq I_1 \int_{-\infty}^{+\infty} \frac{g_k^2}{2(\omega_0+\omega_k)^2} dk_z. \tag{B5}$$

Using the expression of $g_k = \sqrt{\frac{\Gamma c \omega_0}{2\pi\omega_k}}$ and the relation $\int_\varepsilon^\infty dx/[x(1+x)^2] \simeq \ln(\varepsilon)$, we have $I_3 \simeq \sqrt{\frac{1}{2\pi}\frac{\Gamma}{\omega_0}} \ln(\omega_0/\omega_c) I_1$, where ω_c is a low-frequency cutoff because $\Gamma/\omega_0 \ll 1$ and the slow variation in the logarithmic term $I_3 \ll I_1$. This achieves the demonstration that $I_2, I_3 \simeq 0$ for $\omega_1|z - z_j|/c \gg 1$.

APPENDIX C: RELATION BETWEEN PROPAGATING FIELDS AND POPULATION AMPLITUDES

We consider the effective field $A_{\text{eff}}(t,z) = \int_{-\infty}^{+\infty} (\varepsilon_k/\omega_k) B(\omega_k) \alpha_{k_z}(t) e^{-i\omega_k[t - \text{sgn}(k_z)z/c]} dk_z$. Let us consider first the situation $z < 0$. The integration over k_z can be separated into two integrals with $[0,\infty]$ and $[-\infty,0]$ intervals, respectively. We can rewrite $A_{\text{eff}}(t,z)$ as

$$A_{\text{eff}}(t,z) = A_-(t,z) e^{-i\omega_0(t - z/c)} + A_+(t,z) e^{-i\omega_0(t+z/c)}, \quad \text{(C1)}$$

with

$$A_-(t,z) = \int_0^\infty B(\omega_k) \frac{\varepsilon_k}{\omega_k} \left[\alpha_{k_z}(t) e^{-i(\omega_k - \omega_0)(t - z/c)}\right] dk_z, \quad \text{(C2a)}$$

$$A_+(t,z) = \int_0^\infty B(\omega_k) \frac{\varepsilon_k}{\omega_k} \alpha_{-k_z}(t) e^{-i(\omega_k - \omega_0)(t + z/c)} dk_z. \quad \text{(C2b)}$$

We show now that the first reduces to the incident one and the second term corresponds to the reflected wave packet in situations where $\omega_1|z|/c \gg 1$ and $\Delta_0|z|/c \gg 1$, respectively. Indeed, using Eq. (A4), we have

$$A_-(t,z) = \int_0^\infty B(\omega_k) \frac{\varepsilon_k}{\omega_k} \left[\alpha_{k_z}(t \to -\infty) e^{-i(\omega_k - \omega_0)(t - z/c)}\right] dk_z + \int_0^\infty -i B(\omega_k) \frac{\varepsilon_k g_k}{\omega_k} e^{-ik_z z_j} \left(\left\{ \sum_{j=1,2} \int_{-\infty}^t dt' [\beta_j(t') e^{-i(\omega_0 - \omega_k)t'}] \right\} e^{i(\omega_0 - \omega_k)(t - z/c)} \right) dk_z. \quad \text{(C3)}$$

The next step is to show that the second integral in (C3) vanishes for $\omega_1|z|/c \gg 1$. Integration over ω_k is performed first, and we deal with the following integral: $\int_{\omega_1}^{\omega_2} \frac{e^{-i\omega_k T}}{\omega_k} dk_z = g(\omega_2) - g(\omega_1)$ with $T = t - t' - (z - z_j)/c$ and $g(\omega) = \text{Ci}(\omega T) - i\, \text{Si}(\omega T)$. Ci and Si are the cosine and sine integrals functions, respectively [46]. The minimum value for T is obtained for $t' = t$ and is $-(z - z_j)/c$ (> 0). Using the asymptotic values of the Ci and Si functions, we find that the integral vanishes as long as $\omega_1|z|/c \gg 1$. Thus, the amplitude $A_-(t,z)$ reduces to the incident wave packet,

$$A_-(t,z) = A_{\text{inc}}(t - z/c). \quad \text{(C4)}$$

The last step is to show that the negative wavelength contributes [in (C2b)] to a reflected wave packet, e.g., a wave packet propagating with a $t + z/c$ dependence. We use expression (A4) for $\alpha_{-k_z}(t)$ and perform the adiabatic elimination of the continuum technique. By reminding about the initial condition $\alpha_{-k_z}(t \to -\infty) = 0$ (no incident wave packet coming from $z > 0$) and using relation (5a), we obtain

$$A_+(t,z) = -i \frac{g_k \varepsilon_k}{c} \sum_{j=1}^2 \beta_j(t + (z - z_j)/c) \times e^{i(\omega_0/c)z_j} \left(\frac{\pi}{\omega_0} - i \text{P} \int B(\omega_k) \times \frac{\exp\left[-i \frac{(\omega - \omega_0)}{c}(z - z_j)\right]}{\omega_k(\omega_k - \omega_0)} d\omega_k \right). \quad \text{(C5)}$$

The radiated field in this expression can be further simplified using the relation $\text{P}\int B(\omega_k) d\omega_k \frac{e^{-i(\omega/c)(z - z_j)}}{\omega_k(\omega_k - \omega_0)} = f_+[\omega_2, -\omega_0, -(z - z_j)/c] - f_+[\omega_1, -\omega_0, -(z - z_j)/c]$ with the function f_+ given by

$$f_+(\omega,\omega_0,a) = \frac{1}{\omega_0}\{-\cos(\omega_0 a)\text{Ci}[(\omega + \omega_0)a] + \text{Ci}(\omega a) - \sin(\omega_0 a)\text{Si}[(\omega + \omega_0)a]\} + \frac{i}{\omega_0}\{\sin(\omega_0 a)\text{Ci}[(\omega + \omega_0)a] + \text{Si}(\omega a) - \cos(\omega_0 a)\text{Si}[(\omega + \omega_0)a]\}. \quad \text{(C6)}$$

Moreover, for $|z - z_{j'}| \gg c/\omega_1$ (and thus $|z - z_j| \gg c/\omega_2$), we have

$$\text{P}\int B(\omega_k) \frac{e^{-i(\omega/c)(z - z_j)}}{\omega_k(\omega_k - \omega_0)} d\omega_k \simeq i\frac{2}{\omega_0} e^{-i(\omega_0/c)(z - z_j)} \text{Si}(\Delta_0(z - z_j)/2c), \quad \text{(C7)}$$

with $\Delta_0 = \omega_2 - \omega_1$. Since we have $\Delta_0 \gg \Gamma, \Delta \gg c/|z - z_j|$ (and so $\Delta_0|z - l|/c \gg 1$), we obtain $\text{Si}(\Delta_0(z - z_j)/2c) \simeq -\frac{\pi}{2}$. The following relation finally results from (C5):

$$A_+(t,z) = A_{\text{refl}}(t + z/c), \quad \text{(C8)}$$

with

$$A_{\text{refl}}(t + z/c) = -i \frac{g_k \varepsilon_k}{c} \frac{2\pi}{\omega_0} \sum_{j=1}^2 e^{i(\omega_0/c)z_j} \beta_j(t + (z - z_j)/c). \quad \text{(C9)}$$

The field then exhibits a spatial-temporal dependence in $t + z/x$ and can be identified with the reflected field (that necessarily propagates in this way) and is proportional to the population amplitudes of the excited states.

For $z > 0$, the same demonstration can be established for the radiated field but with the difference that no incident field comes from $z > l$.

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