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Acausal measurement-based quantum computing

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In measurement-based quantum computing, there is a natural "causal cone" among qubits of the resource state, since the measurement angle on a qubit has to depend on previous measurement results in order to correct the effect of by-product operators. If we respect the no-signaling principle, by-product operators cannot be avoided. Here we study the possibility of acausal measurement-based quantum computing by using the process matrix framework [Oreshkov, Costa, and Brukner, Nat. Commun. **3**, 1092 (2012)]. We construct a resource process matrix for acausal measurement-based quantum computing restricting local operations to projective measurements. The resource process matrix is an analog of the resource state of the standard causal measurement-based quantum computing. We find that if we restrict local operations to projective measurements the resource process matrix is (up to a normalization factor and trivial ancilla qubits) equivalent to the decorated graph state created from the graph state of the corresponding causal measurement-based quantum computing. We also show that it is possible to consider a causal game whose causal inequality is violated by acausal measurement-based quantum computing.

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I. INTRODUCTION

In Ref. [1], Oreshkov, Costa, and Brukner proposed a novel framework, which is called the process matrix (PM) framework, to study general physics on multipartite systems where locally quantum physics is assumed but globally no restriction, such as the no-signaling and the causality, is set (see also Refs. [2–5]). They showed that this framework can describe general theory beyond the standard quantum physics, including a "mixture" of different time causal orders. Interestingly, they explicitly constructed an example of the PM system whose induced correlation violates a "causal inequality" that is satisfied by all spacelike and timelike correlations [1].

In this Rapid Communication, we study measurement-based quantum computing (MBQC) [6] in the PM framework. MBQC is a new model of quantum computing proposed by Raussendorf and Briegel. In this model, universal quantum computation can be done with only local measurements on each qubit of a certain quantum many-body state, which is called a "resource state." While the computational power of MBQC is equivalent to the traditional circuit model of quantum computation, MBQC provides novel view points to deepen our understanding of quantum computing. In fact, plenty of new results have been obtained by using MBQC, such as relations of quantum computing to condensed matter physics [7–20], the fault-tolerant topological MBQC [21–26], roles of quantum properties (such as entanglement, correlation, and purity) in MBQC [27–32], and secure cloud quantum computing [33–49].

One of the most peculiar things in MBQC is that there is a natural "causal cone" among qubits of the resource state [52–56]. The measurement angle of a qubit has to be determined by the previous measurement results, since we have to correct the effect of the byproduct operators, which cannot be avoided (if we respect the no-signaling [50]). Given the PM framework, it is natural to ask, "Can we describe acausal MBQC in the PM framework?" Here, acausal MBQC means that the measurement angle of each qubit does not depend on measurement results of other qubits, but we can perform correct quantum computing. In the PM framework, a density matrix is generalized to a PM. Therefore, the above question is restated as follows: "Can we find a resource PM (which corresponds to a resource state of the causal MBQC) for acausal MBQC?"

The purpose of this Rapid Communication is to answer the question. We explicitly construct a resource PM for acausal MBQC restricting local operations to projective measurements. In this acausal MBQC, the measurement angle of each qubit can be independent from measurement results of other qubits. Interestingly, if we restrict local operations to projective measurements, the resource PM is (up to a normalization factor and trivial ancilla qubits) equivalent to the decorated graph state of the corresponding causal MBQC. (Here, a decorated graph state is a graph state whose graph is created from the original graph by adding an extra vertex to each vertex of the original graph.) We also consider a causal game whose causal inequality is violated by acausal MBQC.

II. PM FRAMEWORK

Let us quickly review the PM framework. Let us consider a bipartite system, Alice and Bob. (Generalizations to multipartite systems are straightforward.) Alice is in her laboratory, which is isolated from the outer world. In her laboratory, physics is governed by the quantum theory. This means that a measurement by Alice corresponding to the result a is represented by a completely positive (CP) trace-nonincreasing map $\mathcal{M}_a^A : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2})$, where $\mathcal{H}^{A_1}$ and $\mathcal{H}^{A_2}$ are Alice's input and output Hilbert spaces, respectively, $\mathcal{L}(\mathcal{H})$ is the space of operators over a Hilbert space $\mathcal{H}$, and $\sum_a \mathcal{M}_a^A$ is a CP and trace-preserving (CPTP) map. In a similar way, Bob is in his isolated laboratory, and inside of the laboratory the quantum theory is correct. His measurement corresponding

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to the result b is represented by a CP trace-nonincreasing map $\mathcal{M}_b^B : \mathcal{L}(\mathcal{H}^{B_1}) \to \mathcal{L}(\mathcal{H}^{B_2})$, where again $\sum_b \mathcal{M}_b^B$ is a CPTP map. In this way, Alice's and Bob's local systems are explained in the quantum theory. However, no restriction is set on the physics of the outer world where their laboratories are embedded. In particular, the no-signaling and the causality are not assumed between the two laboratories. It was shown [1] that the probability $P(\mathcal{M}_a^A,\mathcal{M}_b^B)$ that Alice's measurement result is a and Bob's measurement result is b is given by

$$P\big(\mathcal{M}_a^A,\mathcal{M}_b^B\big) = \mathrm{Tr}\big[W^{A_1,A_2,B_1,B_2}\big(M_a^{A_1,A_2}\otimes M_b^{B_1,B_2}\big)\big],$$

where $W^{A_1,A_2,B_1,B_2} \in \mathcal{L}(\mathcal{H}^{A_1}\otimes\mathcal{H}^{A_2}\otimes\mathcal{H}^{B_1}\otimes\mathcal{H}^{B_2})$, and

$$\begin{aligned} M_a^{A_1,A_2} &\equiv \big[\big(I\otimes\mathcal{M}_a^A\big)|ME\rangle\langle ME|\big]^T \\ &= \sum_{i,j=1}^{d} |i\rangle\langle j|\otimes\mathcal{M}_a^A(|j\rangle\langle i|) \in \mathcal{L}(\mathcal{H}^{A_1}\otimes\mathcal{H}^{A_2}) \end{aligned}$$

is the positive-semidefinite operator obtained by the Choi-Jamiolkowsky (CJ) isomorphism. Here, d is the dimension of $\mathcal{H}^{A_1}$, T is the matrix transposition, and $|ME\rangle \equiv \sum_{j=1}^d |j\rangle\otimes|j\rangle \in \mathcal{H}^{A_1}\otimes\mathcal{H}^{A_1}$ is the (nonnormalized) maximally entangled state. The operator $M_b^{B_1,B_2} \in \mathcal{L}(\mathcal{H}^{B_1}\otimes\mathcal{H}^{B_2})$ is defined in a similar way. A map $\mathcal{M}^A \equiv \sum_a \mathcal{M}_a^A$ is CPTP if and only if its CJ operator M^{A_1,A_2} satisfies $M^{A_1,A_2} \geqslant 0$ and $\mathrm{Tr}_{A_2} M^{A_1,A_2} = I$. If W^{A_1,A_2,B_1,B_2} satisfies

$$W^{A_1,A_2,B_1,B_2} \geqslant 0 \tag{1}$$

and

$$\mathrm{Tr}\big[W^{A_1,A_2,B_1,B_2}\big(M^{A_1,A_2}\otimes M^{B_1,B_2}\big)\big] = 1 \tag{2}$$

for all M^{A_1,A_2} and M^{B_1,B_2} such that $M^{A_1,A_2}\geqslant 0$, $M^{B_1,B_2}\geqslant 0$, $\mathrm{Tr}_{A_2}M^{A_1,A_2} = I$, and $\mathrm{Tr}_{B_2}M^{B_1,B_2} = I$, we call W^{A_1,A_2,B_1,B_2} the process matrix (PM) [1]. A PM is, in some sense, a generalization of a density matrix in quantum theory. (If operation on A_2 and B_2 are identity, then the PM becomes a density matrix.)

III. MBQC

Before describing our result, we also review the basics of MBQC. Let σ be an $(N+n)$-qubit resource state of MBQC. We divide σ into two subsystems C and O [Fig. 1(a)]. The subsystem C consists of N qubits, and the subsystem O consists of n qubits. Qubits in C are measured in order to implement the *computation*. The *output* of the computation is

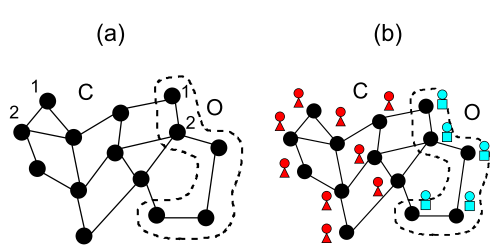

FIG. 1. (Color online) (a) The resource state σ. For example, it is the graph state $|G\rangle$ on the graph G. (b) The distributed MBQC. Red people are Alice$_j$ ($j=1,\ldots,N$) and blue people are Bob$_j$ ($j=1,\ldots,n$).

encoded on qubits in O, and therefore we measure qubits in O to read out the output of the computation. Measurements on C are adaptive: we first measure the first qubit of C in a certain orthonormal basis $\{|\phi_1^0\rangle,|\phi_1^1\rangle\}$. Let $m_1\in\{0,1\}$ be the result of the measurement. We next define an orthogonal basis, $\{|\phi_2^0(m_1)\rangle,|\phi_2^1(m_1)\rangle\}$, which depends on m_1, and measure the second qubit of C in this basis. If the measurement result is $m_2\in\{0,1\}$, we measure the third qubit in the orthonormal basis $\{|\phi_3^0(m_1,m_2)\rangle,|\phi_3^1(m_1,m_2)\rangle\}$, and so on. In this way, we adaptively measure all qubits in C. After measuring all qubits in C, we finally measure each qubit of O in the computational basis $\{|0\rangle,|1\rangle\}$ in order to readout the computation result. Depending on the measurement results on C, some operators (usually Pauli operators) are acted upon O. Such operators are called by-product operators. Because of the effect of the by-product operators, the result on O must be postprocessed.

The canonical example of the resource state is the graph state [6]. Let us consider a graph $G=(V,E)$ of N vertices. The graph state $|G\rangle$ corresponding to the graph G is defined by $|G\rangle \equiv (\bigotimes_{e\in E} CZ_e)|+\rangle^{\otimes N}$, where $|+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, and $CZ_e \equiv |0\rangle\langle 0|\otimes I + |1\rangle\langle 1|\otimes Z$ is the controlled-Z gate between two vertices of the edge e.

IV. RESOURCE PM FOR ACAUSAL MBQC

Now we show the main result. Our acausal MBQC is performed in the distributed way [Fig. 1(b)] by N girls, Alice$_j$ ($j=1,\ldots,N$), and n boys, Bob$_j$ ($j=1,\ldots,n$). They share a certain (possibly superquantum) resource system consists of $N+n$ particles. The system is divided into two subsystems C and O, which consist of N and n particles, respectively. Alice$_j$ possesses the jth particle of C, and Bob$_j$ possesses the jth particle of O [Fig. 1(b)].

In the causal MBQC, Alice$_j$ has to know the measurement results of Alice$_k$ ($k=1,\ldots,j-1$) in order to determine her measurement angle. However, in the present acausal MBQC, we assume that Alice$_j$ measures her system in the fixed orthonormal basis $\{|\phi_j^0\rangle,|\phi_j^1\rangle\}$ irrespective of the measurement results of Alice$_k$ ($k\neq j$) and Bob$_j$ ($j=1,\ldots,n$), where $|\phi_j^m\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle+(-1)^m e^{i\phi_j}|1\rangle)$. In the causal MBQC, we can no longer perform correct quantum computing if Alice$_j$'s measurement is fixed in this way. However, we will see later that in the acausal MBQC, we can perform correct quantum computing in spite of the fact that Alice$_j$'s measurement is fixed.

After Alice$_j$'s measurement, Alice$_j$ sets the system to $|m_j\rangle$, where $m_j\in\{0,1\}$ is Alice$_j$'s measurement result. The CJ operator of such a measurement process is given by

$$\sum_{k=1}^{d}\sum_{l=1}^{d} |k\rangle\langle l|\otimes|m_j\rangle\langle\phi_j^{m_j}|l\rangle\langle k|\phi_j^{m_j}\rangle\langle m_j| = \boxed{\phi_j^{m_j}}\otimes\boxed{m_j}.$$

Here, we have used the convenient notation $\boxed{x} \equiv |x\rangle\langle x|$ [51]. Bob$_j$ measures his system in the computational basis $\{|0\rangle,|1\rangle\}$, and sets the system to $|z_j\rangle$ after Bob$_j$'s measurement, where $z_j\in\{0,1\}$ is Bob$_j$'s measurement result. The CJ operator corresponding to Bob$_j$'s measurement is thus $\boxed{z_j}\otimes\boxed{z_j}$.

Let us consider the decorated graph G' [Fig. 2(a)] of the graph G, which is created by adding an extra vertex to

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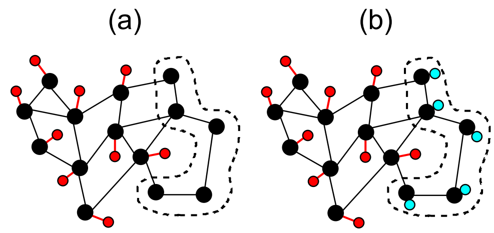

FIG. 2. (Color online) (a) The decorated graph G' created from the graph G of Fig. 1(a). (b) Each blue circle is the completely mixed state $\frac{I}{2}$. The entire state is now $|G'\rangle\langle G'|\otimes(\frac{I}{2})^{\otimes n}$.

each vertex of C in Fig. 1(a). We denote the graph state on the decorated graph G' by $|G'\rangle$. We also add n completely mixed states $(\frac{I}{2})^{\otimes n}$ to $|G'\rangle$ as is shown in Fig. 2(b). Now we have the $(2N+2n)$-qubit state $\boxed{G'}\otimes(\frac{I}{2})^{\otimes n}$. We claim that if we restrict local operations to projective measurements the (unnormalized) $(2N+2n)$-qubit state

$$
\begin{aligned}
W &\equiv 2^{N+n}\boxed{G'}\otimes\left(\frac{I}{2}\right)^{\otimes n}\\
&= 2^{N+n}\left(\bigotimes_{i=1}^{N}V_i\right)\left(\boxed{G}\otimes\boxed{+}^{\otimes N}\right)\left(\bigotimes_{j=1}^{N}V_j\right)\otimes\left(\frac{I}{2}\right)^{\otimes n}
\end{aligned}
$$

is a resource PM for acausal MBQC corresponding to the causal MBQC on $|G\rangle$. Here V_j is the controlled-Z gate, $V\equiv\boxed{0}\otimes I+\boxed{1}\otimes Z$, between the jth black qubit in C and jth red qubit $|+\rangle$ (indicated by red lines) of Fig. 2(a). Note that W satisfies Eq. (1) since W is nothing but an unnormalized quantum state. We will see later that Eq. (2) is also satisfied for measurements used in MBQC.

The probability of obtaining the measurement results $(m_1,\ldots,m_N,z_1,\ldots,z_n)\in\{0,1\}^{N+n}$ by Alice$_j$ $(j=1,\ldots,N)$ and Bob$_j$ $(j=1,\ldots,n)$ in the acausal MBQC is then given by

$$
\begin{aligned}
P\left(\phi_1^{m_1},\ldots,\phi_N^{m_N},z_1,\ldots,z_n\right) &= \mathrm{Tr}\left[W\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{m_s}}\otimes\boxed{m_s}\right)\otimes\left(\bigotimes_{t=1}^{n}\boxed{z_t}\otimes\boxed{z_t}\right)\right]\\
&= 2^{N+n}\mathrm{Tr}\left[\left(\bigotimes_{i=1}^{N}V_i\right)\left(\boxed{G}\otimes\boxed{+}^{\otimes N}\right)\left(\bigotimes_{j=1}^{N}V_j\right)\otimes\frac{I^{\otimes n}}{2^n}\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{m_s}}\otimes\boxed{m_s}\right)\otimes\left(\bigotimes_{t=1}^{n}\boxed{z_t}\otimes\boxed{z_t}\right)\right]\\
&= \sum_{(m_1',\ldots,m_N')\in\{0,1\}^N}\ \sum_{(m_1'',\ldots,m_N'')\in\{0,1\}^N}\mathrm{Tr}\left[\left(\bigotimes_{i=1}^{N}Z_i^{m_i'}\right)\boxed{G}\left(\bigotimes_{j=1}^{N}Z_j^{m_j''}\right)\otimes|m_1',\ldots,m_N'\rangle\langle m_1'',\ldots,m_N''|\right.\\
&\qquad\left.\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{m_s}}\otimes\boxed{m_s}\right)\otimes\left(\bigotimes_{t=1}^{n}\boxed{z_t}\right)\right]\\
&= \mathrm{Tr}\left[\left(\bigotimes_{i=1}^{N}Z_i^{m_i}\right)\boxed{G}\left(\bigotimes_{j=1}^{N}Z_j^{m_j}\right)\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{m_s}}\right)\otimes\left(\bigotimes_{t=1}^{n}\boxed{z_t}\right)\right]\\
&= \mathrm{Tr}\left[\boxed{G}\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{0}}\right)\otimes\left(\bigotimes_{t=1}^{n}\boxed{z_t}\right)\right].
\end{aligned}
$$

In this way, irrespective of the measurement results $(m_1,\ldots,m_N)$ on C, we can always obtain the result of the causal MBQC in the positive branch; i.e., all measurement results are correct $m_j=0$ $(j=0,\ldots,N)$.

Equation (2) is satisfied for measurements used in MBQC, since

$$
\begin{aligned}
\sum_{m\in\{0,1\}^N}\sum_{z\in\{0,1\}^n}P\left(\phi_1^{m_1},\ldots,\phi_N^{m_N},z_1,\ldots,z_n\right) &= \sum_{m\in\{0,1\}^N}\sum_{z\in\{0,1\}^n}\mathrm{Tr}\left[\boxed{G}\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{0}}\right)\otimes\left(\bigotimes_{t=1}^{n}\boxed{z_t}\right)\right]\\
&= 2^N\mathrm{Tr}\left[\boxed{G}\times\left(\bigotimes_{s=1}^{N}\boxed{\phi_s^{0}}\right)\otimes I^{\otimes n}\right]=1,
\end{aligned}
$$

where $m\equiv(m_1,\ldots,m_N)$, $z\equiv(z_1,\ldots,z_n)$, and we have used the fact that every branch of measurement histories occurs with the same probability in MBQC [6].

In Ref. [50] it was shown that the by-product operators cannot be avoided if we respect the no-signaling principle. This is because, if we can avoid by-products, a person who possesses C can create any state in O, and if another far separated person possesses O, then the first person can transmit information to the second person by encoding his message in the created state. Therefore, the acausal MBQC considered in this Rapid Communication should be in a class of signaling theory in the PM framework. In fact, in the acausal MBQC considered here we can always create the correct output quantum state in O without any by-product operators, since we can choose the correct branch. If girls encode a message in the output quantum state, boys can always learn the message by measuring their system. This means that the no-signaling from girls to boys is violated.

V. MBQC CAUSAL GAME

We can consider the following causal game whose causal inequality is violated by the acausal MBQC. Let us again consider the distributed MBQC in Fig. 1(b). Let P_0 be the probability of obtaining the all zero result 0...0 for Bob$_j$ ($j = 1,...,n$). In the causal MBQC, $P_0 \leqslant \frac{1}{2}(1 + \frac{1}{2^n})$, because if all girls are causally past all boys, and all girls are correctly ordered, then girls can steer boys' systems into the state $|0\rangle^{\otimes n}$ up to the by-product operators. If girls send the measurement result to boys, boys can correct the by-product operators, and can obtain $|0\rangle^{\otimes n}$. On the other hand, if all boys are causally past to all girls, and all girls are ordered correctly, then boys' systems are the n-qubit completely mixed state, and therefore the probability of obtaining the all zero result $|0\rangle^{\otimes n}$ is $\frac{1}{2^n}$. As we have seen in the previous section, however, if we consider acausal MBQC, $P_0 = 1$, since all girls and boys can always perform correct MBQC.

VI. CONCLUSION AND DISCUSSION

In this Rapid Communication, we have considered acausal MBQC in the PM framework. Assuming local operations are projective measurements, we have constructed a resource PM for acausal MBQC, and show that it is (up to a normalization factor and trivial ancilla qubits) equivalent to the decorated graph state created from the graph state of the corresponding causal MBQC.

Our result also suggests that acausal MBQC can be simulated on a causal MBQC with postselection [postselecting red qubits in Fig. 2(b)]. Since the simulation of the postselected MBQC is possible for a small-size MBQC, we might be able to experimentally simulate acausal MBQC on a small resource state. (Since the success probability exponentially decreases, larger systems would be hard to simulate.) Such an approach will be connected to recently developed important topics, namely, quantum simulations of phenomena beyond quantum physics [57]. It would be interesting to further explore relations to the result.

In this Rapid Communication we have considered only the qubit graph state MBQC with projective measurements. It would be a future research subject to generalize the present result to more general MBQC including local POVM measurements.

We finally mention that quantum computing without definite causal order was also studied in the circuit model with "quantum switch" [58]. Those authors provided an example of quantum computing which cannot be implemented by inserting a single use of a black box in a quantum circuit with fixed time order. Such quantum computing offers some advantages, such as black box discrimination problems [59] and reducing an unknown black box query complexity [60]. Since circuit models with projective measurements are equivalent to MBQC, it would be an interesting future study to consider relations between the present result and quantum switch.

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