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Radiative transfer equation for media with spatially varying refractive index

Pei Wang*
LMAM, School of Mathematical Sciences, Peking University, Beijing 100871, China

Simon R. Arridge†
Department of Computer Science, University College London, Gower Street, London WC1E 6BT, United Kingdom

Ming Jiang‡
LMAM, School of Mathematical Sciences, and Beijing International Center for Mathematical Research, Peking University, Beijing 100871, China

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The radiative transfer equation is fundamental to multiple physical applications based on light propagation. There have been three different formulations of the radiative transfer equation for media with spatially varying refractive index. Because the radiative transfer equation demonstrates macroscopic phenomena of radiation, the intensity law from geometric optics has been used to validate these formulations of radiative transfer equations. We review the different formulations and compute the steady-state intensity for each of the radiative transfer equations in a nonabsorption and nonscattering medium with a spatially variant refraction index without light sources. By checking each one of the intensities with the intensity law from geometric optics, we find that there is only one formulation that is consistent with the intensity law of geometric optics.

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The radiative transfer equation has found its applications in astrophysics, atmospheric science, biomedical optics, etc. [1–3]. It describes how light propagates through random media via photon packets [4,5]. It is based on the conservation of radiant energy underlying the processes such as absorption, emission, scattering, and refraction, while propagating through random media. Although these processes are due to interactions at the quantum level, they are observable and hence demonstrate macroscopic phenomena of radiation. Therefore, the radiative transfer equation is at the intermediate mesoscale for the radiation phenomena in random media.

Although the interactions due to absorption, emission, and scattering are well understood at this mesoscopic level, the interaction of refraction is not. There are currently three different versions of radiative transfer equations involving refraction for nonhomogeneous media, though these equations reduce to the same equation for homogeneous media with constant refraction indices [6–12]. Because the radiative transfer equation demonstrates macroscopic phenomena of radiation, the intensity law from geometric optics has been used to validate these different formulations [13,14]. We will remark on the approaches in Refs. [13,14].

In this paper we aim to validate the three formulations of radiative transfer equations by checking each one of them with the intensity law of geometric optics. This is done by finding the steady-state specific intensity for each of the radiative transfer equations in a nonabsorption and nonscattering medium with a spatially variant refraction index without light sources. Then the intensity is computed from the specific intensity and checked if it is consistent with the intensity law of geometric optics, for each formulation. We consider only the typical case for monochromatic light propagation in this paper. It is found that the radiative transfer equations from Refs. [10,12] lead to an intensity that follows the intensity law of geometric optics among those in Refs. [6–12]. The paper ends with relevant discussions.

We begin with a review of the current formulations of radiative transfer equations in random media with spatially variant refraction indices. The fundamental quantity of radiative transfer is the specific intensity $L(\mathbf{r},\mathbf{\Omega},t)$ in the phase space $\mathbf{R}^3 \times \mathbf{S}^2$, i.e., the radiant power flux density at position $\mathbf{r} \in \mathbf{R}^3$, direction $\mathbf{\Omega} \in \mathbf{S}^2$ (where $\mathbf{S}^2$ is the unit sphere), and time t. Let the absorption coefficient and the scattering coefficient be $\mu_a(\mathbf{r})$ and $\mu_s(\mathbf{r})$, respectively, in the volume. Then the extinction coefficient is given by as $\mu_t = \mu_a + \mu_s$. Let the light source be $\varepsilon(\mathbf{r},\mathbf{\Omega},t)$, and the refractive index be $n(\mathbf{r})$ inside the volume. The radiative transfer equations in Refs. [6–12] can be written in the form

$$\frac{n}{c}\frac{\partial L}{\partial t} + \mathbf{\Omega}\cdot\nabla L + \Sigma = \varepsilon - \mu_t L + \mu_s \int_{\mathbf{S}^2} p(\mathbf{\Omega},\mathbf{\Omega}')L(\mathbf{r},\mathbf{\Omega}',t)\,d\mathbf{\Omega}', \quad (1)$$

where c is the light speed in vacuum, ∇ the gradient with respect to position $\mathbf{r}$, and $p(\mathbf{\Omega},\mathbf{\Omega}')$ the phase function, describing the probability of scattering from direction $\mathbf{\Omega}'$ to $\mathbf{\Omega}$. The right-hand side of (1) is the accumulative change of the specific intensity because of light sources, absorption, and scattering. The term Σ on the left-hand side is the streaming term, different among the three formulations from Refs. [6–12], which describes the changes of the specific intensity because of refraction. In the following, we label the three different formulations of the radiative transfer equations as RTE_1, RTE_2, and RTE_3. Their corresponding streaming terms Σ_1, Σ_2, and Σ_3 are given, respectively, as follows:

*pei.wng@gmail.com
†s.arridge@cs.ucl.ac.uk
‡Corresponding author: ming-jiang@pku.edu.cn

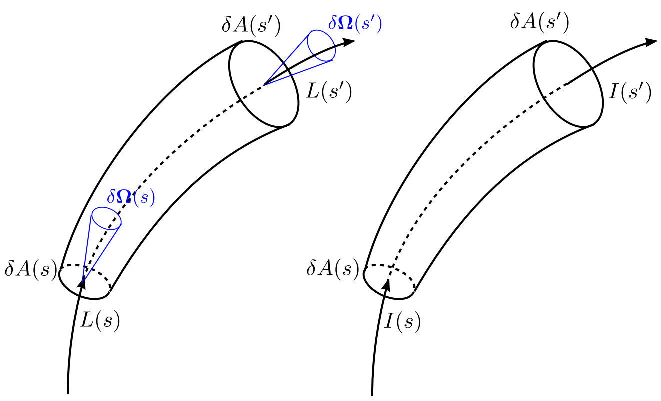

FIG. 1. (Color online) The specific intensity (left) and intensity (right) at two points along the ray path with arc lengths s and s', respectively. At s and s', the transverse sections are $\delta A(s)$ and $\delta A(s')$, respectively; the solid angle elements (in blue) are $\delta\mathbf{\Omega}(s)$ and $\delta\mathbf{\Omega}(s')$, respectively.

RTE$_1$ ([6,8,11]):

$$\Sigma_1 = \frac{1}{n}\left[\nabla n - (\nabla n \cdot \mathbf{\Omega})\mathbf{\Omega}\right] \cdot \nabla_\Omega L - \frac{2}{n}(\nabla n \cdot \mathbf{\Omega})L, \tag{2}$$

RTE$_2$ ([7,9]):

$$\Sigma_2 = \frac{1}{n}\left[\nabla n - (\nabla n \cdot \mathbf{\Omega})\mathbf{\Omega}\right] \cdot \nabla_\Omega L + \Theta L, \tag{3}$$

RTE$_3$ ([10,12]):

$$\Sigma_3 = \frac{1}{n}\left[\nabla n - (\nabla n \cdot \mathbf{\Omega})\mathbf{\Omega}\right] \cdot \nabla_\Omega L - \frac{2}{n}(\nabla n \cdot \mathbf{\Omega})L + \Theta L. \tag{4}$$

In the above equations, ∇_Ω is the gradient with respect to direction $\mathbf{\Omega}$, and Θ is the so-called ray divergence to be explained below. As can be seen, three terms are used to describe the change of the specific intensity because of refraction. The first term, $\frac{1}{n}[\nabla n - (\nabla n \cdot \mathbf{\Omega})\mathbf{\Omega}] \cdot \nabla_\Omega L$, is due to the differential change of the specific intensity along a ray path, which is the same in the three formulations. The second term, $-\frac{2}{n}(\mathbf{\Omega} \cdot \nabla n)L$, is due to the change of the directional differential along the ray path, which is in RTE$_1$ and RTE$_3$ but not in RTE$_2$. The third term ΘL is the ray divergence term due to the refractive variation of the transverse section of a ray tube along the ray path; see Fig. 1. It is given by zero in Refs. [6,8,11]. In Ref. [10] the ray divergence Θ is given by

$$\Theta = \frac{1}{R_1(s)} + \frac{1}{R_2(s)}, \tag{5}$$

where $R_1(s)$ and $R_2(s)$ are the principle radii of curvatures of the wave front, with s being the arc length of the ray path. In Refs. [9,12] the ray divergence Θ is given by

$$\Theta = \frac{\nabla^2 \mathcal{S}}{n} - \frac{\mathbf{\Omega} \cdot \nabla n}{n}, \tag{6}$$

where $\mathcal{S}$ is the eikonal for the eikonal equation [15], [p. 119, Eq. (15a)]:

$$|\nabla \mathcal{S}|^2 = n^2. \tag{7}$$

It can be shown that Eq. (5) and Eq. (6) give the same result for the ray divergence.

Next we are to find the steady-state specific intensity solution for the radiative transfer equations RTE$_1$, RTE$_2$, and RTE$_3$ in a nonabsorption and nonscattering medium with a spatially variant refraction index without light sources. The three radiative transfer equations reduce to

$$\mathbf{\Omega} \cdot \nabla L + \Sigma_i = 0, \tag{8}$$

for $i = 1,2,3$. Light propagates as a collection of photon packets streaming along ray paths. Now consider a ray path parameterized by its arc length s. Along this ray path, the position $\mathbf{r}$ and direction $\mathbf{\Omega}$ of light are functions of the arc length s, by the following equations:

$$\frac{d\mathbf{r}}{ds} = \mathbf{\Omega}, \tag{9}$$

$$\frac{d\mathbf{\Omega}}{ds} = \frac{1}{n}\left[\nabla n - \mathbf{\Omega}(\mathbf{\Omega} \cdot \nabla n)\right]. \tag{10}$$

See, for example, [Ref. 4], [Eqs. (5.30) and (5.32), p. 148]. Thus, on the ray path, L can be written as a function of the arc length by $L(s) = L(\mathbf{r}(s), \mathbf{\Omega}(s))$. In the following, we solve the specific intensity on the ray path from Eq. (8) for RTE$_1$, RTE$_2$, and RTE$_3$, respectively, given the specific intensity $L(s_0)$ at a point s_0 on the ray path. We need the following two equations to reduce the radiative transfer equations. By direct computation, we have

$$n^2 \mathbf{\Omega} \cdot \nabla \left(\frac{L}{n^2}\right) = \mathbf{\Omega} \cdot \nabla L - \frac{2}{n}(\mathbf{\Omega} \cdot \nabla n)L, \tag{11}$$

and because the refractive index n is independent of the direction variable $\mathbf{\Omega}$, we find

$$n^2 \nabla_\Omega \left(\frac{L}{n^2}\right) = \nabla_\Omega L. \tag{12}$$

RTE$_1$: Substituting Σ_1 in Eq. (2) into Eq. (8), we obtain RTE$_1$ in the following form:

$$\mathbf{\Omega} \cdot \nabla L - \frac{2}{n}(\mathbf{\Omega} \cdot \nabla n)L + \frac{1}{n}\left[\nabla n - \mathbf{\Omega}(\mathbf{\Omega} \cdot \nabla n)\right] \cdot \nabla_\Omega L = 0. \tag{13}$$

By Eqs. (11) and (12), Eq. (13) becomes

$$n^2 \mathbf{\Omega} \cdot \nabla \left(\frac{L}{n^2}\right) + \frac{1}{n}\left[\nabla n - \mathbf{\Omega}(\mathbf{\Omega} \cdot \nabla n)\right] \cdot n^2 \nabla_\Omega \left(\frac{L}{n^2}\right) = 0. \tag{14}$$

Dividing both sides of Eq. (14) by n^2, by Eqs. (9) and (10), yields

$$\frac{d\mathbf{r}}{ds} \cdot \nabla \left(\frac{L}{n^2}\right) + \frac{d\mathbf{\Omega}}{ds} \cdot \nabla_\Omega \left(\frac{L}{n^2}\right) = 0. \tag{15}$$

By the chain rule of differentiation, Eq. (15) reduces to

$$\frac{d}{ds}\left(\frac{L}{n^2}\right) = 0. \tag{16}$$

The specific intensity L for RTE$_1$ along the ray path is then given by

$$L(s) = \frac{n^2(s)}{n^2(s_0)} L(s_0). \tag{17}$$

RTE_2: Substituting Σ_2 in Eq. (3) into Eq. (8), we obtain RTE_2 in the following form:

$$\mathbf{\Omega}\cdot\nabla L+\frac{1}{n}[\nabla n-\mathbf{\Omega}(\mathbf{\Omega}\cdot\nabla n)]\cdot\nabla_{\Omega}L+\Theta L=0. \tag{18}$$

By using Eq. (9) and Eq. (10), Eq. (18) becomes

$$\frac{dL}{ds}+\Theta L=0. \tag{19}$$

The specific intensity L for RTE_2 along the ray path is then given by

$$L(s)=L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\,ds\right). \tag{20}$$

RTE_3: Substituting Σ_3 in Eq. (4) into Eq. (8), we obtain RTE_3 in the following form:

$$\mathbf{\Omega}\cdot\nabla L+\frac{1}{n}[\nabla n-\mathbf{\Omega}(\mathbf{\Omega}\cdot\nabla n)]\cdot\nabla_{\Omega}L-\frac{2}{n}(\mathbf{\Omega}\cdot\nabla n)L+\Theta L=0. \tag{21}$$

It has one more term, the ray divergence term ΘL, than Eq. (13). Similarly, we can reduce Eq. (21) to the following form:

$$\frac{d}{ds}\left(\frac{L}{n^2}\right)+\Theta\frac{L}{n^2}=0. \tag{22}$$

The specific intensity L for RTE_3 along the ray path is then given by

$$L(s)=\frac{n^2(s)}{n^2(s_0)}L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\,ds\right). \tag{23}$$

Note that Eq. (23) has been obtained in Ref. [10] by using Eq. (5).

We proceed to compute the intensity from Eq. (17), Eq. (20), and Eq. (23). The intensity $I(\mathbf{r})$ is the radiant power flux density at a position $\mathbf{r}$, i.e., the (time average) amount of radiant energy per unit time through a unit area perpendicular to the propagation direction. Along the ray path, the intensity is a function of the arc length, $I(\mathbf{r}(s))=I(s)$. Let $\delta A(s)$ be the transverse section of the ray tube at s. It is in the order of the wavelength. Let $\delta\mathbf{\Omega}(s)$ be the solid angle element at $\mathbf{r}(s)$ surrounding $\mathbf{\Omega}(s)$. The intensity $I(s)$ is equal to the illumination $\delta E(s)$ onto the surface element $\delta A(s)$ [15], [p. 195, footnote]. Because the illumination $\delta E(s)$ and the specific intensity $L(s)=L(\mathbf{r}(s),\mathbf{\Omega}(s))$ have the following relationship [15], [p. 195, Eq. (5)]:

$$\delta E(s)=L(s)\delta\mathbf{\Omega}(s), \tag{24}$$

the intensity $I(s)$ and the specific intensity $L(s)$, as shown in Fig. 1, satisfy

$$I(s)=L(s)\delta\mathbf{\Omega}(s). \tag{25}$$

To find the steady-state intensity for each of the radiative transfer equations, we need the following theorem about the variation of the solid angle element $\delta\mathbf{\Omega}(s)$ along the ray path.

Theorem 1. Along a ray path, at the initial point s_0, given an element $\delta\mathbf{\Omega}(s_0)$ of the solid angle surrounding $\mathbf{\Omega}(s_0)$, the solid angle element at s surrounding $\mathbf{\Omega}(s)$ is

$$\delta\mathbf{\Omega}(s)=\frac{n^2(s_0)}{n^2(s)}\delta\mathbf{\Omega}(s_0), \tag{26}$$

which holds to the first order of the differential element of the solid angle.

Proof. Let $d\mathbf{\Omega}(s_0)$ and $d\mathbf{\Omega}(s)$ be the differential elements of the solid angle surrounding, respectively, $\mathbf{\Omega}(s_0)$ and $\mathbf{\Omega}(s)$. The variation of $d\mathbf{\Omega}(s)$ along the ray path has been studied in Refs. [6,8,10,12] and is given by

$$\frac{d}{ds}(d\mathbf{\Omega})=-\frac{2}{n}(\mathbf{\Omega}\cdot\nabla n)\,d\mathbf{\Omega}. \tag{27}$$

By Eq. (9), we obtain

$$\frac{d}{ds}(d\mathbf{\Omega})+\frac{2}{n}\left(\frac{d\mathbf{r}}{ds}\cdot\nabla n\right)d\mathbf{\Omega}=0. \tag{28}$$

By the chain rule of differentiation, it follows that

$$\frac{d\mathbf{r}}{ds}\cdot\nabla n=\frac{dn}{ds}. \tag{29}$$

Therefore, Eq. (28) reduces to

$$\frac{d}{ds}(d\mathbf{\Omega})+\frac{2}{n}\frac{dn}{ds}\,d\mathbf{\Omega}=0. \tag{30}$$

Multiplying n^2 on both sides and applying the chain rule of differentiation again yields

$$\frac{d}{ds}(n^2 d\mathbf{\Omega})=0. \tag{31}$$

Hence $n^2(s)d\mathbf{\Omega}(s)=n^2(s_0)d\mathbf{\Omega}(s_0)$, and it follows that

$$d\mathbf{\Omega}(s)=\frac{n^2(s_0)}{n^2(s)}\,d\mathbf{\Omega}(s_0). \tag{32}$$

Now we approximate the solid angle element $\delta\mathbf{\Omega}$ by the differential element $d\mathbf{\Omega}$ of the solid angle, at, respectively, s and s_0. This is a first order approximation. With Eq. (32), then we immediately obtain

$$\delta\mathbf{\Omega}(s)=\frac{n^2(s_0)}{n^2(s)}\delta\mathbf{\Omega}(s_0). \tag{33}$$

■

Then we multiply both sides of Eq. (17), Eq. (20), and Eq. (23), respectively, with $\delta\mathbf{\Omega}(s)$, and apply Theorem 1. We obtain the following intensity solutions from the three steady-state radiative transfer equations RTE_1, RTE_2, and RTE_3, respectively, as follows:

RTE_1:

$$L(s)\delta\mathbf{\Omega}(s) \tag{34}$$

$$=\frac{n^2(s)}{n^2(s_0)}L(s_0)\delta\mathbf{\Omega}(s) \tag{35}$$

$$=\frac{n^2(s)}{n^2(s_0)}L(s_0)\frac{n^2(s_0)}{n^2(s)}\delta\mathbf{\Omega}(s_0) \tag{36}$$

$$=L(s_0)\delta\mathbf{\Omega}(s_0). \tag{37}$$

By Eq. (25), the intensity at $\mathbf{r}(s)$ is

$$I(s)=I(s_0). \tag{38}$$

RTE_2:

$$L(s)\delta\mathbf{\Omega}(s) \tag{39}$$
$$= L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right)\delta\mathbf{\Omega}(s) \tag{40}$$
$$= L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right)\frac{n^2(s_0)}{n^2(s)}\delta\mathbf{\Omega}(s_0) \tag{41}$$
$$= \frac{n^2(s_0)}{n^2(s)}L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right)\delta\mathbf{\Omega}(s_0). \tag{42}$$

By Eq. (25), the intensity at $\mathbf{r}(s)$ is

$$I(s) = \frac{n^2(s_0)}{n^2(s)}I(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right). \tag{43}$$

RTE_3:

$$L(s)\delta\mathbf{\Omega}(s) \tag{44}$$
$$= \frac{n^2(s)}{n^2(s_0)}L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right)\delta\mathbf{\Omega}(s) \tag{45}$$
$$= \frac{n^2(s)}{n^2(s_0)}L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right)\frac{n^2(s_0)}{n^2(s)}\delta\mathbf{\Omega}(s_0) \tag{46}$$
$$= L(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right)\delta\mathbf{\Omega}(s_0). \tag{47}$$

By Eq. (25), the intensity at $\mathbf{r}(s)$ is

$$I(s) = I(s_0)\exp\left(-\int_{s_0}^{s}\Theta\, ds\right). \tag{48}$$

Finally, we check if the intensity solutions in Eq. (38), Eq. (43), and Eq. (48), respectively, from the three steady-state radiative transfer equations RTE_1, RTE_2, and RTE_3, are consistent with the intensity law of geometric optics. By the intensity law of geometric optics [15, p. 125, Eq. (40)], the intensity ratio

$$\frac{I(s)}{I(s_0)} = \frac{n(s)}{n(s_0)}\exp\left(-\int_{s_0}^{s}\frac{\nabla^2\mathcal{S}}{n}\, ds\right), \tag{49}$$

where $\mathcal{S}$ is the eikonal. For RTE_2 and RTE_3, we compute the exponential integral of the ray divergence Θ by Eq. (6) as follows:

$$\exp\left(-\int_{s_0}^{s}\Theta\, ds\right) = \exp\left(-\int_{s_0}^{s}\frac{\nabla^2\mathcal{S}}{n} - \frac{\mathbf{\Omega}\cdot\nabla n}{n}\, ds\right). \tag{50}$$

Because

$$\exp\left(\int_{s_0}^{s}\frac{\mathbf{\Omega}\cdot\nabla n}{n}\, ds\right) = \exp\left(\int_{s_0}^{s}\frac{\frac{dn}{ds}}{n}\, ds\right) = \frac{n(s)}{n(s_0)}, \tag{51}$$

we have

$$\exp\left(-\int_{s_0}^{s}\Theta\, ds\right) = \frac{n(s)}{n(s_0)}\exp\left(-\int_{s_0}^{s}\frac{\nabla^2\mathcal{S}}{n}\, ds\right). \tag{52}$$

By Eq. (38), Eq. (43), and Eq. (48), the intensity ratios from the three steady-state radiative transfer equations RTE_1, RTE_2, and RTE_3 are the following, respectively:
RTE_1:

$$\frac{I(s)}{I(s_0)} = 1, \tag{53}$$

RTE_2

$$\frac{I(s)}{I(s_0)} = \frac{n(s_0)}{n(s)}\exp\left(-\int_{s_0}^{s}\frac{\nabla^2\mathcal{S}}{n}\, ds\right), \tag{54}$$

RTE_3

$$\frac{I(s)}{I(s_0)} = \frac{n(s)}{n(s_0)}\exp\left(-\int_{s_0}^{s}\frac{\nabla^2\mathcal{S}}{n}\, ds\right). \tag{55}$$

It can be seen that only the intensity ratio in Eq. (55) from RTE_3 is consistent with the intensity law of geometric optics in Eq. (49). This result is different from other work [13,14]. In Ref. [13], RTE_2 is shown to be consistent with the intensity law from geometric optics. In Refs. [13,14], the intensity is by the average of the specific intensity, $I = \int_{\mathbf{S}^2} L(\mathbf{r},\mathbf{\Omega})\, d\mathbf{\Omega}$, rather than by Eq. (25). Thus the refractive variation of the solid angle element along the ray path diminishes. In Ref. [14], under the assumption that the specific intensity is peaked in one direction, the intensity is "defined as equal to the average diffuse intensity." The difference of our result from Refs. [13,14] is because the difference of the intensity definition.

In conclusion, we find that only the radiative transfer equation RTE_3 is consistent with the intensity law of geometric optics.

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