Solution to the Lorentzian quantum reality problem

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The quantum reality problem is that of finding a mathematically precise definition of a sample space of configurations of beables, events, histories, paths, or other mathematical objects, and a corresponding probability distribution, for any given closed quantum system. Given a solution, we can postulate that physical reality is described by one randomly chosen configuration drawn from the sample space. For a physically sensible solution, this postulate should imply quasiclassical physics in realistic models. In particular, it should imply the validity of Copenhagen quantum theory and classical dynamics in their respective domains. A Lorentzian solution applies to relativistic quantum theory or quantum field theory in Minkowski space and is defined in a way that respects Lorentz symmetry. We outline a solution to the nonrelativistic and Lorentzian quantum reality problems and associated generalizations of quantum theory.

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I. INTRODUCTION

Quantum theory is a mathematically beautiful theory that unifies all of known physics with the exception of gravity. Its probabilistic predictions for experimental outcomes have been verified for a very large range of physical phenomena and contradicted by no experiment. Yet, as John Bell so eloquently and persuasively argued [1], we do not know what precisely it is that quantum probabilities are probabilities of. We do not have a mathematically precise description of what Bell called [2,3] the "beables" for quantum theory. That is, we do not have a sample space of events, or histories, or paths, or other mathematical objects, on which the quantum probability distribution is defined. This is the quantum reality problem, sometimes referred to as the measurement problem, rather misleadingly from a modern perspective, since few physicists now believe that the fundamental laws of nature involve measuring devices *per se* or that progress can be made by analyzing them. As Bell emphasized, the quantum reality problem becomes particularly conceptually problematic when we impose the natural condition that any solution should respect the symmetries of special relativity. We focus here on solutions to the *Lorentzian quantum reality problem*, i.e., solutions that have this property.

As Bell also stressed [1], mathematical aesthetics are not the main motivation for solving the quantum reality problem. The motivation is the following. On the one hand, the impressive successes of quantum theory and the lack of compelling alternatives make it natural to try to treat quantum theory as fundamental and so to derive everything else in physics from quantum theory. On the other hand, it appears to us that we live in a quasiclassical world, in which macroscopic variables are most of the time approximately governed by deterministic equations of motion, but are also affected by random events of quantum origin. Moreover, it appears as though this quasiclassical world emerged from an initial quantum state with no initial quasiclassical properties. Given a well-defined probabilistic version of the quantum theory of closed systems, we can hope to explain these features from within quantum theory and indeed to sketch a coherent and unified account of cosmology, classical and quasiclassical dynamics, and quantum theory. Without one, we cannot rigorously derive classical or quasiclassical physics from quantum theory nor give a coherent treatment of cosmology from within quantum theory.

The once-standard Copenhagen interpretation of quantum theory explicitly accepted these limitations. It is the hope of going further and giving a unified framework that includes all of modern physics that motivates the ongoing search for a solution.

The first well-known attempt to address the quantum reality problem directly was the pilot wave theory of de Broglie and Bohm [4,5], in which the beables are particle trajectories whose evolution is defined by the quantum wave function by a guidance equation. However, de Broglie and Bohm's models apply to nonrelativistic quantum mechanics and are inconsistent with special relativity. No fundamentally relativistic generalization of the models has been found, nor is there a convincing extension to quantum field theory. Many (though not all) physicists also find de Broglie and Bohm's trajectories and guidance equations rather mathematically unnatural and inelegant additions to quantum theory.

Nonrelativistic dynamical collapse models [6,7] attempt to give another story about physical reality that is consistent with experiment to date at the price of changing the dynamics and hence the experimental predictions of quantum theory. (For some attempts in the direction of relativistic collapse models, see [8–11].) While scientifically interesting, these and other generalizations of quantum theory do not address the main question we focus on here, namely, whether we can find a mathematically precise description of reality consistent with standard quantum theory.

Another line of thought, initiated by Everett, suggests that quantum theory is deterministic and that pure unitary quantum evolution holds at all times. The problems with this idea and with the many incompatible proposals for some form of "many worlds" quantum theory that it has inspired continue to be

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debated [12]. Still, two relatively uncontroversial points can be made. First, since, according to most of those who advocate some version of many-worlds quantum theory, quantum theory is fundamentally deterministic and the appearance of quasiclassical physics is supposed to arise as an approximation via decoherence, no mathematically precise sample space and probability distribution emerges. Second, many-worlds theories are radically different types of scientific theory from standard "one-world" versions of quantum theory (or indeed from anything previously considered in science) and give a qualitatively different (and fantastically weird) description of reality.

Many critics are also unpersuaded either that the appearance of quasiclassical physics can, in fact, be explained by decoherence or that a fundamentally deterministic theory can account for an approximate higher level description that explains the empirical appearance of Born rule probabilities (see, e.g., [13–15]). However, even convinced advocates of many-worlds quantum theory, who believe that many-worlds theories can give a complete explanation of the Born rule, the appearance of probabilities and of quasiclassicality and everything else described by standard physics should still be (and generally are) interested in whether we *need* to invoke many-worlds ideas to do all this.

In short, there remains an unresolved and intellectually fascinating question fundamental to our understanding of quantum theory. Namely, is there a mathematically precise solution to the quantum reality problem, consistent with the symmetries of special relativity, that gives a probabilistic description of one physical world, consistent with the quasiclassical combination of classical and quantum physics that we actually observe? Or, if this is too much to presently hope, given that any fully realistic model of quasiclassical physics would need to describe the real-time evolution of complex physical structures within quantum field theory and quantum gravity, is there at least a conceptually clear route to defining such a solution?

This paper answers this last question positively. The solution described here uses the strategy of inferring finite time beables from asymptotic behavior and many of the other ideas set out in Ref. [16], but is simpler than the proposals made in that paper.

II. THE REALITY PROBLEM FOR NONRELATIVISTIC QUANTUM MECHANICS

We should note at the start that the intuitions underlying our proposed solution come from the properties of relativistic quantum field theories and from the current tentative understanding of the likely asymptotic state of the universe within presently favored cosmological models. Thus, although the ideas are described more simply in nonrelativistic quantum mechanics than in relativistic theories, nonrelativistic quantum mechanics is not ultimately the most natural setting for them, and our solution does not necessarily give intuitively appealing descriptions of reality for simple models of nonrelativistic systems without further assumptions. We discuss further the underlying intuitions and the assumptions that need to be introduced into models to reflect them, below, after giving the proposal in general form.

Suppose that we have a system of N particles, including b indistinguishable bosons and f indistinguishable fermions, with $b + f \leqslant N$, and $(N - b - f)$ distinguishable particles. (These choices are made purely to simplify our discussion, which can easily be extended to allow M_b types of bosons and M_f types of fermions for any integers $M_b, M_f \geqslant 0$.) We take $b, f > 1$ (otherwise, we treat the relevant particle as distinguishable) and label the bosons by $\{1, \ldots, b\}$ and the fermions by $\{b + 1, \ldots, b + f\}$; if $N - b - f > 0$, we label the remaining distinguishable particles by $\{b + f + 1, \ldots, N\}$. We also suppose that the N particles have some natural division into two classes, which we label 1 and 2. All indistinguishable particles of the same type belong to the same class. Both classes contain significant numbers of particles, and there are significant interactions between the particles in class 1 and those in class 2, which allow the final states of the class 1 and class 2 particles to be highly correlated.

Our proposal involves (as mathematical abstractions) final and intermediate time measurements of the masses at points in space, which are functions of particle position operators. Position (more precisely, the mass at a given position) thus plays a special role as a mathematically preferred observable. We thus suppress spin and any other internal degrees of freedom to simplify the notation; note that our proposal requires no additional spin or other measurements to be introduced in systems where they are relevant.

The system's position space wave function has the appropriate statistics. If ρ_b is a permutation of $\{1, \ldots, b\}$ and ρ_f is a permutation of $\{b + 1, \ldots, b + f\}$, write $\rho = \rho_b \otimes \rho_f \otimes I_{N-b-f}$ for the corresponding permutation of $\{1, \ldots, N\}$. Then we have

$$\psi(x_{\rho(1)}, \ldots, x_{\rho(N)}) = \epsilon(\rho_f)\psi(x_1, \ldots, x_N),$$

where ϵ is the sign of ρ_f.

We treat this as a closed system without external intervention, which comes into existence at $t = 0$ and continues to a final time $t = T$, at which point time and physics end. This is a mathematical device, not a fundamental assumption about nature. Later we consider the limit $T \to \infty$, which gives a more conventional (although in this model still nonrelativistic) picture in which physics begins at some point in the past and continues forever thereafter.

Suppose we are given the initial state $|\psi(0)\rangle$ at $t = 0$ with wave function

$$\psi(x_1, \ldots, x_N; 0) = \langle x_1, \ldots, x_N \mid \psi(0) \rangle$$

and that we are given a Hamiltonian H. Quantum theory then gives the Schrödinger evolution

$$|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(0)\rangle,$$

or, in terms of wave functions,

$$\psi(x_1, \ldots, x_N; t) = \exp(-iHt/\hbar)\psi(x_1, \ldots, x_N; 0).$$

Now consider, for each of classes 1 and 2, the corresponding mass-density function defined by a mass-weighted sum of position operators,

$$\rho^{(j)}(x; t) = \sum_{i \in C^j} m_i \rho_i(x, t),$$

for $j = 1,2$. Here the set C^j denotes the labels of all particles in class j, and

$$\rho_i(x,t) = \int dx_1 \cdots dx_{i-1} dx_{i+1} \cdots dx_N \; |\psi(x_1,\ldots x_{i-1},x,x_{i+1},\ldots,x_N;t)|^2 = \langle \psi(t) \,|\, P_i^x \,|\, \psi(t)\rangle, \tag{1}$$

where

$$P_i^x = I_1 \otimes \cdots I_{i-1} \otimes |x\rangle_i \langle x|_i \otimes I_{i+1} \otimes \cdots \otimes I_N$$

is the formal projection operator onto the value $x_i = x$ of the ith coordinate. Note that, while P_i^x is not even formally well defined if i is a bosonic or fermionic label, the sums $\sum_{i=1}^{b} m_i P_i^x$ and $\sum_{i=b+1}^{b+f} m_i P_i^x$ are. (The identical bosons have equal masses, $m_1 = m_2 = \cdots = m_b$; similarly, $m_{b+1} = m_{b+2} = \cdots = m_{b+f}$.)

The operators $\sum_i m_i P_i^x$ and $\sum_i m_i P_i^y$ commute. So, formally, we can consider the effect of a simultaneous measurement of all such operators at time $t = T$. This produces a possible final time distribution

$$\rho_f(x,T) = \rho_f^{(1)}(x,T) + \rho_f^{(2)}(x,T) = \sum_i m_i \delta(x - y_i),$$

randomly chosen from the sample space of all distributions with total mass $\sum_i m_i$, via the probability distribution defined by $|\psi(0)\rangle, H$ and the Born rule. This is the first ingredient in our construction of a beable description of nonrelativistic quasiclassical reality. We take the randomly chosen pair $\rho_f^{(1)}(x,T)$ and $\rho_f^{(2)}(x,T)$ to define the "real world" that is chosen from among the sample space of possible worlds that could arise given the initial state and Hamiltonian. This final time measurement is not meant to be thought of as carried out by any external system: We treat the N particle universe as a closed system with no external observers or devices. It is simply a mathematical operation that allows a precise description of the sample space of possible worlds and a corresponding probability distribution.

Next we consider the expected value of $\rho^{(j)}(x,t)$ (for $0 < t < T$), given the initial state $|\psi(0)\rangle$ and Hamiltonian H, when we condition on the outcome of our final time measurements producing the final distribution $\rho_f^{(\bar{j})}(x,T)$. Here $\bar{j}$ denotes the other class to j; thus, we consider expected values of $\rho^{(1)}(x,t)$ given the final distribution $\rho^{(2)}(x,T)$ and vice versa.

Since we have a postselected final outcome, this expectation value depends on precisely which set of commuting operators are simultaneously measured. For each point in time t, we consider a simultaneous measurement of $\rho^{(j)}(x,t)$ at all points $x \in R^3$. The measurements for each t are considered separately.

Formally, the relevant expectation value is then given by the Aharonov-Bergmann-Lebowitz rule [17], extended to the case where the intermediate and final projection operators may be degenerate. We write $\{M_k^j : k \in K\}$ for the possible nonzero outcomes of measuring the mass of particles in class j at a point in our system, so that each $M_k^j = \sum_{l\in L} m_l$ for some nonzero subset $L \subseteq C^{(j)} \subseteq \{1,\ldots,N\}$. Write $P_M^{x,dV}$ for the projection onto the space of states with mass M in a volume element dV around the point x. Then we have

$$\langle \rho^{(j)}(x,t)\rangle = \lim_{dV\to 0} \frac{1}{dV} \sum_k M_k \frac{A^j(M_k,x,t,T)}{B^j(t,T)}, \tag{2}$$

where

$$\begin{aligned} A^j(M_k,x,t,T) &= \sum_{\{k_1,\ldots,k_r : k_i \in C^{(j)}\,\forall\, i \text{ and } \sum_{i=1}^{r} M_{k_i} + M_k = M^{(j)}\}} \int dy_1 \cdots dy_r \mathrm{Tr}\Big\{ P_f \exp[-iH(T-t)/\hbar] P_{M_k}^{x,dV} \\ &\times \prod_{i=1}^{N-1} P_{M_{k_i}}^{y_i} \exp(-iHt/\hbar) P_0 \exp(iHt/\hbar) P_{M_k}^{x,dV} \\ &\times \prod_{i=1}^{N-1} P_{M_{k_i}}^{y_i} \exp[iH(T-t)/\hbar] \Big\}, \end{aligned} \tag{3}$$

with the y_i integrals taken over all of space outside dV, and

$$\begin{aligned} B^j(t,T) = &\sum_{\{k_1,\ldots,k_{r+1} : k_i \in C^{(j)}\,\forall i \text{ and } \sum_{i=1}^{r+1} M_{k_i} = M^{(j)}\}} \int dy_1 \ldots dy_{r+1} \\ &\times \mathrm{Tr}\Big\{ P_f \exp[-iH(T-t)/\hbar] \\ &\times \prod_{i=1}^{r+1} P_{M_i}^{y_i} \exp(-iHt/\hbar) P_0 \exp(iHt/\hbar) \\ &\times \prod_{i=1}^{r+1} P_{M_i}^{y_i} \exp[iH(T-t)/\hbar] \Big\}. \end{aligned} \tag{4}$$

Here $P_0 = |\psi(0)\rangle\langle\psi(0)|$ and P_f is the projection onto the space of states for which the class $\bar{j}$ particles have final mass density $\rho_f^{\bar{j}}(x,T)$, and

$$M^{(j)} = \sum_{l\in C^{(j)}} m_l.$$

That is, our definition of an expectation value for the mass density of class j particles at intermediate times uses only the postselected data from the other class, $\bar{j}$.

That is, for an infinitesimal volume element dV around x

$$\langle \rho^{(j)}(x,t)\rangle dV = \frac{\sum_k M_k \sum \mathrm{wt}(\text{all outcomes including mass } M_k \text{ in volume } dV \text{ around } x)}{\sum \mathrm{wt}(\text{all outcomes})},$$

where wt( ) denotes the pre- and postselected probability weights used in the above expression, and the outcomes considered are of simultaneous mass measurements of all the class j particles at all points $y \in R^3$. We could also include projections onto the zero mass eigenspaces at all points other than $y_1, \ldots, y_{r-1}, y_r$ in the denominator, and at all points other than $y_1, \ldots, y_{r-1}, x$ in the numerator. However, these would not change the expressions here, since we have a fixed number of positive mass particles of total mass $\sum_{l \in C^{(j)}} m_l$.

This is the second ingredient in our construction. Given the final outcomes $\rho_f^{(j)}(x,T)$, we take the expressions

$$\rho_T^{(j)}(x,t) = \langle \rho^{(j)}(x,t) \rangle$$

just calculated (using the postselected final conditions for the complementary class $\bar{j}$ to define the beable for class j) to define the beables at position x and time t for a universe in which physics runs from time 0 to time T.

The full set of beables describing reality for our first model, in which all physics takes place between times 0 and T, is thus given by

$$\left\{ \rho_T^{(j)}(x,t) : 0 < t < T, x \in R^3, \; j = 1,2 \right\}. \qquad (5)$$

To make further progress, we need a key assumption. This is that quantum physics in our model universe involves nontrivial interactions between the particles in the two classes at finite times, creating effective records, but becomes, in a sense to be characterized more precisely, asymptotically trivial as $t \to \infty$.

A possible intuition that would support this assumption is that, while initially the particles often are localized in the same region and interact, eventually all particles that can decay will have decayed, all particles that are capable of interacting with one another either do interact or become more and more widely separated, nongravitational interactions become rarer and rarer, and the asymptotic evolution is effectively described by a free quantum field theory. This intuition relies on being able to think of the asymptotic physical state as composed of elementary particles, or at least as behaving qualitatively as though it were. It is supported by some cosmological scenarios that are presently taken seriously, for example (and most cleanly), in "big rip" scenarios. To model something like this in the nonrelativistic setting requires in particular that interactions between particles in class 1 and those in class 2 are initially significant but are switched off, or become negligible, at large times.

A weaker intuition, still adequate to justify the assumption, is that the outcome of a typical indeterministic quasiclassical event leaves an indelible asymptotic record in the mass densities of one or both classes (both being required if the event itself is quasiclassical with respect to variables defined by both classes). That is, in principle, a measurement of the class mass densities at large final time T allows one to infer all initially undetermined outcomes, and thereby the entire history of quasiclassical physics, which is encoded in the inferred mass-density distributions at times between 0 and T.

For example, suppose that the Schrödinger equation creates what is traditionally thought of as a measurement event at time $t < T$. That is, suppose that, around time t, a quantum system interacts with an apparatus whose pointer initially has a single approximately localized position and creates a superposition of two states corresponding to macroscopically separated approximately localized pointer positions at time $t + \delta$. Suppose also that the pointer comprises particles in class 1 and its environment contains particles in class 2 that interact with it. In a fairly general class of such models, the position degrees of freedom of many particles in the environment typically become coupled to the pointer positions, and produce effective records (i.e., multiply redundant subsystems that are persistently correlated with the original data) of those positions in the environment. The class 2 mass-density measurement at the final time T distinguishes different states of these records and so indirectly measures the pointer position at times soon after $t + \delta$, whether or not the pointer itself remains intact or quasiclassical indefinitely.

Of course, the point of giving a precise definition of reality in terms of beables is to go beyond intuition. In our models, a definite quasiclassical measurement event is ultimately a higher level description, which can generally only be approximately characterized in the quasiclassical theory based on the beables. Such an event occurs if and only if it leaves effective records in the final time mass-density measurement(s) for the relevant class(es). Thus, a hypothetical experiment successfully demonstrating interference between two paths of a macroscopic object would not produce definite events selecting one of the paths, since the interference implies near-perfect isolation of the beams from all other particles, and hence a typical final time mass-density measurement outcome will give almost no path information.

Intuitions aside, then, the precise assumption we need to make is that the probability distribution for the possible configurations of beables describing reality (5) within each fixed time interval $[0,t]$, for any $t < T$, has a well-defined limit as $T \to \infty$. Given models in which this holds, we can then test whether typical beable distributions represent quasiclassical reality appropriately [18].

Our asymptotic assumption translates as follows. Let $C_t^{(j)}$ (for $j = 1,2$) be any coarse-grained subsets of the sets of continuous functions

$$\{\rho^{(j)}(x,t') : x \in R^3, 0 \leqslant t' \leqslant t\}$$

obeying (in our nonrelativistic model) the constraint

$$\int d^3x \rho^{(j)}(x,t') = \sum_{l \in C^{(j)}} m_l$$

for any t'. Let

$$\mathrm{Prob}_T(C_t^{(1)}, C_t^{(2)})$$

be the joint probability that $\{\rho_T^{(j)}(x,t') : x \in R^3, 0 \leqslant t' \leqslant t_f\}$ belong to $C_t^{(j)}$ for $j = 1$ and 2, given our constructed probability density function on the set of possible $\rho_T^{(j)}$. Then, assuming that

$$\mathrm{Prob}_\infty\left(C_t^{(1)}, C_t^{(2)}\right) = \lim_{T \to \infty} \mathrm{Prob}_T\left(C_t^{(1)}, C_t^{(2)}\right)$$

exists, we define this expression to be the probability that reality up to time t is described by a time-evolving mass distribution for class j particles belonging to $C_t^{(j)}$, for $j = 1$ and 2. This, together with the additivity of the probability measure on finite disjoint measurable subsets of the sample space, completes the definition of the probability distribution on the beable configurations, i.e., on the possible descriptions of reality, in this nonrelativistic model.

We consider other nonrelativistic beable models below. First, though, we discuss relativistic generalizations of the above model.

III. THE REALITY PROBLEM FOR RELATIVISTIC QUANTUM THEORY

It is not presently possible to give a completely rigorous discussion of the reality problem for any physically relevant relativistic quantum field theory in Minkowski space, because no version of relativistic quantum field theory is well-enough understood to allow quasiclassical equations to be rigorously derived from first principles. We cannot even give a fully mathematically rigorous quantum field-theoretic description of any realistic physical experiment, for example, of electrons passing from a source through a two-slit region and registering at detectors. Evidently, then, we cannot hope to prove rigorously that a particular mathematical construction attached to such a description gives a description of physical reality with any given desired property.

However, we *can* aim to separate the conceptual issue posed by the reality problem from the technical issues that prevent us from carrying out complete calculations describing realistic experiments, or other phenomena characterized by quasiclassical physics, in quantum field theory. We can also hope to make it plausible that a proposed solution correctly describes quasiclassical reality in realistic models. This is the strategy we follow here.

We now suppose that the initial state $|\psi_0\rangle$ is given on some spacelike hypersurface S_0 and that some relativistic unitary evolution law is given. The Tomonaga-Schwinger formalism allows us to define formally the evolved state $|\psi_S\rangle$ on any hypersurface S in the future of S_0 via a unitary operator $U_{S_0 S}$. These future hypersurfaces S play the same role in our relativistic formalism as the final time coordinate, $t = T$, does in the nonrelativistic case.

As in the nonrelativistic case, we may assume that the relevant fields are naturally divided into two (or more) classes. Our reason for employing this construction in the nonrelativistic case is the unphysical nature of nonrelativistic propagators, which imply that propagations from a single space point x to any other point y in any time t are equally probable.

Since relativistic propagators encode the causal structure of the underlying space-time, and tend rapidly to zero outside the future light cone, it seems to us an open question—which depends on the details of the fields and their interactions and the underlying assumptions about the initial state—whether dividing the particles up into classes is necessarily required in the case of relativistic field theory. Our relativistic constructions could be considered for a single class of particles, in which case the stress-energy expectations at intermediate points would be defined by postselecting on the total final hypersurface mass-energy density operator. We intend to explore this possibility further in future.

We discuss here the case of two classes, $j = 1$ and 2, analogous to the discussion given explicitly in the nonrelativistic case above. Our definitions can easily be applied to the case of a single class. Both the relativistic and nonrelativistic definitions can also easily be extended to other postselection rules, involving more than two classes; we discuss these possibilities in the next section.

We use the following natural generalization of the final time measurements of mass density in our nonrelativistic models. For any given smooth hypersurface S in the future of the initial hypersurface S_0, we consider the effect of joint measurements of the local mass-energy density operators for classes 1 and 2, $T_S^{(j)}(x) = T_{\mu\nu}^{(j)}(x) n_\mu n_\nu$, carried out at each point $x \in S$, where n_μ is the forward-pointing timelike unit 4-vector orthogonal to the tangent plane of S at x. We assume here the two mass-density operators commute.

This gives us a probability distribution on possible mass-energy distributions $t_S^{(j)}(x)$ on S. Conditioned on any given outcome of the $t_S^{(j)}(x)$, we wish to calculate expectation values of the stress-energy tensors for the field classes, $\langle T_{\mu\nu}^{(\bar{j})}(y)\rangle$, at each point y between S_0 and S.

We again define these expectation values using the Aharonov-Bergmann-Lebowitz formalism. As in the nonrelativistic case, we need to take appropriate limits. Again, because we have a postselected final outcome, the expectation value $\langle T_{\mu\nu}^{(\bar{j})}(y)\rangle$ depends on the other commuting observables that we consider as jointly measured. Because we no longer have an absolute time coordinate, we need to define the relevant measurement more carefully.

For any point y in the future of S_0, define the *effective past boundary* $\Lambda(y)$ of y in our model to be $\Lambda_0(y) \cup S_0(y)$, where $\Lambda_0(y)$ is the set of points in the lightlike past of y and the future of S_0, and $S_0(y)$ is the set of points in S_0 not in the past light cone of y. Let $\{S_i(y)\}$ be a sequence of smooth spacelike hypersurfaces that include y such that

$$\lim_{i\to\infty} S_i(y) = \Lambda(y).$$

Consider a joint measurement of $T_{\mu\nu}^{(\bar{j})}(y)$ and of $T_{S_i(y)}^{(\bar{j})}(x)$ for all $x \in S_i(y)$ other than y. Given the initial state $|\psi_0\rangle$ on S_0 and the relevant postconditioned final measurement outcomes $t_S^{(j)}(x)$ on S, the ABL rule gives a value

$$\left\langle T_{\mu\nu}^{(\bar{j})}(y)\right\rangle_{S_i(y)}$$

for the pre- and postselected stress-energy tensor expectation value which, as our notation suggests, may, in general, depend on $S_i(y)$. It is important to note that, as in the nonrelativistic case, this expectation value depends on the full specification of the measurement. To apply the ABL rule, we need to include all possible outcomes of all the measurements of $T_{S_i(y)}^{(\bar{j})}(x)$. We comment further on this later.

Finally, we define

$$\left\langle T_{\mu\nu}^{(\bar{j})}(y)\right\rangle = \lim_{i\to\infty}\left\langle T_{\mu\nu}^{(\bar{j})}(y)\right\rangle_{S_i(y)},$$

assuming both that this limit exists and that it is independent of the chosen limit sequence $\{S_i(y)\}$.

For a toy model in which all of physics takes place between S_0 and S, the two functions $t_S^{(j)}(x)$ on S define the particular real world that was randomly selected. The pre- and postselected expectation values $\langle T_{\mu\nu}^{(\bar{j})}(y)\rangle$, for y between S_0 and S and $j = 1$ and 2, are the beables corresponding to the given real world and define physical reality between S_0 and S in our model.

We then consider the asymptotic limit in which S tends to the infinite future of S_0. Suppose that S_1 is some fixed hypersurface in the future of S_0. Let $C^{(k)}_{S_1}$ (for $k = 1$ and 2) be any coarse-grained subsets of the sets of continuous tensor functions $\{t^{(k)}_{\mu\nu}(x) : x \in R^4, S_0 < x < S_1\}$, where the notation $S_0 < x < S_1$ means that x lies in the future of some point in S_0 and the past of some point in S_1. Let

$$\mathrm{Prob}_S\big(C^{(1)}_{S_1}, C^{(2)}_{S_1}\big)$$

be the probability that $\{t^{(k)}_{\mu\nu}(x) : x \in R^4, S_0 < x < S_1\}$ belongs to $C^{(k)}_{S_1}$, for $k = 1$ and 2, given our constructed probability density function on the set of possible functions $T^{(j)}(x) : S \to R$. Then, assuming that

$$\mathrm{Prob}_\infty\big(C^{(1)}_{S_1}, C^{(2)}_{S_1}\big) = \lim_{S\to\infty}\big[\mathrm{Prob}_S\big(C^{(1)}_{S_1}, C^{(2)}_{S_1}\big)\big]$$

exists, we define this to be the probability that reality between S_0 and S_1 is described by time-evolving mass distributions belonging to $C^{(k)}_{S_1}$ for $k = 1$ and 2. This completes our proposed description of reality in this relativistic model.

Note that an interesting alternative model can be defined by using the effective future boundary (defined analogously) in place of the effective past boundary. Again, this requires defining the expectation value at x via a limit using measurements on spacelike hypersurfaces that tend to the effective future boundary of x.

Another interesting possibility to explore, in discrete finite lattice models of space-time, would be to define the expectation value at x via a measurement on the first (or last) spacelike hypersurface through x from a foliation defined by stochastic forward time evolution from S_0, as in [19].

IV. DISCUSSION

We have described a way of defining a mathematically precise description of physical reality that is not only consistent with standard nonrelativistic quantum mechanics, but also involves only familiar quantities that are simply defined within the theory, namely, expectation values of mass density. In contrast to Everett's ideas, this solution to the reality problem describes a randomly chosen single real physical real world, selected from a well-defined probability distribution in an entirely standard and unproblematic way. In contrast to de Broglie-Bohm theory, we believe our solution will appear mathematically natural to anyone familiar with quantum theory. In contrast to dynamical collapse models, our solution requires no change to quantum dynamics.

We have also extended this to a Lorentz covariant solution of the quantum reality problem for quantum field theory in Minkowski space. As in the nonrelativistic case, this solution requires assumptions about the asymptotic behavior of solutions to quantum dynamics given a realistic unitary evolution law and initial conditions.

Our asymptotic assumptions can be tested directly in reasonably complex models in the nonrelativistic case, with the caveat we noted above: To be reasonable tests, such models need to include assumptions that reflect the underlying field-theoretic and cosmological intuitions. Relativistic quantum field theory is not itself rigorously enough developed to allow either our asymptotic assumptions or the beable configurations they are intended to define to be directly calculated for complex systems. Our solution to the reality problem in this case thus involves formal definitions. It could, however, still be tested in hybrid toy models in which, for example, the asymptotic early and late time states are taken to have fixed finite particle number.

Of course, *no* proposal for solving the reality problem in relativistic quantum field theory can be fully rigorously tested, given our presently limited understanding of the latter. At present, the best one can hope for is to show that there is a route to a solution with no evident conceptual obstacles, and this we claim to have achieved. Our proposal's ability to reproduce quasiclassical physics, and the limiting behaviors it requires, can be tested in toy models.

Relativistic quantum theory has been, purportedly, one of our two fundamental theories of nature and may yet subsume the other—general relativity—in some future quantum theory of gravity. Yet, to date, it has been completely unclear whether it admits *any* conceptually clear description of physical reality or allows a conceptually clear derivation of classical dynamics or other higher-level theories. This has left serious questions over its status as a fundamental theory—in Bell's words, it has seemed to "carry the seeds of its own destruction"—and led Bell and many others to suspect that these problems can only be solved by a deeper theory with different dynamics and experimental predictions. Replacing these fundamental conceptual problems with technical questions about asymptotic behavior—in a theory that has in any case always been understood to have deep unresolved technical questions—seems to us a considerable advance.

We do not know for certain whether some appropriately further extended version of our asymptotic assumptions holds true in realistic cosmologies that include a theory of gravity, or, *a fortiori*, whether it holds true in our universe. However, the essential idea that final states are asymptotically well-defined superpositions of states of different mass density configurations is at least consistent with some standard cosmological pictures. It is also consistent with the standard intuition that quantum field theory should be understood as describing processes from which asymptotically well-defined particle states emerge.

Modulo these caveats, we believe our solution method is currently the most promising way of obtaining a physically sensible description of a single quasiclassical world consistent with quantum theory and special relativity and plausibly consistent with gravity and cosmology.

The method could, of course, be applied to other physical quantities, and so our solution is not unique. For example, probability or charge densities could be used instead of mass densities in the nonrelativistic case. In the relativistic case, the final measurements could be of $J_\mu n_\mu$, and the pre- and postselected expectation values of the electromagnetic 4-current J_μ could be used instead of that of the stress-energy tensor to define the real beables.

Other possibilities could also be considered. Nonetheless, relatively few options seem particularly natural, and among these, mass density (in the nonrelativistic case) and the stress-energy tensor (in the relativistic case) seem to us the most natural. A strong additional motivation for focusing on these options is that they suggest new ideas to explore in unifying quantum theory with gravity. We discuss these further below.

There are also various ways in which the particles (in the nonrelativistic case) or fields (in the relativistic case) could naturally be divided into two or more classes, and various postselection rules that could be considered. For example, one might take the classes to be bosons and fermions, or massive and massless particles (or, more speculatively, matter and gravitational fields, or ordinary and dark matter, in the appropriate contexts). Which choice(s) of classes are most natural depends on the Hamiltonian and the asymptotic form of the final state (and thus also on the initial state). Any given physical theory in which these are specified should allow relatively few options that seem particularly natural.

In the variants of our model in which several classes are considered, one natural rule for defining the class j mass densities is to postselect on the final outcomes for all the other classes; again, other rules could be considered. Once again, though, relatively few options for choosing classes, or postselection rules, are likely to seem particularly natural, given a specific theory.

One other variant of our model that is worth noting is that in which there are two classes of particles or fields, but only the mass density (respectively, mass-energy density) for one of them defines beables. While this seems less natural if the ultimate aim is to couple the mass(-energy) beables directly to a quasiclassical gravitational field, it seems adequate as a solution to the quantum reality problem *per se*. A quasiclassical picture of reality can seemingly be described adequately in terms of the mass-energy densities of fermions or massive particles, for example.

A. Implications for earlier approaches to the quantum reality problem

Although admittedly incomplete, this work raises, in our view, significant questions about previous approaches to the reality problem. For example, why resort to de Broglie-Bohm theory, with its inelegant combination of particlelike trajectories guided by an evolving quantum wave function, if a solution to the nonrelativistic quantum reality problem exists that uses only simple quantities that arise naturally in quantum theory? The case against de Broglie-Bohm theory seems all the stronger when we consider the relativistic reality problem, and the fundamental conceptual problems that arise when one tries to define any fully Lorentz covariant version of de Broglie-Bohm field theory.

Similarly, why resort to many-worlds ideas, if there is a simple one-world solution to the reality problem? Why try to deal with the problem of the appearance of quasiclassicality in many-worlds quantum theory, and the necessary imprecision in defining the branching worlds, when we can give a simple picture with a single, precisely defined quasiclassical world? Why struggle with what seems to many (e.g., [13–15]) the hopeless task of trying to make sense of probability in a deterministic many-worlds theory if a straightforwardly probabilistic one-world description is available?

Moreover, if there is a reasonably natural way of solving the reality problem within standard quantum mechanics, do we need to consider collapse models, with their *ad hoc* assumptions and extra parameters? The question seems even more apt given that this solution also extends naturally to relativistic quantum theory and—while admittedly not rigorously defined in this context—still appears to pose fewer technical or conceptual problems than attempts at relativistic generalizations of collapse models.

Bell said [20] of Ghirardi-Rimini-Weber's original discrete dynamical collapse model [7]: "I am particularly struck by the fact that the model is as Lorentz invariant as it could be in the nonrelativistic version. It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance." The ideas for a solution to the reality problem outlined in this paper take away the ground of my own prior hunch that any exact Lorentz invariant formulation of quantum theory must *necessarily* alter the dynamical equations (as the Ghirardi-Rimini-Weber theory and other dynamical collapse models do). Given the extraordinary beauty of both special relativity and quantum theory this prompts the following question: If we can solve the reality problem and retain both theories intact, (why) would we want to consider alternatives that break one or the other?

B. Generalizations of quantum theory

1. Generalizations using beable guided quantum theory

This last question has some real force. In particular, if the ideas outlined here work, then the case for dynamical collapse models does seem weakened. However, it is by no means purely rhetorical. There *are* still good reasons for continuing to explore generalizations of quantum theory, and indeed the solutions to the quantum reality problem described above also suggest intriguing new directions for such exploration.

One entirely uncontroversial motivation is that, however beautiful quantum theory appears, and even if the reality problem and all other conceptual and technical issues can be resolved within standard quantum theory, it still may turn out not to be the final theory of nature. We want to test our best current theories as strongly as we can. To do this, we need alternatives against which to test it; ideally, we would like parametrized classes of alternatives to quantify the extent to which it has been tested. Such alternatives need not be as beautiful or compelling as our best theory; indeed, almost by definition, they will not be. They can still serve a valuable role as foils, or, to be open minded about it, as ways of pointing out domains in which our best theory might, in fact, break down, even if not necessarily quite for the reasons those alternative theories suggest.

There is, though, also a strong case for taking generalizations of quantum theory seriously on their own terms [21]. Any solution to the quantum reality problem defines a probability distribution on configurations of beables. It is perfectly logically consistent for this distribution to be defined only by the initial conditions and quantum dynamics, as our solutions are. However, there is, arguably, something oddly epiphenomenal about the status of the beables in such a theory. On the one hand, they are the building blocks of physical reality. On the other hand, they seem to play a mathematically secondary role to that of the evolving quantum state. It determines their probability distribution, while they have no effect on it.

Of course, it could be that nature is described this way. It is hard to know just how much weight to put on the intuition that physically crucial quantities in a fundamental theory should play a more central role in the mathematics [22]. Still, the intuition is there. It also motivates a class of generalizations of quantum theory, which moreover suggest a new way of thinking about quantum theory and gravity. So, whether or not the underlying intuition is fundamentally right, it suggests potentially valuable new directions for theoretical physics.

Recall that, given initial conditions and dynamics, our solution defines a probability distribution on configurations of beables—in the form of space and time-dependent mass-density or stress-energy tensor expectation values—that define reality. Any such construction can be generalized by taking the probability distribution to depend not only on the initial conditions and dynamics but also on the beable configuration itself [21]. To give just one example among countless possibilities, the probability of configurations could be enhanced or suppressed depending on some global measure of its uniformity over time. Different generalizations can be obtained from versions of our solution involving charge density or other quantities.

The choice of stress-energy tensor expectation values as beables for relativistic quantum field theory is particularly suggestive if we allow the probability distribution for configurations of these beables to depend on laws not defined only by the evolving quantum state. This suggests the thought that it might be possible to unify gravity and quantum theory via probabilistic quasiclassical laws that couple the background geometry directly to the quasiclassical beables defining a matter distribution, without necessarily requiring any quantized gravitational field.

The idea here is not to restrict to defining versions of semiclassical gravity by equations of the form

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi \langle T_{\mu\nu} \rangle,$$

where the expectation value on the right-hand side is defined by one of the recipes given above (summing over the expectation values for the classes if there are two or more classes). Such equations need not generally be everywhere consistent. Instead, the goal is to extend the probability distributions defined above on beable configurations, represented by $\langle T_{\mu\nu} \rangle$, to joint probability distributions on Riemannian manifolds and tensor fields defined on such manifolds, with the property that the quasiclassical Einstein equations emerge as approximately valid in appropriate domains. We intend to explore this further in future work.

C. Relation to previous work

The solution to the reality problem outlined here uses the strategy of inferring finite time beables from asymptotic behavior and many of the other ideas set out in Ref. [16], but is simpler than the proposals made in that paper. While the simplicity of the solution given here is particularly appealing and its suggestion of a relationship to gravity is particularly intriguing, those earlier proposals still remain potentially interesting alternatives in our view. Both the solution proposed here and those in Ref. [16] have features in common with other earlier ideas in the literature.

The fundamental significance of the quantum reality problem and the possibility of finding a mathematical solution was perhaps first realized by de Broglie and Bohm [4,5]. The concept of beable is due to Bell [2,3], who also illustrated the variety of possible types of beable solution to the reality problem and focused attention on the Lorentzian quantum reality problem [20].

Aharonov and collaborators [17,23,24] have long stressed the value of considering both initial and final states in order to illuminate the properties of and interpret quantum theory from various perspectives. Related ideas were previously considered by Watanabe [25]. Suggestions for interpretations of quantum theory in terms of initial and final states have been made by Davidon [26] and Aharonov and Gruss [27]. From the perspective adopted here, one major limitation of these latter ideas is their reliance on intuitive definitions of measurement and classicality, which are unsatisfactorily imprecise in any setting and especially problematic in the context of cosmology; see [1,16] for further discussion. Another fundamental problem, from the perspective of those looking for a new one-world solution to the reality problem, is that they define an ontology that is larger than that used by Everett, since it includes backward evolving data as well as the standard wave function unitarily evolving forward in time.

The idea of defining cosmological models with independent initial and final boundary conditions, using a decoherent-histories version of the ABL rule, was discussed by Gell-Mann and Hartle [28]. The possibility of defining cosmological models and other generalizations of the quantum theory of closed systems by going beyond boundary conditions and considering a sequence of constraints on the system's evolution was proposed in Ref. [29] and developed and discussed further in Ref. [21]. The potential uses of environmental records in making sense of quantum theory and quantum cosmology have been stressed by Zurek and collaborators [30–32] and by Gell-Mann and Hartle [28], among others.

While all of these contributions have been influential and relevant to our discussion, none of the above authors has proposed a mathematically precise solution to the Lorentzian quantum reality problem in the sense defined by Bell and considered here.

Mass-density beable ontologies were first proposed for nonrelativistic collapse models by Pearle and Squires ([33]; see also [34]). An extension of these ontologies to relativistic collapse models using constructions previously defined in [35,36] was proposed in [8]. These proposals apply to generalizations of quantum theory rather than to quantum theory itself. We presently see our relativistic solution as more natural and see the path to rigorizing it as having fewer (although still considerable) technical obstacles.

We also see our proposed solutions as calling into question part of the original motivation for dynamical collapse models. It should be noted, though, that one possible motivation for dynamical collapse models is the desire for a theory that effectively *ensures* that macroscopic superpositions essentially never take place, even in hypothetical future experiments in which technology allows us either to isolate macroscopic systems for long times or to control their environments, in such a way that quantum theory would predict a genuine macroscopic superposition and an ensuing quantum interference pattern. Our solutions suggest an ontology in which all significant components of the macroscopic superposition have corresponding beable trails in such experiments. This does not seem evidently problematic: There is no logical inconsistency in such a description, nor any contradiction with experiment or observation to date. Still, those who prefer the hunch that nature abhors a macroscopic superposition will prefer collapse models or others with this feature.

Of course, all these various questions certainly deserve further analysis. We also wish to stress that, whether or not they ultimately prove relevant to nature, dynamical collapse models remain, in our view, a landmark intellectual achievement in the development of work on the quantum reality problem and that there remains a scientific case for exploring them simply because they are testable generalizations of quantum theory.

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APPENDIX A: BEABLE MODELS BASED ON OTHER DEFINITIONS

In this appendix we consider alternative strategies for defining beables, applicable to both nonrelativistic and relativistic models. We make the same assumptions as previously about the initial state and asymptotically defined final conditions.

1. Expectation values defined by measurements at a single point

In our nonrelativistic model, we defined the beable at the point (x,t) in terms of an expectation value $\langle \rho(x,t) \rangle$. This expectation value was defined via the ABL rule for a joint measurement of mass density at all points y on the surface of constant time t. We could, instead, have defined an expectation value, which we denote $\langle \rho(x,t) \rangle_x$, by applying the ABL rule for a single measurement of the operator $\rho(x,t)$.

As Aharonov and Vaidman's box "paradoxes" [37] illustrate, there are final states for which this would give a different beable distribution, with somewhat counterintuitive properties. The beables for a single particle in a three-box example would suggest that its entire mass was simultaneously in two distinct regions (with some further nonzero mass-density expectation in a third). Mass would thus not generally be conserved at the beable level.

This is aesthetically worrying, and even more troubling if one hopes that the mass-density beables play a significant role in combining quantum theory and gravity (a possibility we consider further below). Nonetheless, we should note that such a description is not logically inconsistent, and it is not immediately evident to us that it is incapable of reproducing quasiclassical physics, at least in presently familiar contexts. In principle, a beable version of quantum theory might give a counterintuitive picture of reality in microscopic experiments (or indeed in macroscopic experiments that are to date unperformed) and still allow the derivation of the correct higher-level quasiclassical laws in the right regime.

Similar comments apply to the relativistic models. We could define a stress-energy tensor beable $\langle T_{\mu\nu}(y) \rangle_y$ as the ABL expectation value for a measurement of $T_{\mu\nu}(y)$ alone. This has what might be seen as the advantage of dispensing with a definition based on a limit of spacelike hypersurfaces that approximate and tend to the past light cone. It has the same counterintuitive features as its nonrelativistic counterpart in its description of three-box experiments, however.

We should also note that, while the beable models we defined earlier give more intuitively sensible descriptions of three-box experiments, we would not expect them to agree with all prior intuitions in their descriptions of every closed quantum system. For example, the stress-energy tensor beable $\langle T_{\mu\nu}(y) \rangle$ need not generally satisfy conservation laws everywhere and cannot be directly used as a source in Einstein's equations for a semiclassical treatment of gravity. (Indeed, no quasiclassical derivation of Einstein's equations involving macroscopic quantum experiments can be valid everywhere [38].)

In summary, models in which the beables are defined by single-point expectation values have decidedly odd features. While these make such models seem presently less attractive, we do not feel they presently logically exclude them. More analysis of the relationship between the beable distributions in these models and quasiclassical variables is needed.

2. Comments on weak operator values and beables

Another candidate beable in our relativistic models could be the modulus of a generalized form of the so-called weak value [27,37] of the stress-energy tensor,

$$\mathcal{A}_{\mu\nu}(x)_w = \left| \left\{ \frac{\mathrm{Tr}[T_{\mu\nu}(x) P_S T_{\mu\nu}(x) P_0]}{\mathrm{Tr}(P_S P_0)} \right\} \right|^{1/2} .$$

Here P_S is the projection onto the space of states whose mass-energy distributions $t_s(x)$ agree with that randomly selected on the final hypersurface S, $P_0 = |\psi_0\rangle\langle\psi_0|$ is the projection onto the initial state on S_0. These are unitarily evolved backwards and forwards, respectively, to any chosen spacelike hypersurface S' through x, and the inner products are calculated on S': We suppress these details in our notation.

Similar comments apply to this and other quantities derived from weak expectation values or decoherence functions. Such quantities behave similarly to the single-point expectation values in three-box experiments so that a single particle would appear in the beable description to be located in more than one box and have other peculiar properties (see, e.g., [39]). While these observations do not logically exclude them as candidate beables capable of describing quasiclassical physics, they do motivate further careful scrutiny and analysis.

APPENDIX B: FURTHER GENERALIZATIONS OF QUANTUM THEORY

1. Generalizations by taking finite limit parameters

Consider again our nonrelativistic model, in which a mass distribution is obtained at final time T and then used to define mass-density distributions at times $0 < t < T$. These expressions may be calculated by using projectors $P_{\rho_f}^{\Delta}$ onto the set of states with final mass distribution in a neighborhood Δ of $\rho_f(x,T)$ and taking the limit as the size of Δ tends to zero. Finally, we take the limit $T \to \infty$. These limits are intended to reproduce a quasiclassical reality consistent with standard quantum theory in realistic models.

To produce generalizations of quantum theory, we can take T, and if we wish also Δ, to be fixed finite parameters. Intuitively, if T is large compared to the duration of a quantum experiment, one expects this to give predictions almost indistinguishable from those of standard quantum theory for that experiment.

Of course, taking the finite T version of the model literally suggests that reality ceases after time T has elapsed, even though (on a standard reading) the quantum dynamics may continue to be eventful long afterward. Our recommended attitude to this is not to take the model literally on this point. A commonly held view of dynamical collapse models is that although the mathematical details of their collapse mechanisms are *ad hoc* and it is hard to believe that either they or the associated ontologies are fundamentally correct, the models are nonetheless interesting generalizations of quantum theory. They make an intellectually significant point—altering quantum dynamics somewhat radically alters the ontology and gives alternative solutions to the quantum reality problem—and also point to interesting experimental tests. A model does not need to be completely right in order to point in a direction that is theoretically or experimentally fruitful to explore. Similarly, finite T and Δ versions of our models show that altering quantum theory gives a well-defined realist ontology without any assumption about the asymptotic dynamics and in a way that could affect the predictions for quasiclassical dynamics and experiment so subtly as to be essentially undetectable [40].

Another possible approach to finite T models would be to construct versions in which "final measurements" are made repeatedly on time scales of order T, and the chosen reality depends on a sequence of final measurement outcomes in such a way that it evolves smoothly. At first sight, such models look mathematically rather *ad hoc*, since one can imagine many recipes of this type, none of which seems particularly natural. They also look likely to have physically peculiar consequences, in which reality is something like a real superposition (with time-evolving weights) of a sequence of independently randomly chosen realities defined by measurements at times $\approx T, \approx 2T, \ldots$. Perhaps, though, there is scope to construct more natural models by variations on these ideas. We leave this for future exploration.

2. Comments on relativistic generalizations

One might also investigate "finite proper time" generalizations of our relativistic models. Given initial data on a spacelike hypersurface S, and a time parameter τ, we can define a finite version of the models given any Lorentz covariant rule that produces a final hypersurface S' that depends (stochastically or deterministically) on S and τ, with the property that S' is always in the future of S and that the maximum proper time between points on S and S' is a function of τ. Such rules can be naturally defined in finite lattice models [19]. It would be interesting to explore continuous versions in Minkowski space or, indeed, analogous rules in quantum gravity using natural definitions of cosmological time [41].

Again, a literal reading of such models suggests that reality exists only between S and S'; again, our preferred attitude would be not to take any such model so literally. Another feature is that the choice of hypersurfaces S,S' suggests some sort of preferred coordinate choice, even if the rules relating S' to S are Lorentz covariant. This may not necessarily be problematic—after all, standard general relativistic cosmological models can also include both preferred proper time coordinates and associated spacelike hypersurfaces—but it requires careful discussion. Again, we leave these preliminary ideas for future exploration.

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