Long-time correlated quantum dynamics of phonon cooling

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We investigate the steady-state cooling dynamics of vibrational degrees of freedom related to a nanomechanical oscillator coupled with a laser-pumped quantum dot in an optical resonator. Correlations between phonon-cooling and quantum-dot photon emission processes occur, respectively, when a photon laser absorption together with a vibrational phonon absorption is followed by photon emission in the optical resonator. Therefore, the detection of photons generated in the cavity mode concomitantly contributes to phonon cooling detection of the nanomechanical resonator.

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I. INTRODUCTION

The nanomechanical resonator (NMR) is a relevant tool for building ultrasensitive measurement devices [1–3]. Therefore, its properties have been and are continuously investigated. Outstanding works toward NMR cooling to quantum regimes have already been reported [4–10]. An important issue here is how to detect experimentally the mechanical vibrations of the NMR. One option is the superconducting quantum interferometer, in which the vibrations of the NMR are detected via variation of the magnetic field [11]. The mechanical vibrations can be detected as well via a single-electron transistor which is extremely sensitive to electrical charges [12]. Additionally one can use interference effects among the incident light on the NMR and the reflected one [13]. Furthermore, high-sensitivity optical monitoring of a micromechanical resonator with a quantum-limited optomechanical sensor was reported in [14], while fast sensitive displacement measurements were reported in [15]. Remarkably, the quantum motion of a nanomechanical resonator was experimentally observed in [16]. The possibility of real time displacement detection by the luminescence signal and of displacement fluctuations by the degree of a second-order coherence function was recently demonstrated in [17].

Here, we look for a regime in which cooling of a nanomechanical oscillator is correlated with emission processes such that the maximum photon detection corresponds to the vibrational phonon minimum. For this, we investigate a laser-pumped two-level quantum dot which is fixed on a nanomechanical beam while suspended in an optical resonator [see Fig. 1(a)]. If the quantum dot dynamics is faster than that of nanobeam and cavity dynamics, respectively, one arrives at a situation in which laser photon and phonon absorption processes are accompanied with photon emission in the cavity mode [see Fig. 1(b)]. Therefore, the cavity photon detection assures the cooling of the nanomechanical resonator.

The article is organized as follows. In Sec. II we describe the theoretical framework used to obtain the master equation characterizing the correlated cooling dynamics of nanomechanical degrees of freedom. Section III deals with the corresponding equations of motion and discussion of the obtained results, while the summary is given in the last section, i.e., Sec. IV.

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II. THEORETICAL FRAMEWORK

Let us consider the setup represented in Fig. 1(a): Inside an optical resonator is placed a NMR incorporating a laser pumped two-level quantum dot. The laser beam wave vector is $\vec{k}_L$ while its frequency is ω_L. The frequency of the optical resonator is ω_C and the nanomechanical vibrational frequency is ω. The energy separation between the excited bare-state $|e\rangle$ and the ground-state one, $|g\rangle$, is denoted by $\hbar\omega_0$. The master equation describing the whole system in the Born-Markov approximations and in a frame rotating at the laser frequency ω_L is

$$\frac{d}{dt}\rho(t) + \frac{i}{\hbar}[H,\rho] = -\gamma[S^+,S^-\rho] - \gamma_c[S_z,S_z\rho] - \kappa_a[a^\dagger,a\rho] - \kappa_b(1+\bar{n})[b^\dagger,b\rho] - \kappa_b\bar{n}[b,b^\dagger\rho] + \text{H.c.}, \quad (1)$$

where S_z and $S^\pm$ are the qubit operators satisfying the standard commutation relations, while $\{a^\dagger,a\}$ and $\{b^\dagger,b\}$ are the generation and annihilation operators for photon and phonon subsystems, respectively, and obey the boson commutation relations [18]. γ and γ_c are the single-qubit spontaneous decay and dephasing rates, respectively, whereas $\kappa_a(\kappa_b)$ is the photon (phonon) resonator damping rate, and $\bar{n}$ is the mean-phonon number corresponding to temperature T and vibrational frequency ω. The Hamilton operator, i.e., H, is given by the following expression:

$$H = \hbar\Delta S_z - \hbar\Delta_1 a^\dagger a + \hbar\omega b^\dagger b + \hbar\Omega(S^+ + S^-) + \hbar g(a^\dagger S^- + aS^+) + \hbar\lambda S_z(b^\dagger + b). \quad (2)$$

In Eq. (2), the first three terms describe the free energies of the artificial two-level system as well as of the optical and mechanical modes. The fourth and the fifth terms characterize the interaction of the quantum dot with the laser field and optical resonator mode, respectively. The last term takes into account the interaction of the vibrational degrees of freedom with the radiator [4]. Correspondingly, g and λ denote the interaction strengths among the two-level emitter and the involved optical and mechanical modes, while Ω is the corresponding Rabi frequency due to external laser pumping. $\Delta = \omega_0 - \omega_L$ describes the detuning of the laser frequency from the two-level transition frequency, while $\Delta_1 = \omega_L - \omega_C$ accordingly is the detuning of the cavity frequency from the laser one.

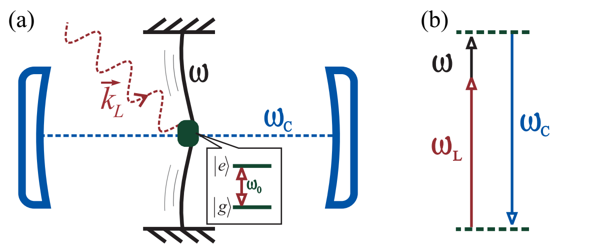

FIG. 1. (Color online) Schematic model: (a) a laser-pumped two-level quantum dot with transition frequency ω_0 is fixed on a nanomechanical resonator vibrating at frequency ω and is interacting also with the quantized resonator optical mode of frequency ω_C, and (b) correlated cooling dynamics occurs when a laser photon absorption together with a vibrational phonon absorption is accompanied by the emission of a cavity photon.

For our purpose, it is more appropriate to use the laser-qubit dressed-state representation given by [19]: $|g\rangle = \sin\theta|+\rangle + \cos\theta|-\rangle$ and $|e\rangle = \cos\theta|+\rangle - \sin\theta|-\rangle$, with $|+\rangle$ and $|-\rangle$ being the corresponding states in the dressed-state picture. Here, 2θ is the angle in the right triangle, drawn in the imaginative space of frequencies, with adjoining cathetus $\Delta/2$ and opposite cathetus Ω, and, therefore, $\cot 2\theta = \Delta/2\Omega$. In the case in which $\Omega \gg \{\gamma,\gamma_c\} \gg \kappa_{a,b}$ while $\Omega \gg \{g,\lambda\} > \{\gamma,\gamma_c\}$, meaning that the dynamics of the cavity photon and NMR phonon subsystems are slower than the quantum dot dynamics, one can eliminate the quantum dot variables (see also [19–22]). Thus, the master equation describing the cavity and NMR degrees of freedom can be represented as

$$\begin{aligned}\frac{d}{dt}\rho(t) &+ \frac{i}{2}(\Delta_1+\omega)[b^\dagger b - a^\dagger a,\rho]\\ &= -A_1^*[a,a^\dagger\rho] - B_1^*[a^\dagger,a\rho] - A_2^*[b,b^\dagger\rho]\\ &\quad - B_2^*[b^\dagger,b\rho] + C_1^*[b,a^\dagger\rho] + D_1^*[b^\dagger,a\rho]\\ &\quad + C_2^*[a^\dagger,b\rho] + D_2^*[a,b^\dagger\rho] + \text{H.c.} \qquad (3)\end{aligned}$$

Here "$*$" means complex conjugation, whereas

$$\begin{aligned}A_1^* &= \frac{1}{4}\frac{g^2\sin^2 2\theta}{\Gamma - i\Delta_1} + \frac{g^2 P_-\sin^4\theta}{\Gamma_\perp + i(2\Omega_R - \Delta_1)}\\ &\quad + \frac{g^2 P_+\cos^4\theta}{\Gamma_\perp - i(2\Omega_R + \Delta_1)},\\ A_2^* &= \frac{1}{4}\left(\frac{\lambda^2\cos^2 2\theta}{\Gamma + i\omega} + \frac{\lambda^2 P_-\sin^2 2\theta}{\Gamma_\perp + i(2\Omega_R + \omega)}\right.\\ &\quad \left. + \frac{\lambda^2 P_+\sin^2 2\theta}{\Gamma_\perp - i(2\Omega_R - \omega)}\right) + \kappa_b\bar{n},\\ C_1^* &= \frac{P_+}{2}\frac{g\lambda\sin 2\theta\cos^2\theta}{\Gamma_\perp - i(2\Omega_R + \Delta_1)} - \frac{P_-}{2}\frac{g\lambda\sin 2\theta\sin^2\theta}{\Gamma_\perp + i(2\Omega_R - \Delta_1)}\\ &\quad - \frac{1}{4}\frac{g\lambda\sin 2\theta\cos 2\theta}{\Gamma - i\Delta_1},\\ C_2^* &= \frac{P_-}{2}\frac{g\lambda\sin 2\theta\cos^2\theta}{\Gamma_\perp + i(2\Omega_R - \omega)} - \frac{P_+}{2}\frac{g\lambda\sin 2\theta\sin^2\theta}{\Gamma_\perp - i(2\Omega_R + \omega)}\\ &\quad - \frac{1}{4}\frac{g\lambda\sin 2\theta\cos 2\theta}{\Gamma - i\omega}.\end{aligned}$$

Other parameters are $\Omega_R = \sqrt{\Omega^2 + (\Delta/2)^2}$, $\Gamma = \gamma(1-\cos^2 2\theta) + \gamma_c\sin^2 2\theta$, $\Gamma_\perp = 4\gamma_0 + \gamma_+ + \gamma_-$, $\gamma_+ = \gamma\cos^4\theta + \frac{\gamma_c}{4}\sin^2 2\theta$, $\gamma_- = \gamma\sin^4\theta + \frac{\gamma_c}{4}\sin^2 2\theta$, and $\gamma_0 = \frac{1}{4}(\gamma\sin^2 2\theta + \gamma_c\cos^2 2\theta)$. The dressed-state populations are given by

$$P_+ = \frac{\gamma_-}{\gamma_+ + \gamma_-} \quad \text{and} \quad P_- = \frac{\gamma_+}{\gamma_+ + \gamma_-}.$$

In Eq. (3), B_i^* can be obtained from A_i^* via $P_\mp \leftrightarrow P_\pm$ as well as by adding κ_a to B_1^* and κ_b to B_2^*, correspondingly. Respectively, D_i^* can be obtained from C_i^* through $P_\mp \leftrightarrow P_\pm$, and $\{i \in 1,2\}$. Notice that in obtaining Eq. (3) we have ignored rapidly oscillating terms at frequencies $\pm 2\Delta_1$, $\pm(\Delta_1 - \omega)$, and $\pm 2\omega$, that is, we are interested in a situation in which $\omega \approx -\Delta_1$.

III. RESULTS AND DISCUSSION

In the following, we shall describe the steady-state correlated cooling dynamics of the vibrational degrees of freedom. Actually, adjusting the laser frequency to fulfill the condition $\omega + \Delta_1 \approx 0$, one shall look at situations in which simultaneously photon laser and vibrational phonon absorption processes are accompanied by a photon emission in the cavity mode [see Fig. 1(b)]. Using the master equation (3), the following equations of motion can be obtained for the mean photon and phonon numbers, respectively:

$$\begin{aligned}\frac{d}{dt}\langle a^\dagger a\rangle &= \langle a^\dagger a\rangle(A_1 - B_1 + A_1^* - B_1^*) + \langle a^\dagger b\rangle(C_2^* - D_2)\\ &\quad + \langle b^\dagger a\rangle(C_2 - D_2^*) + A_1 + A_1^*,\\ \frac{d}{dt}\langle b^\dagger b\rangle &= \langle b^\dagger b\rangle(A_2 - B_2 + A_2^* - B_2^*) - \langle a^\dagger b\rangle(C_1^* - D_1)\\ &\quad - \langle b^\dagger a\rangle(C_1 - D_1^*) + A_2 + A_2^*,\\ \frac{d}{dt}\langle a^\dagger b\rangle &= \langle a^\dagger b\rangle[A_1^* - B_1 + A_2 - B_2^* - i(\Delta_1 + \omega)]\\ &\quad - \langle a^\dagger a\rangle(C_1 - D_1^*) + \langle b^\dagger b\rangle(C_2 - D_2^*) - C_1 - D_2^*,\\ \frac{d}{dt}\langle b^\dagger a\rangle &= \langle b^\dagger a\rangle[A_1 - B_1^* + A_2^* - B_2 + i(\Delta_1 + \omega)]\\ &\quad - \langle a^\dagger a\rangle(C_1^* - D_1) + \langle b^\dagger b\rangle(C_2^* - D_2) - C_1^* - D_2.\end{aligned} \qquad (4)$$

Based on Eqs. (4), Figs. 2 and 3 show the steady states of the cavity mean-photon number and the vibrational NMR mean-phonon number, respectively. As was mentioned before, the maximum photon detection corresponds to the NMR phonon minimum around $\Delta_1 + \omega \approx 0$. Quantum cooling occurs for vibrational-mean-phonon numbers below those values imposed by the environmental incoherent reservoir, i.e., $\langle b^\dagger b\rangle < \bar{n}$. Furthermore, the quantum cooling is still efficient while increasing the temperature, i.e., $\bar{n}$. These behaviors can be understood also by taking into account that for certain positive laser-qubit frequency detunings the qubit population is mostly in the lower dressed state $|-\rangle$. This means that the phonon generation processes are minimized while phonon absorption followed simultaneously by photon laser absorption processes are accompanied by photon emission

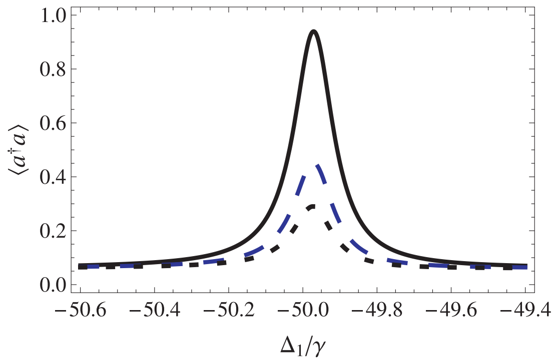

FIG. 2. (Color online) The steady-state mean value of the photon number $\langle a^\dagger a\rangle$ as a function of Δ_1/γ. Here, $\gamma_c/\gamma = 0.3$, $g/\gamma = 2$, $\lambda/\gamma = 4$, $\Omega/\gamma = 50$, $\omega/\gamma = 50$, $\Delta/(2\Omega) = 0.5$, $\kappa_a/\gamma = 0.01$, and $\kappa_b/\gamma = 0.001$. The solid line corresponds to $\bar{n} = 10$, the long-dashed line corresponds to $\bar{n} = 4$, and the short-dashed line corresponds to $\bar{n} = 2$.

in the cavity mode. Therefore, the cavity maximum photon detection signifies also the minimum of the vibrational quanta, that is, the cooling detection of the NMR vibrational degrees of freedom. Notice a small shift between photon maximum and phonon minimum detections for arbitrary larger qubit-cavity and qubit-NMR coupling strengths [see, also, Eq. (5)]. Finally, efficient cooling occurs as well for uncorrelated regimes of phonon-photon detection processes, i.e., when $\Delta_1 + \omega \gg \gamma$ (while correlated regimes occur for $\Delta_1 + \omega \ll \gamma$). However, in this case, i.e., the uncorrelated regime, it is hard to manage to have maximum photon cavity emission corresponding to vibrational NMR phonon cooling processes simply because these processes are uncorrelated.

In what follows, we shall represent approximative analytical expressions for the variables of interest in the steady state. This will help one to understand the behaviors shown in Figs. 2 and 3. If one performs further approximations $\{\Delta_1,\omega\} \gg \Gamma$, $2\Omega_R \pm \Delta_1 \gg \Gamma_\perp$ and $2\Omega_R \pm \omega \gg \Gamma_\perp$, the master equation (3) simplifies considerably, namely,

$$\frac{d}{dt}\rho(t) = \frac{i}{4}(\Delta_1 + \omega - \bar{\delta}_a + \bar{\delta}_b)[a^\dagger a - b^\dagger b,\rho] + i\eta[ab^\dagger,\rho] - \kappa_a[a^\dagger,a\rho] - \kappa_b(1+\bar{n})[b^\dagger,b\rho] - \kappa_b\bar{n}[b,b^\dagger\rho] + \text{H.c.} \quad (5)$$

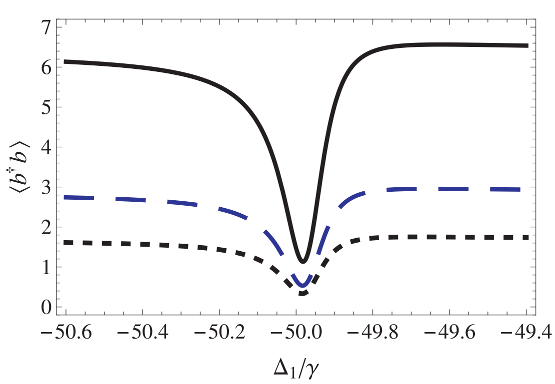

FIG. 3. (Color online) The same as in Fig. 2 but for the vibrational phonon mean number $\langle b^\dagger b\rangle$.

Here, the frequency shift $\bar{\delta}_a - \bar{\delta}_b$ observed also in Figs. 2 and 3 is described by the following expressions: $\bar{\delta}_a = g^2(P_+ - P_-)\{\sin^4\theta/(2\Omega_R + \omega) + \cos^4\theta/(2\Omega_R - \omega)\}$ and $\bar{\delta}_b = \Omega_R\lambda^2\sin^2 2\theta(P_+ - P_-)/(4\Omega_R^2 - \omega^2)$, whereas $\eta = g\lambda\sin 2\theta(P_+ - P_-)(\Omega_R\cos 2\theta + \omega/2)/(4\Omega_R^2 - \omega^2)$. The last term of the first line in Eq. (5) with its H.c. part describe the vibrational phonon emission followed by cavity-photon absorption processes, and vice versa, mediated by the laser field, and it is responsible for cooling and detection of the vibrational degrees of freedom. Consequently, based on Eq. (5), Eqs. (4) reduce to

$$\frac{d}{dt}\langle a^\dagger a\rangle = -i\eta\langle x\rangle - 2\kappa_a\langle a^\dagger a\rangle,$$
$$\frac{d}{dt}\langle x\rangle = 2i\eta(\langle b^\dagger b\rangle - \langle a^\dagger a\rangle) - (\kappa_a + \kappa_b)\langle x\rangle, \quad (6)$$
$$\frac{d}{dt}\langle b^\dagger b\rangle = i\eta\langle x\rangle - 2\kappa_b\langle b^\dagger b\rangle + 2\kappa_b\bar{n},$$

where $x = ab^\dagger - a^\dagger b$. We have assumed also that $\Delta_1 + \omega = \bar{\delta}_a - \bar{\delta}_b$; i.e., we are interested in the maximal values of the mean-photon number corresponding to minimal values of the vibrational mean-phonon number, respectively (see, also, Figs. 2 and 3). In the steady state, one immediately obtains from Eqs. (6) that

$$\kappa_a\langle a^\dagger a\rangle + \kappa_b\langle b^\dagger b\rangle = \kappa_b\bar{n}. \quad (7)$$

This expression can help us to estimate the mean-vibrational-phonon number if the mean-photon number is known (i.e., detected). The explicit expressions for the steady-state values of the photon and phonon mean numbers are, respectively,

$$\langle a^\dagger a\rangle = \frac{\bar{n}\kappa_b\eta^2}{(\kappa_a + \kappa_b)(\kappa_a\kappa_b + \eta^2)},$$
$$\langle b^\dagger b\rangle = \frac{\bar{n}\kappa_b}{\kappa_a + \kappa_b}\left(1 + \frac{\kappa_a^2}{\kappa_a\kappa_b + \eta^2}\right), \quad (8)$$

or

$$\langle b^\dagger b\rangle = \langle a^\dagger a\rangle(1 + \kappa_a(\kappa_a + \kappa_b)/\eta^2). \quad (9)$$

Equations (8) characterize the quantum steady-state vibrational-phonon cooling dynamics under performed approximations which may be considered as conditions for the cooling effect to take place in this particular setup. Furthermore, Eqs. (7) and (9) describe the efficiency of the proposed vibrational-phonon cooling method. In particular, if $\kappa_a \gg \kappa_b$ while $(\kappa_a/\eta)^2 \ll 1$, then $\langle b^\dagger b\rangle \approx \langle a^\dagger a\rangle \approx (\kappa_b/\kappa_a)\bar{n}$, which can be as well below unity, i.e., $\langle b^\dagger b\rangle < 1$. Finally, optical photon detection schemes are well developed nowadays. There is also a good progress toward ultrasmall mechanical vibrations detection [2].

IV. SUMMARY

In summary, we have proposed a scheme to detect the vibrational phonon cooling of a nanomechanical oscillator in the steady state. The idea is based on correlating the vibrational degrees of freedom with those of a laser-pumped quantum dot when fixed on a nanomechanical beam while interacting with an optical resonator. More concretely, when the quantum dot dynamics is faster than the corresponding ones of other involved subsystems, one needs to adjust the laser frequency such that both photon laser and NMR phonon absorption processes are accompanied by photon emission in the resonator mode. Therefore, detection of the cavity photons is followed in parallel by cooling of the nanomechanical oscillator. Finally, we give approximative analytical expressions for the variables of interest which describe also the method efficiency.

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