Aharonov-Bohm effect for light in a moving medium

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In this paper we propose a way to detect the Aharonov-Bohm effect for light in a moving medium. We use the position-dependent photon wave function to describe the propagation of the photon in order to evaluate the phase shift acquired by the photon while it passes around a rotating cylinder embedded in a viscous fluid. We show that this phase depends on the speed of the fluid as well as the electromagnetic properties of the cylinder.

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I. INTRODUCTION

Bialynicki-Birula [1–3] has described a photon as a quantum-mechanical particle and proposed a theory based on the first quantization for this particle. This approach resulted in obtaining of the wave function describing the photon. The theory by Bialynicki-Birula is based on the fact that the photon is a quantum particle; thus its pure states are described by a wave function. Bialynicki-Birula has claimed that a complex combination of electric and magnetic field vectors represented by the Riemann-Silberstein vector is a most natural and convenient way to describe photons. This affirmation is supported by the fact that the Riemann-Silberstein vector contains all information about the classical electromagnetic field and can be used in a quantum theory of a photon as a wave function of this particle. Independently, Sipe [4] also obtained a photon wave function. This theory has been denominated as the theory of a photon wave, or the Bialynicky-Birula–Sipe theory of photons. Recently, this theory has been used in solving a number of problems in physics. Among these applications, we note the study of photonic tunneling [5], the analysis of two-photon wave mechanics [6] with application to the Lamb shift [7], the study of interaction between light and matter [8], the analysis of propagation of photons in resting and moving media [9], the study of the equivalence between the wave functions of photon wave mechanics and mode functions of quantum field theory, evolving via identical equations of motion and completely describing the quantum state [10], and the development of the covariant formulation of photon wave mechanics and proof of its equivalence with QED photonic states [11]. In this paper we investigate the Aharonov-Bohm effect for light in moving media using the theory of a position-dependent [9] photon wave function to describe the propagation of the photon.

Aharonov and Bohm [12] demonstrated that the electromagnetic vector potential has a physical meaning in quantum mechanics. Charged particles are affected by this potential, even if they are localized in regions with no magnetic fields. The best known form of the Aharonov-Bohm effect describes a charged particle affected by the potential vector corresponding to a solenoid; the particle acquires a quantum phase proportional to the magnetic flux inside the solenoid. Aharonov and Casher [13] have demonstrated that, in certain circumstances, a neutral particle with a permanent magnetic moment also exhibits an Aharonov-Bohm effect. When the neutral particle makes a closed path around a line of electric charges, its wave function acquires a phase shift proportional to the magnetic moment of the neutral particle and to the charge density of the line. He and McKellar [14] and Wilkens [15] independently have investigated a quantum phase acquired by a neutral particle with a permanent electric dipole moment. In the same way as within the Aharonov-Casher (AC) effect, the wave function of the neutral particle acquires a phase shift due to the coupling between the electric dipole moment of the neutral particle and the magnetic field of the monopoles. The He-McKellar-Wilkens (HMW) phase is the Maxwell dual of the AC phase. A more realistic field configuration of the HMW effect has been proposed by Wei, Han, and Wei (WHW) [16]. In their work, the neutral particle is exposed to a nontrivial field configuration characterized by the simultaneous action of the electric field and a uniform magnetic field. This field configuration induces an electric dipole moment for the neutral particle. The WHW proposal is physically more realistic because it avoids the necessity of the field generated by magnetic monopoles. The quantum phase for electric dipoles has been investigated in [17,18], where a quantum phase different from that proposed by Wilkens by the presence of an extra term was obtained. Analogs of the Aharonov-Bohm effect have been investigated in several contexts such as gravitation [19,20], dynamics of quasiparticles in a superfluid [21,22], propagation of light in moving media [23], dynamics of a quasiparticle in a medium with disclination [24], and the quantum dynamics of quasiparticles in graphene in the presence of a topological defect [25]. Cook *et al.* [26] have investigated the Aharonov-Bohm effect for light using the electromagnetic wave equations for a moving medium and have demonstrated that the Fizeau experiment for the velocity of light in moving media is an example of this classical analog of the Aharonov-Bohm effect for light. In this paper we investigate the Aharonov-Bohm effect for

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propagation of photons around a spinning cylinder immersed in a fluid. Recently, a series of articles have considered the analogs of the Aharonov-Bohm effect for an electromagnetic wave [27,28] in an optical medium.

In this paper we investigate the Aharonov-Bohm (AB) effect using the Minkowski nonrelativistic constitutive relations for moving media [29]. We study the geometric phase acquired by the wave function of the photon in the interferometric path around of a rotating cylinder in a viscous medium. This paper is organized in the following form: in Sec. II we present the description of electromagnetism in moving media; in Sec. III, we investigate the AB effect for light in a moving medium. Finally, in Sec. IV we present the concluding remarks.

II. ELECTROMAGNETIC WAVES IN MOVING MEDIA

Now, we use the Maxwell equation in a moving medium [29] to obtain a position-dependent photon wave function [9]. We follow the procedure of Zaleśny [9,30] to describe the propagation of an electromagnetic wave in moving media and to define the concept of vector potential associated with the velocity of a medium. The aim of this paper is the investigation of the Aharonov-Bohm effect for classical electromagnetic waves. We start with the Maxwell equations for moving media without sources:

$$\nabla \cdot \mathbf{D} = 0, \tag{1a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1b}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{E}}{\partial t}, \tag{1c}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}. \tag{1d}$$

The constitutive relations for nonrelativistic moving media are given by Minkowski relations [29],

$$\mathbf{D} = \epsilon_0\epsilon\mathbf{E} + (\epsilon\mu - 1)\frac{\mathbf{v}}{c^2} \times \mathbf{H}, \tag{2a}$$

$$\mathbf{B} = \mu_0\mu\mathbf{H} - (\epsilon\mu - 1)\frac{\mathbf{v}}{c^2} \times \mathbf{E}, \tag{2b}$$

where ϵ and μ are, respectively, the dielectric constant and the magnetic permeability of the medium and $\mathbf{v}$ is the velocity field associated with the motion of the medium.

For monochromatic waves, the electric and magnetic fields are given, respectively, by

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{-i\omega t}, \tag{3a}$$

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})e^{-i\omega t}. \tag{3b}$$

Assuming that the velocity field depends only on the position $\mathbf{v} = \mathbf{v}(\mathbf{r})$ and combining (2b), (3a), and (3b) in the Maxwell equations, one can write the curl equations as

$$\left(\nabla + i\omega\frac{\epsilon\mu - 1}{c^2}\mathbf{v}(\mathbf{r})\right) \times \mathbf{E}(\mathbf{r}) = i\omega\mu_0\mu\mathbf{H}(\mathbf{r}), \tag{4a}$$

$$\left(\nabla + i\omega\frac{\epsilon\mu - 1}{c^2}\mathbf{v}(\mathbf{r})\right) \times \mathbf{H}(\mathbf{r}) = -i\omega\epsilon_0\epsilon\mathbf{E}(\mathbf{r}). \tag{4b}$$

We can make an analogy between the electromagnetic and quantum approaches by multiplying Eqs. (4a) and (4b) by $-i\hbar$, where $\hbar$ is the Planck constant. Thus we get the following set of equations:

$$[\hat{\mathbf{p}} + q\mathbf{v}(\mathbf{r})] \times \mathbf{E}(\mathbf{r}) = \hbar\omega\mu_0\mu\mathbf{H}(\mathbf{r}), \tag{5a}$$

$$[\hat{\mathbf{p}} + q\mathbf{v}(\mathbf{r})] \times \mathbf{H}(\mathbf{r}) = -\hbar\omega\epsilon_0\epsilon\mathbf{E}(\mathbf{r}), \tag{5b}$$

where $\hat{\mathbf{p}} = -i\hbar\nabla$ is the momentum operator and $q = \hbar\omega\frac{\epsilon\mu-1}{c^2}$. The parameter q is identified as "effective charge for the radiation." Zaleśny [9] showed that when a relative movement is introduced into the medium, the velocity field acts as an effective potential vector for the radiation. This conclusion can be made on the basis of Eqs. (5a) and (5b). It allows us to identify the vector potential for the electromagnetic radiation as

$$\mathbf{A}(\mathbf{r}) \equiv \mathbf{v}(\mathbf{r}). \tag{6}$$

In the next section, we present an explicit expression for the vector potential $\mathbf{A}(\mathbf{r})$, as well as a description of a physical system that displays the Aharonov-Bohm effect for an electromagnetic wave.

III. THE AHARONOV-BOHM EFFECT FOR THE PHOTON

In the context of quantum mechanics, the Aharonov-Bohm effect occurs when an electrically charged particle is affected by the presence of a magnetic field $\mathbf{B}$, despite this particle being confined to a region in which $\mathbf{B}$ is zero but the vector potential $\mathbf{A}$ is not zero. It is necessary only that $\nabla \times \mathbf{A} = \mathbf{0}$ in regions where the particle is confined.

How can the Aharonov-Bohm effect be observed in the case of the photon? In order to answer this question, we ensure that $\nabla \times \mathbf{A} = \mathbf{0}$ and propose the following physical system as a laboratory. Let us consider a rotating, rigid, and long cylinder. This cylinder has a radius R and rotates with a constant angular frequency $\Omega\hat{\mathbf{z}}$. Moreover, let us suppose that the cylinder is inserted in a viscous medium with uniform and constant viscosity (see Fig. 1). The situation is similar to that of an electron beam which is separated into two beams passing around the solenoid and then recombining. The same occurs with the photons around the spinning cylinder described above. To show how this happens, we need to choose

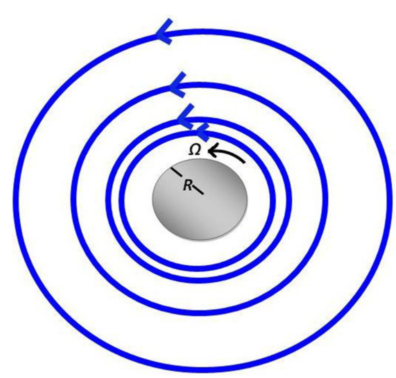

FIG. 1. (Color online) A cylinder of radius R, rotating about an axis with angular velocity Ω immersed in a viscous fluid. The field of velocity in the fluid is represented by blue circles.

properly the velocity field of the fluid. In the closest layers, the angular velocity caused by the rotating cylinder is simply ΩR. However, for more distant layers, this velocity drops off with distance from the center of the cylinder. Thus it is reasonable to write the velocity field as follows:

$$\vec{v}(\vec{r}) = \begin{cases} \Omega r\hat{\theta}, & 0 \leqslant r \leqslant R, \\ \frac{\Omega a^2}{r}\hat{\theta}, & r > R. \end{cases} \tag{7}$$

Thus the effective vector potential (6) produced by the medium has the form

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \Omega r\hat{\theta}, & 0 \leqslant r \leqslant R, \\ \frac{\Omega a^2}{r}\hat{\theta}, & r > R. \end{cases} \tag{8}$$

In this way, we can calculate the effective *magnetic field* produced by moving medium, calculating $\nabla \times \mathbf{A}(\mathbf{r})$ using Eq. (9),

$$\bar{\mathbf{H}} = \begin{cases} \Omega\hat{\mathbf{z}}, & 0 \leqslant r \leqslant R, \\ \mathbf{0}, & r > R, \end{cases} \tag{9}$$

and the associated *magnetic flux* is $\bar{\Phi} = \pi R^2 \Omega$. We have noticed that the angular velocity field of the medium plays the role of a effective *magnetic field*. Suppose then that a beam of photons is divided into two along a circumference of radius b and then recombines (see Fig. 2), similar to the case of the Aharonov-Bohm effect. Now, we consider the wave equation in the presence of the photon vector potential (6). Let us suppose the following ansatz for the solutions of the Maxwell equations:

$$\mathbf{E}(\mathbf{r}) = e^{i\varphi(\mathbf{r})}\mathbf{E}'(\mathbf{r}), \tag{10}$$

$$\mathbf{H}(\mathbf{r}) = e^{i\varphi(\mathbf{r})}\mathbf{H}'(\mathbf{r}). \tag{11}$$

In the regions where the radiation beam is confined, $\nabla \times \mathbf{A}(\mathbf{r}) = \mathbf{0}$; then $\varphi(\mathbf{r})$ is given by

$$\varphi(\mathbf{r}) = \frac{q}{\hbar}\int \mathbf{A}\cdot d\mathbf{r}. \tag{12}$$

In the expression above, the integral does not depend on the path. Note that $\mathbf{E}'$ and $\mathbf{H}'$ are the fields for the medium in the rest frame, when $\mathbf{v} = \mathbf{0}$. Thus, substituting (10) and (11) in

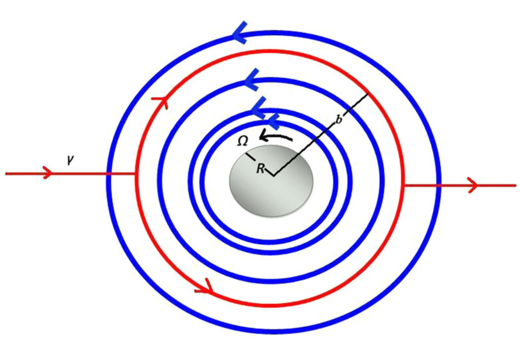

FIG. 2. (Color online) A pictorial representation of a beam of photons γ in a closed path of radius b around the rotating cylinder.

Eq. (6), we obtain the following wave equation:

$$\nabla \times \mathbf{E}'(\mathbf{r}) = i\omega\mu_0\mu\mathbf{H}'(\mathbf{r}), \tag{13}$$

$$\nabla \times \mathbf{H}'(\mathbf{r}) = -i\omega\epsilon_0\epsilon\mathbf{E}'(\mathbf{r}). \tag{14}$$

The above equations correspond to the field equations for a linear, isotropic, and nondispersive medium in the rest frame. Now, we obtain the geometric phase acquired by the electromagnetic fields. The analog of the Aharonov phase of a photon is given by

$$\varphi = \frac{q}{\hbar}\oint_C \mathbf{A}\cdot d\vec{r}. \tag{15}$$

Substituting the expression for $\mathbf{A}$ and q, we obtain

$$\varphi = \frac{1}{\hbar}\int_0^{2\pi} \frac{\hbar\omega}{c^2}(\epsilon_f\mu_f - 1)\Omega\frac{R^2}{b}\hat{\theta}\cdot bd\theta\hat{\theta}. \tag{16}$$

Explicitly calculating the phase shift, we find the result

$$\varphi = 2\pi\frac{\omega}{c^2}(\epsilon\mu - 1)\Omega R^2; \tag{17}$$

this is the phase acquired by the wave function of the photon transported along a closed path which encircles the rotating cylinder in the fluid.

IV. CONCLUDING REMARKS

We have investigated the Aharonov-Bohm effect starting from Maxwell equations, using the Minkowski relations for nonrelativistic moving media. We have proposed a possible experiment to measure the geometric phase for the photon. We have considered photon beam propagation in a fluid where a cylinder rotates around its axis with an angular velocity Ω. This result shows that even when one considers the photon beam propagating in a region outside the rotating cylinder, the phase shift depends directly on the electromagnetic properties of the cylinder, characterized by electric permittivity and magnetic permeability of the material composing the cylinder. With this we inferred that the rotation of the cylinder acts on the photon beam at a "distance" in a way analogous to was the magnetic vector potential acts on an electron beam within the electromagnetic Aharonov-Bohm effect. This influence in the present case is mediated by the presence of a viscous fluid. Again, we draw an analogy between the usual AB effect and the analog AB effect for the photon. The phase difference between the two beams is the usual AB effect, which is given by

$$\varphi = \frac{e}{\hbar}\oint_C \mathbf{A}\cdot d\vec{r} = \frac{e\Phi_m}{\hbar}, \tag{18}$$

where e is the charge of an electron, $\hbar$ is the Planck constant, and Φ_m is the magnetic flux. The path of integration C is a curve around the solenoid. Now, let us consider the photon beam. The phase difference obtained when the beam circulates around the cylinder is given by

$$\varphi = \frac{q}{\hbar}\oint_C \mathbf{A}\cdot d\vec{r} = \frac{q\Phi_L}{\hbar}. \tag{19}$$

The flux $\Phi_L = \Omega\pi R^2$ is generated by the curl of the field of velocities, and $q = \frac{\hbar\omega}{c^2}(\epsilon\mu - 1)$.

We can also note that if the cylinder is metallic, the phase difference can be complex. This means that the phase shift causes a damping of the amplitude fields, even when a constructive interference occurs. We also noted that this difference also depends on the rotation of the viscous fluid. In conclusion we claim that this experiment can be performed and the Aharonov-Bohm effect for a photon in this moving medium can be detected.

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