Universal quantum gates on microwave photons assisted by circuit quantum electrodynamics

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Based on a microwave-photon quantum processor with two superconducting resonators coupled to one transmon qutrit, we construct the controlled-phase (c-phase) gate on microwave-photon-resonator qudits, by combination of the photon-number-dependent frequency-shift effect on the transmon qutrit by the first resonator and the resonant operation between the qutrit and the second resonator. This distinct feature provides us a useful way to achieve the c-phase gate on the two resonator qudits with a higher fidelity and a shorter operation time, compared with the previous proposals. The fidelity of our c-phase gate can reach 99.51% within 93 ns. Moreover, our device can be extended easily to construct the three-qudit gates on three resonator qudits, far different from the existing proposals. Our controlled-controlled-phase gate on three resonator qudits is accomplished with the assistance of a transmon qutrit and its fidelity can reach 92.92% within 124.64 ns.

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I. INTRODUCTION

Quantum computation has attracted much attention in recent years [1]. Many important schemes have been proposed for quantum computation by using different quantum systems, such as photonic systems [2–6], nuclear magnetic resonance [7,8], quantum dots [9–12], and diamond nitrogen-vacancy (NV) centers [13–15]. Universal quantum gates are the key elements in constructing a universal quantum computer. Moreover, they can be used to produce the entanglement of multipartite quantum systems. The controlled-phase (c-phase) gate is one of the important universal two-qubit gates. It has the same role as the controlled-not gate in quantum computation. The controlled-controlled-phase (cc-phase) gate is an important three-qubit gate which can play the same role as the three-qubit Toffoli gate which can be used to construct a universal quantum computation with single-qubit Hadamard operations [1].

Circuit quantum electrodynamics (QED), which combines superconducting circuits and cavity QED, provides a good platform for quantum computation [16–20]. A superconducting Josephson junction can act as a perfect qubit and it has some good features, such as the large-scale integration [21], a relatively long coherence time of about 0.1 ms [22], the versatility in its energy-level structure with Ξ, Λ, V, and even Δ types [23] which cannot be found in atom systems, and the superconducting qubit with tunable coupling strength [24–26]. All these characteristics have attracted much attention focused on quantum information processing on superconducting qubits in circuit QED. Some interesting proposals for quantum information processing on qubits have been presented, such as the reset of a superconducting qubit [27–29], universal quantum gates and entanglement generation [30–35], and single-shot individual qubit measurement and the joint qubit readout [36–38].

A superconducting coplanar resonator whose quality factor Q can be increased to be 10^6 [39–42] can act as a qudit because it contains some microwave photons whose lifetimes are much longer than that of a superconducting qubit [42–44]. The coupling strength between a resonator and a transmission line is tunable [45]. Moreover, the strong and even ultrastrong coupling [16,46–51] in circuit QED affords a strong nonlinear interaction between a superconducting qubit and a microwave-photon qudit. These good features make resonators a powerful platform for quantum computation as well. There are some interesting studies on resonator qudits. For example, Moon and Girvin [52] studied theoretically the parametric down-conversion and squeezing of microwaves inside a transmission line resonator, resorting to circuit QED in 2005. In 2007, Marquardt [53] presented an efficient scheme for the generation of microwave photon pairs by parametric down-conversion in a superconducting resonator coupled to a superconducting qubit. In 2008, Hofheinz *et al.* [54] demonstrated the preparation of pure Fock states with a microwave resonator, resorting to a superconducting phase qubit. In the next year, they synthesized arbitrary quantum states in a superconducting resonator [55]. In 2009, Rebić *et al.* [56] introduced a scheme for generating giant Kerr nonlinearities in circuit QED. In 2010, Bergeal *et al.* [57] proposed a practical microwave device for achieving parametric amplification. In this year, Johnson *et al.* [58] demonstrated a quantum nondemolition detection scheme that measures the number of photons inside a high-quality-factor microwave cavity on a chip and Strauch *et al.* [59] presented an effective method to synthesize an arbitrary quantum state of two superconducting resonators. In 2011, Mariantoni *et al.* [60] used a three-resonator circuit to shuffle one- and two-photon Fock states between the three resonators and demonstrated qubit-mediated vacuum Rabi swaps between two resonators. In addition, there are some interesting works to generate the entanglement between the resonator qudits [61–68].

To realize the quantum computation based on resonator qudits, people should construct the universal quantum gates on qudits. In 2007, Schuster *et al.* [69] proposed the effect of the number-state-dependent interaction between a superconducting qubit and resonator qudits, which provides an interesting way to achieve the state-selective qubit rotation. Based on this effect, Strauch [70] presented an interesting scheme to construct the c-phase gate on two superconducting resonator qudits in 2011. In his work, each of two resonators (A and B) is coupled to an auxiliary three-level transmon or phase qutrit

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(a and b), and each qutrit is coupled to each other directly. The operation time of the c-phase gate on two resonator qudits is 150 ns. In 2012, Wu *et al.* [71] gave an effective scheme for the construction of the c-phase gate on two resonators by using the number-state-dependent interaction between a two-energy-level charge qubit and two resonator qudits for one-way quantum computation, and the operation time of the c-phase gate was 125 ns.

In this paper, we give a microwave-photon quantum processor with two resonators which are coupled to just one transmon qutrit which has the characteristics of a lesser anharmonicity energy level and a long coherence time [72], and we construct an effective c-phase gate on two resonator qudits, resorting to the combination of the number-state-dependent interaction between the qutrit and one-resonator-qudit subsystem and the simple resonant operation between the qutrit and another resonator-qudit subsystem. This different physical mechanism provides us a faster way to achieve a higher-fidelity c-phase gate on the two resonator qudits without increasing the difficulty of its implementation, compared with the previous proposals [70,71]. The fidelity of our c-phase gate is 99.51% within the operation time of 93 ns. Moreover, our device can be extended easily to construct the three-qudit cc-phase gate on three resonator qudits, by using a resonator to complete the simple resonant operation and the other two resonators to achieve the number-state-dependent interaction on the transmon qutrit, far different from the existing proposals. Its fidelity is 92.92% within the operation time of 124.64 ns.

II. CONTROLLED-PHASE GATE ON TWO RESONATORS IN CIRCUIT QED

Let us first consider a system composed of a perfect two-level superconducting qubit q and a resonator (r_1), whose schematic diagram is the same as that shown in the dashed-line box in Fig. 1 (by replacing the three-energy-level qutrit with

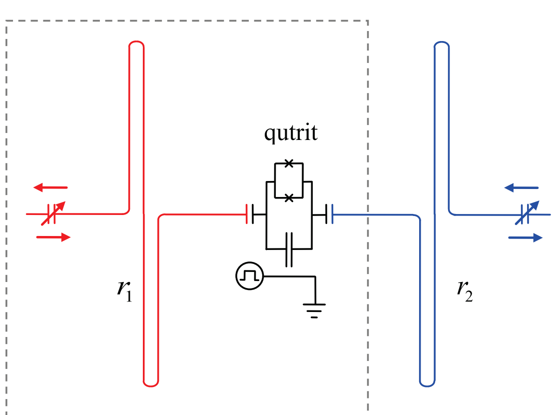

FIG. 1. (Color online) Schematic diagram for our c-phase gate on two microwave-photon-resonator qudits, by combination of the number-state-dependent interaction between the transmon qutrit and the left resonator (r_1) and the simple resonant operation between the transmon qutrit and the right resonator (r_2). The two resonators are capacitively coupled to the qutrit whose transition frequency can be tuned by an external flux.

a two-energy-level qubit). The Hamiltonian for this system under the rotating-wave approximation ($\hbar = 1$) is

$$H_1 = \omega_{r_1} a^\dagger a + \omega_q \sigma^+\sigma^- + g(a\sigma^+ + a^\dagger\sigma^-), \qquad (1)$$

where $\sigma^+ = |1\rangle\langle 0|$ and $a^\dagger$ are the creation operators of the superconducting qubit q and the resonator r_1, respectively. g is the coupling strength between the qubit and the resonator. ω_{r_1} and ω_q are the transition frequencies of the resonator r_1 and the qubit, respectively. In the dispersive regime ($\frac{g^2}{\Delta} \leqslant 1$) in circuit QED, by making the unitary transformation $U = \exp[\frac{g}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)]$, the Hamiltonian H_1 becomes [16]

$$H_1' = UH_1U^\dagger \approx \omega_{r_1} a^\dagger a + \frac{1}{2}\left[\omega_q + \frac{g^2}{\Delta}(2a^\dagger a + 1)\right]\sigma_z. \qquad (2)$$

The effect of the photon-number-dependent transition frequency of the qubit can be described as

$$\omega_q'^n = \omega_q + \frac{g^2}{\Delta}(2n+1). \qquad (3)$$

Here $\Delta = \omega_{r_1} - \omega_q$. $n = a^\dagger a$ is the photon number in a resonator. $\omega_q'^n$ is the changed transition frequency of the qubit due to the different photon numbers in the resonator.

In the dispersive strong regime ($\frac{g^2}{\Delta} \ll 1$) [69], the photon-number-dependent transition frequency of the qubit is too small to distinguish the different transition frequencies of the qubit due to the different photon numbers in the resonator. By keeping $\frac{g}{\Delta}$ to be a small value to make Eq. (2) work well, one can increase the coupling strength to make the transition frequency of the qubit depend on largely the photon number, shown in Eq. (3). That is, if we apply a drive field with the frequency equivalent to the transition frequency of the qubit when $n = 1$, and take the proper amplitude $|\Omega| \ll \frac{g^2}{\Delta}$ to suppress the error generated by off-resonant transitions sufficiently, the field will flip the qubit only if there is one microwave photon in the resonator. On the other hand, if we apply a drive field with the frequency equivalent to the transition frequency of the qubit when $n = 0$, and take the proper amplitude, the field will flip the qubit only if there is no microwave photon in the resonator. To describe this effect, we consider a system with the resonator r_1 coupled to a practical transmon qutrit [73], whose Hamiltonian is (under the rotating-wave approximation)

$$H_2 = \sum_{l=g,e,f} E_l\, |l\rangle_q \langle l| + \omega_{r_1} a_1^\dagger a_1 + g_{r_1}^{g,e}(a_1^\dagger \sigma_{g,e}^- + a_1\sigma_{g,e}^+)$$
$$+ g_{r_1}^{e,f}(a_1^\dagger \sigma_{e,f}^- + a_1\sigma_{e,f}^+), \qquad (4)$$

where $|g\rangle_q$, $|e\rangle_q$, and $|f\rangle_q$ are the first three lower-energy levels of the qutrit. $\sigma_{g,e}^+$ and $\sigma_{e,f}^+$ are the creation operators for the transitions $|g\rangle_q \to |e\rangle_q$ and $|e\rangle_q \to |f\rangle_q$ of the qutrit q, respectively. $a_1^\dagger$ is the creation operator of the resonator r_1. The energy for the level l of q is E_l, and ω_{r_1} is the transition frequency of r_1. $g_1^{g,e}$ and $g_1^{e,f}$ are the coupling strengths between these two transitions of q and r_1.

A microwave drive field $H_d = \Omega(|f\rangle_q\langle e|e^{-i\omega_d t} + |e\rangle_q \langle f|e^{i\omega_d t})$ with a proper amplitude Ω is applied to interact with the qutrit, and here the frequency ω_d is chosen to be equivalent to the transition frequency ($|e\rangle_q \leftrightarrow |f\rangle_q$) of the qutrit q when

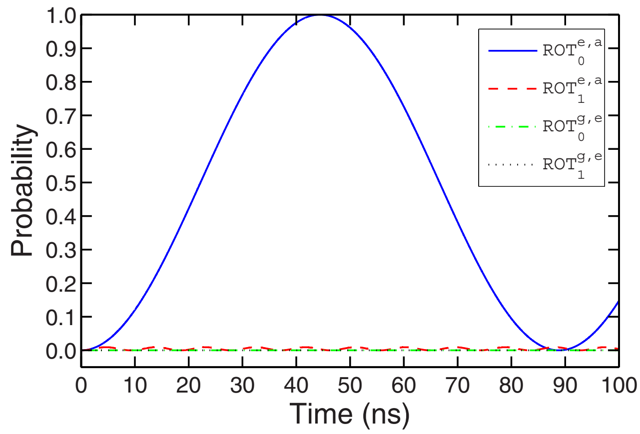

FIG. 2. (Color online) The expectation values of the probability distributions of the quantum Rabi oscillations $\mathrm{ROT}_0^{g,e}$, $\mathrm{ROT}_0^{e,f}$, $\mathrm{ROT}_1^{g,e}$, and $\mathrm{ROT}_1^{e,f}$. $\mathrm{ROT}_0^{g,e}$ and $\mathrm{ROT}_0^{e,f}$ represent the oscillations of $(|0\rangle_{r_1}|g\rangle_q)_{\mathrm{dress}} \leftrightarrow (|0\rangle_{r_1}|e\rangle_q)_{\mathrm{dress}}$ and $(|0\rangle_{r_1}|e\rangle_q)_{\mathrm{dress}} \leftrightarrow (|0\rangle_{r_1}|f\rangle_q)_{\mathrm{dress}}$, respectively. $\mathrm{ROT}_1^{g,e}$ and $\mathrm{ROT}_1^{e,f}$ represent the oscillations of $(|1\rangle_{r_1}|g\rangle_q)_{\mathrm{dress}} \leftrightarrow (|1\rangle_{r_1}|e\rangle_q)_{\mathrm{dress}}$ and $(|1\rangle_{r_1}|e\rangle_q)_{\mathrm{dress}} \leftrightarrow (|1\rangle_{r_1}|f\rangle_q)_{\mathrm{dress}}$, respectively.

there is no microwave photon in the resonator. Due to the realistic quantum Rabi oscillation (ROT) occurring between the dress states of the system [74], we simulate the expectation value of $\mathrm{ROT}_0^{e,f}$ $(|0\rangle_{r_1}|e\rangle_q)_{\mathrm{dress}} \leftrightarrow (|0\rangle_{r_1}|f\rangle_q)_{\mathrm{dress}}$ and $\mathrm{ROT}_1^{e,f}$ $(|1\rangle_{r_1}|e\rangle_q)_{\mathrm{dress}} \leftrightarrow (|1\rangle_{r_1}|f\rangle_q)_{\mathrm{dress}}$, shown in Fig. 2. The transition frequencies of $|g\rangle_q \leftrightarrow |e\rangle_q$ and $|e\rangle_q \leftrightarrow |f\rangle_q$ of the qutrit are chosen to be $\omega_{g,e}/(2\pi) = E_e - E_g = 8.7$ GHz and $\omega_{e,f}/(2\pi) = E_f - E_e = 8.0$ GHz, respectively. $\omega_{r_1}/(2\pi) = 7.5$ GHz. The coupling strengths between two transitions of the qutrit and r_1 are taken in convenience with $g_1^{g,e}/(2\pi) = g_1^{e,f}/(2\pi) = 0.2$ GHz. The frequency and amplitude of the drive field are $\omega_d/(2\pi) = 8.043$ GHz and $\Omega = 0.0115$ GHz, respectively.

As shown in Fig. 2, the maximal probability of $\mathrm{ROT}_0^{e,f}$ can reach 100%. After a period of $\mathrm{ROT}_0^{e,f}$, a π phase shift can be generated in the state $(|0\rangle_1|e\rangle_q)_{\mathrm{dress}}$, and $\mathrm{ROT}_1^{e,f}$ and the other oscillations take place with a very small probability, which indicates that the final state of the system composed of r_1 and q becomes

$$|\phi_f\rangle = \tfrac{1}{2}[(|0\rangle_1|g\rangle_q)_{\mathrm{dress}} - (|0\rangle_1|e\rangle_q)_{\mathrm{dress}} + (|1\rangle_1|g\rangle_q)_{\mathrm{dress}} + (|1\rangle_1|e\rangle_q)_{\mathrm{dress}}] \tag{5}$$

after the state-selective qubit rotation if the initial state of the system is

$$|\phi_0\rangle = \tfrac{1}{2}[(|0\rangle_1|g\rangle_q)_{\mathrm{dress}} + (|0\rangle_1|e\rangle_q)_{\mathrm{dress}} + (|1\rangle_1|g\rangle_q)_{\mathrm{dress}} + (|1\rangle_1|e\rangle_q)_{\mathrm{dress}}]. \tag{6}$$

This is just the outcome of a hybrid c-phase gate on r_1 and q by using r_1 as the control qubit and q as the target qubit. At the beginning and the end of the algorithm, one can turn on and off the coupling between r_1 and q to evolve the dress states into the computational states [74]. On one hand, one can tune the transition frequency of the resonator or the qutrit to make them resonate or largely detune with each other, in order to turn on or off the interaction between q and a noncomputational resonator. On the other hand, one can tune on or off the coupling between the qutrit and the noncomputational resonator [75,76].

The principle of our c-phase gate on two resonator qudits based on the state-selective qubit rotation is shown in Fig. 1. The matrix representation of the c-phase gate can be written as

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{7}$$

in the basis of a two-resonator-qudit system $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$. The Hamiltonian of the system composed of the resonators r_1, r_2, and q can be written as (under the rotating-wave approximation)

$$H_2 = \sum_{l=g,e,f} E_l|l\rangle_q\langle l| + \sum_{i=1,2}\left[\omega_{r_i}a_i^\dagger a_i + g_i^{g,e}(a_i^\dagger\sigma_{g,e}^- + a_i\sigma_{g,e}^+) + g_i^{e,f}(a_i^\dagger\sigma_{e,f}^- + a_i\sigma_{e,f}^+)\right]. \tag{8}$$

Suppose that the initial state of the system is

$$|\psi_0\rangle = \tfrac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)\otimes|g\rangle_q. \tag{9}$$

Our c-phase gate on the two resonators can be accomplished with three steps as follows.

First, by resonating r_2 and q with $g_2^{g,e}t = \frac{\pi}{2}$, and turning off the interaction between r_1 and q, the system evolves from the initial state $|\psi_0\rangle$ to the state

$$|\psi_1\rangle = \tfrac{1}{2}(|0\rangle_1|g\rangle_q - i|0\rangle_1|e\rangle_q + |1\rangle_1|g\rangle_q - i|1\rangle_1|e\rangle_q)\otimes|0\rangle_2. \tag{10}$$

Second, by turning on the coupling between r_1 and q, and turning off the coupling between r_2 and q, the state of the system becomes

$$|\psi_1'\rangle = \tfrac{1}{2}[(|0\rangle_1|g\rangle_q)_{\mathrm{dress}} - i(|0\rangle_1|e\rangle_q)_{\mathrm{dress}} + (|1\rangle_1|g\rangle_q)_{\mathrm{dress}} - i(|1\rangle_1|e\rangle_q)_{\mathrm{dress}}]\otimes|0\rangle_2. \tag{11}$$

By applying a drive field $H_d = \Omega(|f\rangle_q\langle e|e^{-i\omega_d t} + |e\rangle_q\langle f|e^{i\omega_d t})$ with the frequency equivalent to the transition frequency $(|e\rangle_q \leftrightarrow |f\rangle_q)$ of the qutrit when there is no microwave photon in the resonator r_1, after an operation time of $\Omega t = \pi$, the state of the system is changed to be

$$|\psi_2\rangle = \tfrac{1}{2}[(|0\rangle_1|g\rangle_q)_{\mathrm{dress}} + i(|0\rangle_1|e\rangle_q)_{\mathrm{dress}} + (|1\rangle_1|g\rangle_q)_{\mathrm{dress}} - i(|1\rangle_1|e\rangle_q)_{\mathrm{dress}}]\otimes|0\rangle_2. \tag{12}$$

Third, by turning off the coupling between r_1 and q, the state of the system evolves from $|\psi_2\rangle$ into

$$|\psi_2'\rangle = \tfrac{1}{2}(|0\rangle_1|g\rangle_q + i|0\rangle_1|e\rangle_q + |1\rangle_1|g\rangle_q - i|1\rangle_1|e\rangle_q)\otimes|0\rangle_2. \tag{13}$$

By resonating r_2 and q with $g_2^{g,e}t = \frac{\pi}{2}$ again, and turning off the interaction between r_1 and q, the state of the system

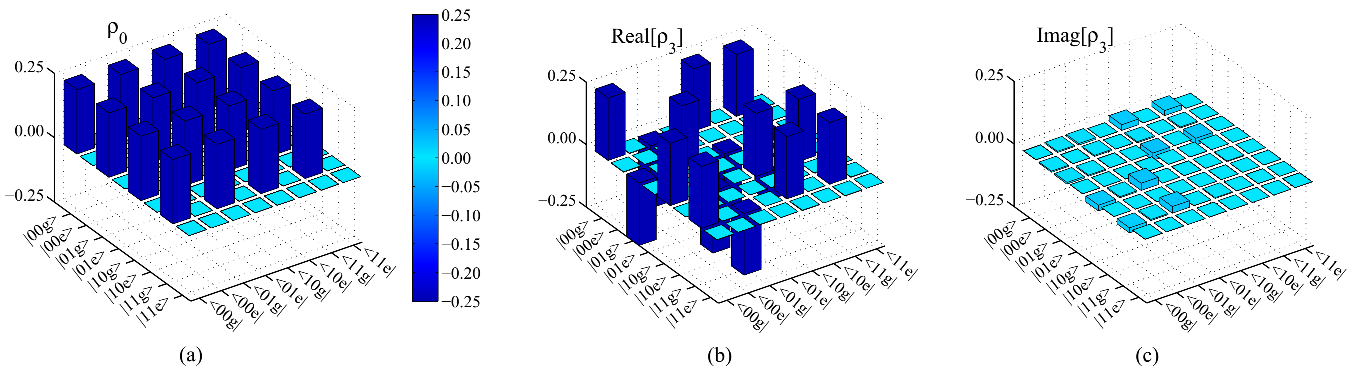

FIG. 3. (Color online) (a) The density operator (ρ_0) of the initial state $|\psi_0\rangle$ of the system composed of the two resonators and the qutrit in our c-phase gate. Panels (b) and (c) are the real part (Real[ρ_3]) and the imaginary part (Imag[ρ_3]) of the final state $|\psi_f\rangle$ of the system, respectively.

becomes

$$|\psi_f\rangle = \tfrac{1}{2}(|0\rangle_1\,|0\rangle_2 + |0\rangle_1\,|1\rangle_2 + |1\rangle_1\,|0\rangle_2 - |1\rangle_1\,|1\rangle_2)\otimes|g\rangle_q. \tag{14}$$

This is just the result of the c-phase gate on r_1 and r_2 by using r_1 as the control qubit and r_2 as the target qubit.

The reduced density operators of the two-resonator system in the initial state $|\psi_0\rangle$ in Eq. (9) and the final state $|\psi_f\rangle$ in Eq. (14) are shown in Fig. 3. One can see that the fidelity of our c-phase gate on two microwave-photon qudits is about 99.51% within about 93 ns. Here the fidelity is defined as $F = \mathrm{Tr}(|\sqrt{\rho_f}\,\rho_{\mathrm{ideal}}\sqrt{\rho_f}|)$ [77]. ρ_f is the density operator of the final state of the two-microwave-photon-qudit system $|\psi_f\rangle$ and ρ_{ideal} is the density operator of the final state of the system after an ideal c-phase gate operation is performed with the initial state $|\psi_0\rangle$.

III. CONTROLLED-CONTROLLED-PHASE GATE ON THREE RESONATORS

The principle of our cc-phase gate on a three-resonator system is shown in Fig. 4. Here the resonator r_1 has the same role as r_2 and they both are used to provide the effect of the photon-number-dependent transition frequency of the Ξ-type three-level qutrit to accomplish the state-selective qubit rotation, different from the resonator r_3. The photon-number-dependent transition frequency between $|e\rangle_q$ and $|f\rangle_q$ of the qutrit can be written as [59,71,74]

$$\omega'^{\,n_1,n_2}_{e,f} \approx \omega_{e,f} + \frac{\left(g_1^{e,f}\right)^2}{\omega_{e,f}-\omega_{r_1}}(2n_1+1) + \frac{\left(g_2^{e,f}\right)^2}{\omega_{e,f}-\omega_{r_2}}(2n_2+1). \tag{15}$$

Here n_1 and n_2 are the photon numbers in the resonators r_1 and r_2, respectively. The photon-number-dependent transition frequency of q depends on the relationship of the photon numbers in two resonators. That is, one can afford a drive field with the frequency equivalent to the changed transition frequency of the qutrit to achieve the state-selective qubit rotation with different relations between n_1 and n_2.

Suppose that $\frac{3(g_1^{e,f})^2}{\omega_{e,f}-\omega_{r_1}} = \frac{(g_2^{e,f})^2}{\omega_{e,f}-\omega_{r_2}}$, one can obtain the relation $\omega'^{\,n_1,n_2}_{e,f} = \omega_{e,f} + \frac{(g_1^{e,f})^2}{\omega_{e,f}-\omega_{r_1}}(N+4)$, where $N = 2n_1 + 6n_2$. The transition frequency of q can be divided into four groups, according to the photon-number relations between r_1 and r_2. That is, $|0\rangle_1\,|0\rangle_2$, $|1\rangle_1\,|0\rangle_2$, $|0\rangle_1\,|1\rangle_2$, and $|1\rangle_1\,|1\rangle_2$ with $N = 0$, 2, 6, and 8, respectively. Considering $N = 8$, a drive field with the frequency $\omega_d = \omega_{e,f} + \frac{12(g_1^{e,f})^2}{\omega_{e,f}-\omega_{r_1}}$ can flip the qutrit between $|e\rangle_q$ and $|f\rangle_q$ only if there is one microwave photon in each of the two resonators r_1 and r_2.

If we take the initial state of the hybrid system composed of r_1, r_2, and q as

$$\begin{aligned}|\Phi_0\rangle = \frac{1}{2\sqrt{2}}[&(|0\rangle_1\,|0\rangle_2\,|g\rangle_q)_{\mathrm{dress}} + (|0\rangle_1\,|0\rangle_2\,|e\rangle_q)_{\mathrm{dress}}\\ &+ (|0\rangle_1\,|1\rangle_2\,|g\rangle_q)_{\mathrm{dress}} + (|0\rangle_1\,|1\rangle_2\,|e\rangle_q)_{\mathrm{dress}}\\ &+ (|1\rangle_1\,|0\rangle_2\,|g\rangle_q)_{\mathrm{dress}} + (|1\rangle_1\,|0\rangle_2\,|e\rangle_q)_{\mathrm{dress}}\\ &+ (|1\rangle_1\,|1\rangle_2\,|g\rangle_q)_{\mathrm{dress}} + (|1\rangle_1\,|1\rangle_2\,|e\rangle_q)_{\mathrm{dress}}],\end{aligned} \tag{16}$$

applying a drive field to complete a state-selective qubit rotation operation on the transition between $|e\rangle_q$ and $|f\rangle_q$ when there is one microwave photon in each of the two resonators

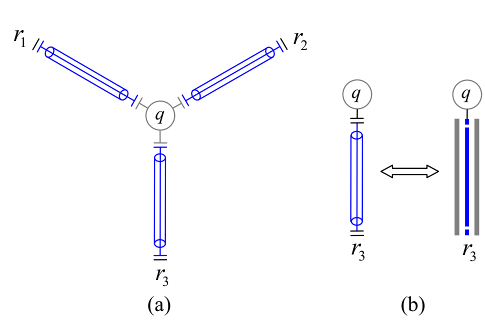

FIG. 4. (Color online) (a) The schematic diagram for our cc-phase gate on a three-qudit microwave-photon system. (b) The schematic diagram for the coupling between a transmon qutrit and a microwave-photon resonator. Here q represents a transmon qutrit. r_1, r_2, and r_3 are three microwave-photon resonators which have the same structure as those shown in Fig. 1.

r_1 and r_2, the state of the hybrid system becomes

$$\begin{aligned}|\Phi_f\rangle = \frac{1}{2\sqrt{2}}[&(|0\rangle_1|0\rangle_2|g\rangle_q)_{\text{dress}} + (|0\rangle_1|0\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|0\rangle_1|1\rangle_2|g\rangle_q)_{\text{dress}} + (|0\rangle_1|1\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|1\rangle_1|0\rangle_2|g\rangle_q)_{\text{dress}} + (|1\rangle_1|0\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|1\rangle_1|1\rangle_2|g\rangle_q)_{\text{dress}} - (|1\rangle_1|1\rangle_2|e\rangle_q)_{\text{dress}}]\end{aligned} \tag{17}$$

after the operation time $t = \frac{\pi}{\Omega}$. This is just the result of a hybrid cc-phase gate on the system composed of r_1, r_2, and q by using r_1 and r_2 as the control qudits and q as the target qubit.

With the hybrid cc-phase gate above, we can construct the cc-phase gate on three resonator qudits, shown in Fig. 4. The Hamiltonian of the hybrid system composed of the three resonators r_1, r_2, and r_3 and the transmon qutrit q is

$$\begin{aligned}H_3 = \sum_{l=g,e,f} E_l|l\rangle_q\langle l| + \sum_{i=1,2,3}\big[&\omega_i^r a_i^\dagger a_i + g_i^{g,e}(a_i^\dagger\sigma_{g,e}^- + a_i\sigma_{g,e}^+)\\ &+ g_i^{e,f}(a_i^\dagger\sigma_{e,f}^- + a_i\sigma_{e,f}^+)\big].\end{aligned} \tag{18}$$

Suppose that the initial state of the system is

$$\begin{aligned}|\Psi_0\rangle = \frac{1}{2\sqrt{2}}(&|0\rangle_1|0\rangle_2|0\rangle_3 + |0\rangle_1|0\rangle_2|1\rangle_3\\ &+ |0\rangle_1|1\rangle_2|0\rangle_3 + |0\rangle_1|1\rangle_2|1\rangle_3\\ &+ |1\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|0\rangle_2|1\rangle_3\\ &+ |1\rangle_1|1\rangle_2|0\rangle_3 + |1\rangle_1|1\rangle_2|1\rangle_3)\otimes|g\rangle_q.\end{aligned} \tag{19}$$

The cc-phase gate can be achieved with three steps as follows.

First, we turn off the interaction between the two resonators r_1r_2 and q, and then resonate r_3 and the qutrit q in the transition between $|g\rangle_q$ and $|e\rangle_q$. After the operation time $t = \frac{\pi}{2g_3^{g,e}}$, the state of the system evolves from $|\Psi_0\rangle$ into

$$\begin{aligned}|\Psi_1\rangle = \frac{1}{2\sqrt{2}}(&|0\rangle_1|0\rangle_2|g\rangle_q + i\,|0\rangle_1|0\rangle_2|e\rangle_q\\ &+ |0\rangle_1|1\rangle_2|g\rangle_q + i\,|0\rangle_1|1\rangle_2|e\rangle_q\\ &+ |1\rangle_1|0\rangle_2|g\rangle_q + i\,|1\rangle_1|0\rangle_2|e\rangle_q\\ &+ |1\rangle_1|1\rangle_2|g\rangle_q + i\,|1\rangle_1|1\rangle_2|e\rangle_q)\otimes|0\rangle_3.\end{aligned} \tag{20}$$

Second, we turn off the interaction between r_3 and q, and turn on the interactions between r_1 and q and between r_2 and q. The state $|\Psi_1\rangle$ is changed to be

$$\begin{aligned}|\Psi_1'\rangle = \frac{1}{2\sqrt{2}}[&(|0\rangle_1|0\rangle_2|g\rangle_q)_{\text{dress}} + i(|0\rangle_1|0\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|0\rangle_1|1\rangle_2|g\rangle_q)_{\text{dress}} + i(|0\rangle_1|1\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|1\rangle_1|0\rangle_2|g\rangle_q)_{\text{dress}} + i(|1\rangle_1|0\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|1\rangle_1|1\rangle_2|g\rangle_q)_{\text{dress}} + i(|1\rangle_1|1\rangle_2|e\rangle_q)_{\text{dress}}]\otimes|0\rangle_3.\end{aligned} \tag{21}$$

By taking the hybrid cc-phase gate on r_1, r_2, and q, we can get

$$\begin{aligned}|\Psi_2\rangle = \frac{1}{2\sqrt{2}}[&(|0\rangle_1|0\rangle_2|g\rangle_q)_{\text{dress}} + i(|0\rangle_1|0\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|0\rangle_1|1\rangle_2|g\rangle_q)_{\text{dress}} + i(|0\rangle_1|1\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|1\rangle_1|0\rangle_2|g\rangle_q)_{\text{dress}} + i(|1\rangle_1|0\rangle_2|e\rangle_q)_{\text{dress}}\\ &+ (|1\rangle_1|1\rangle_2|g\rangle_q)_{\text{dress}} - i(|1\rangle_1|1\rangle_2|e\rangle_q)_{\text{dress}}]\otimes|0\rangle_3.\end{aligned} \tag{22}$$

Third, we turn off the coupling between r_1r_2 and q, the state of the system becomes

$$\begin{aligned}|\Psi_2'\rangle = \frac{1}{2\sqrt{2}}[&|0\rangle_1|0\rangle_2|g\rangle_q + i\,|0\rangle_1|0\rangle_2|e\rangle_q\\ &+ |0\rangle_1|1\rangle_2|g\rangle_q + i\,|0\rangle_1|1\rangle_2|e\rangle_q\\ &+ |1\rangle_1|0\rangle_2|g\rangle_q + i\,|1\rangle_1|0\rangle_2|e\rangle_q\\ &+ |1\rangle_1|1\rangle_2|g\rangle_q - i\,|1\rangle_1|1\rangle_2|e\rangle_q]\otimes|0\rangle_3.\end{aligned} \tag{23}$$

By resonating r_3 and q, we can get the final state of the system as

$$\begin{aligned}|\Psi_f\rangle = \frac{1}{2\sqrt{2}}[&|0\rangle_1|0\rangle_2|0\rangle_3 + |0\rangle_1|0\rangle_2|1\rangle_3\\ &+ |0\rangle_1|1\rangle_2|0\rangle_3 + |0\rangle_1|1\rangle_2|1\rangle_3\\ &+ |1\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|0\rangle_2|1\rangle_3\\ &+ |1\rangle_1|1\rangle_2|0\rangle_3 - |1\rangle_1|1\rangle_2|1\rangle_3]\otimes|g\rangle_q.\end{aligned} \tag{24}$$

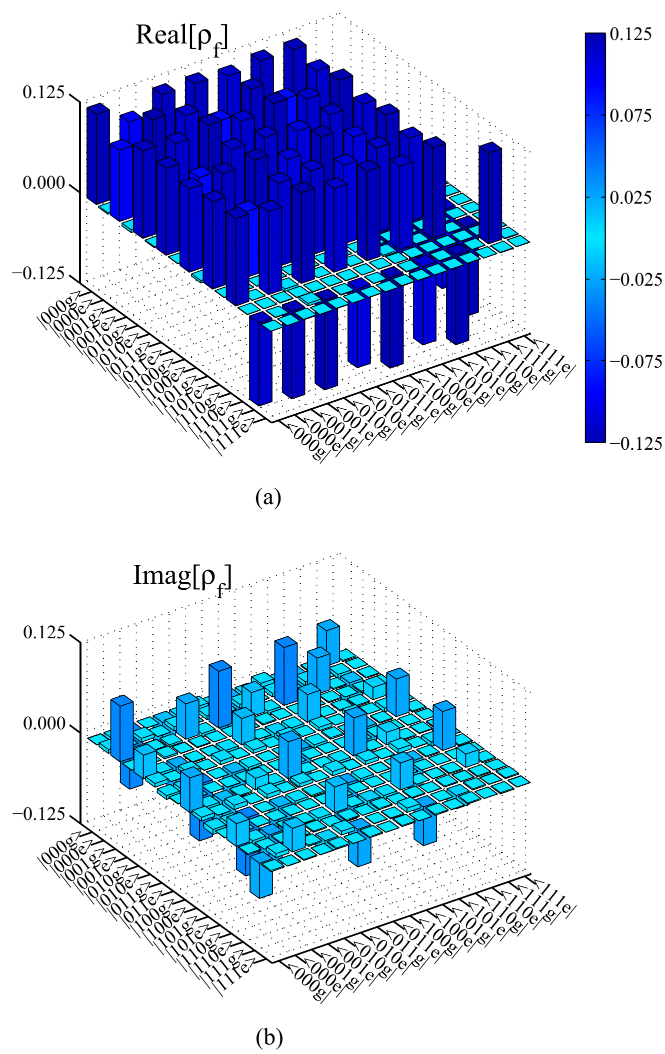

FIG. 5. (Color online) The real part (a) and the imaginary part (b) of the final state $|\Psi_f\rangle$ of the system composed of the three microwave-photon resonators and the superconducting qutrit in our cc-phase gate, respectively.

This is just the outcome of the cc-phase gate operation on r_1, r_2, and r_3 by using r_1 and r_2 as the control qudits and r_3 as the target qudit.

We simulate the evolution for the density operator of the system with the initial state $|\Psi_0\rangle$, and the reduced density operator of the final state $|\Psi_f\rangle$ shown in Eq. (24) is shown in Fig. 5. Here, $\omega^{r_1}/(2\pi) = 6.5$ GHz, $\omega^{r_2}/(2\pi) = 7.5$ GHz, $\omega^{r_3}/(2\pi) = 7.5$ GHz, $\omega^{g,e}/(2\pi) = E_e - E_g = 8.7$ GHz, $\omega^{e,f}/(2\pi) = E_f - E_e = 8.0$ GHz, $g_1^{g,e}/(2\pi) = g_1^{e,f}/(2\pi) = 0.2$ GHz, $g_2^{g,e}/(2\pi) = g_2^{e,f}/(2\pi) = 0.2$ GHz, $g_3^{g,e}/(2\pi) = g_3^{e,f}/(2\pi) = 0.12$ GHz, $\omega^d/(2\pi) = 8.1768$ GHz, and $\Omega = 0.0266$ GHz. From Fig. 5, one can see that the fidelity of our cc-phase gate can reach 92.92% within about 124.64 ns, without considering the decoherence and leakage of the resonators.

IV. DISCUSSION AND SUMMARY

The deterministic approaches to realize the nonlinear interaction between two photons for quantum computation are usually based on the Kerr effect. Here we constructed the local c-phase and cc-phase gates on the resonator qudits in a microwave-photon quantum processor assisted by only one transmon qutrit, resorting to the combination of the number-state-dependent interactions between the transmon qutrit and the resonator qudits and the simple resonant interaction between the qutrit and one resonator qudit. Usually, the processor on microwave-photon systems needs a tunable coupling superconducting qubit or some tunable resonators [59,70,71,74]. The experiments showed that a tunable coupling strength between a superconducting qubit and a superconducting resonator is feasible [24–26]. Some recent experiments were demonstrated for tuning the frequency of a resonator [78–80]. In order to avoid shortening the relaxation time of the qutrit, the processor needs some high-Q resonators. That is, the present c-phase and cc-phase gates are feasible, similar to those in Refs. [59,63,67,71].

In our calculation, the parameters of the transmon qutrit are chosen as the same as those in Ref. [73]. Actually, the coupling strength for the two different transitions of a transmon qutrit and a microwave-photon resonator is asymptotically increased as $(E_J/E_C)^{1/4}$ (for a transmon qubit, $20 < E_J/E_C < 5\times10^4$) [72]. That is, it is reasonable to use the same coupling strength for the two different transitions of a transmon qutrit and a resonator for convenience. The amplitudes of the drive fields for constructing the c-phase and cc-phase gates are too small (compared with the anharmonicity between the two transitions of the qutrit) to induce the influences coming from the higher excited energy levels of the transmon qutrit.

The coherence time of a transmon qubit approaches 0.1 ms [22] and the lifetime of microwave photons contained in resonators are always longer than that of a qutrit [42], which means our gates can be operated several hundreds of times within the lifetime of the processor. In order to evolve the systems from the dress states to the computational states in our schemes for constructing the gates, one needs to tune on or off the interaction between the resonators and the qutrit, the same as in Refs. [59,74]. The quantum error coming from this method in experiment is determined by the technique of the tunable transition frequency of a superconducting resonator or the tunable coupling strength between the qutrit and the resonators. In our calculation, we don't consider the error coming from the preparation of the initial states shown in Eqs. (9) and (19). To prepare the states shown in Eqs. (9) and (19) from the state of the system composed of multiple resonators coupled to a qutrit $\otimes_i|0\rangle_i|g\rangle_q$, one needs to take the single-qubit gate by appling a $\frac{\pi}{2}$ pulse on the qutrit, and the state of the system is changed into $\otimes_i\frac{1}{\sqrt{2}}|0\rangle_i(|g\rangle_q - i|e\rangle_q)$. By resonating the qutrit and the resonator j with the time $t = \frac{3\pi}{2g_j^{g,e}}$, one can obtain the state $\otimes_{i,i\neq j}\frac{1}{\sqrt{2}}(|0\rangle_j + |1\rangle_j)|0\rangle_i|g\rangle_q$ [71]. By repeating the steps to the rest resonators, one can obtain the states shown in Eqs. (9) and (19) $\otimes_i\frac{1}{\sqrt{2^i}}(|0\rangle_i + |1\rangle_i)|g\rangle_q$. The single-qubit gate can be realized with the quantum error of 0.007 ± 0.005 [81] and even smaller than 0.0009 [35], which is too small to influence the fidelity of our gates. By using a qubit to read out the state of the resonator [54,55,58], one can complete fully the tomography of the resonator logic gates [70].

Now, let us compare our c-phase gate on two resonators with the one constructed in Ref. [70]. In Ref. [70], Strauch presented an interesting scheme to construct the c-phase gate on two superconducting resonator qudits. In his work, each of two resonators (A and B) is coupled to an auxiliary three-level transmon or phase qutrit (a and b), and each qutrit should be coupled to each other directly. Moreover, the c-phase gate on two resonators based on the Fock states $|0\rangle$ and $|1\rangle$ is constructed by first using the number-state-dependent interactions between a superconducting qutrit and a resonator qudit (A to a and B to b) twice and then turning on the interaction between two superconducting qutrits. Finally, the gate is completed by repeating the first step. The operation time of this gate is 150 ns. In our work, the c-phase gate is accomplished with two resonators which are coupled to just one transmon qutrit. It is easier to extend our two-resonator c-phase gate to a three-resonator processor assisted by a transmon qutrit. The effects used for constructing our c-phase gate on two resonators include the number-state-dependent interaction between the qubit and one resonator-qudit subsystem and the simple resonant operation between the qubit and another resonator-qudit subsystem. These different characteristics make us get a higher fidelity c-phase gate with a faster operation time. The fidelity of our gate is 99.51% within the operation time of 93 ns.

Different from the effective c-phase gate constructed by Wu *et al.* [71] in which both two resonators are coupled to a two-energy-level charge qubit by the number-state-dependent interactions and it is completed without considering the existence of the third energy level of the charge qubit (the operation time of this gate is 125 ns), our c-phase gate on the two resonators (1 and 2) is accomplished by combination of the photon-number-dependent frequency-shift effect on the transmon qutrit by the first resonator and the simple resonant operation between the qutrit and the second resonator. That is, resonator 2 is used to resonate with the qutrit and resonator 1 is used to complete the selective rotation on the qutrit by using the effect that the transition frequency of the qutrit is determinated by the photon number in only resonator 1, which is simpler than the effect used in Ref. [71]. This different physical mechanism makes us obtain the higher fidelity and faster c-phase gate on the two resonators. Moreover, there are no works about the construction of the cc-phase gate on three microwave-photon-resonator qudits. The different devices in our work make it possible to construct the cc-phase gate on three resonators, far different from the previous proposals [70,71]. The fidelity of our cc-phase gate is 92.92% within the operation time of 124.64 ns.

In summary, we have constructed two universal quantum gates, i.e., the c-phase and cc-phase gates in a microwave-photon quantum processor which contains multiple superconducting microwave-photon-resonator qudits coupled to a Ξ-type transmon qutrit. Our gates are based on the combination of the number-state-dependent interaction between a transmon qutrit and a resonator-qudit subsystem and the simple resonant operation between the qutrit and another resonator-qudit subsystem, and they have a high fidelity in a short operation time. The algorithms of our gates are based on the Fock states of the resonators, and the microwave photon number in each resonator is limited to none or just one. Our universal quantum processor can deal with the quantum computation with microwave photons in resonators.

It is worth pointing out that the techniques for catching and releasing microwave-photon states from a resonator to the transmission line [45] and the single-photon router in the microwave regime [73] have been realized in experiments. The microwave-photon quantum processor can act as an important platform for quantum communication as well.

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