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Gravity effects on light scattering from a simple fluid near its critical point

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The effect of the earth's gravity on light scattering from a simple fluid near its critical point is investigated. It is found that, in general, gravity influences the angular distribution of the scattered irradiance in three ways. First, the forward scattering at the critical point does not diverge. Second, the irradiance in any but the forward direction is not greatest at the critical temperature, but slightly above it. Third, Ornstein-Zernike-Debye plots of the inverse irradiance versus the square of the wave number are concave downward at small scattering angles. This last feature is usually attributed to a nonvanishing of the critical exponent η and thus indicates that the experimental evidence for this conjecture is misleading. Computations are peformed using an approximate equation of state to exhibit these deviations from the traditional Ornstein-Zernike theory.

I. INTRODUCTION

The divergence of the isothermal compressibility of a simple fluid at its critical point results in a large gravity-induced variation of density with height when the fluid is in the critical region. Even though this phenomenon has been well known for many years,[1] and in fact has been utilized in the measurement of critical exponents,[2] it is usually ignored in the analysis of scattering experiments. To recognize that this neglect is an oversimplification, recall that the scattered irradiance depends on the pair-correlation length which, in turn, is a rapidly varying function of the density.

The problem of taking into account a macroscopic density variation in calculating the scattered irradiance can be cleanly divided into two parts. First there is the electromagnetic aspect of the problem: owing to the change in local refractive index accompanying the density variation, the usual scattering Green's function for a uniform fluid must be modified.[3] Under the assumption that the density is continuous and its variation is small over distances comparable to both a wavelength of the exciting radiation (assumption of geometric optics) and the pair-correlation length, we have found a compact expression for the scattered irradiance in terms of the pair-correlation function.[4] This assumption is valid for those parts of the supercritical region ($T > T_c$) that are presently accessible in the laboratory. The statistical mechanics and thermodynamics needed to find an explicit expression for the pair-correlation function in terms of the macroscopic parameters of the system constitute the remainder of the problem.

In this article the pair-correlation function is evaluated at different heights in the sample by using the classical Ornstein-Zernike (OZ) theory[5,6] and a local-equilibrium argument. This provides us with an expression for the scattered irradiance in terms of the local compressibility of the fluid. From straightforward arguments we conclude that, in general, for any reasonable equation of state in the critical region, the gravity-induced inhomogeneity affects the scattering in three ways. First, the forward scattering no longer diverges at the critical point. Second, the scattered irradiance in any other direction achieves its maximum value at a temperature higher than T_c, the critical temperature. Third, even with the OZ expression for the correlation function, Ornstein-Zernike-Debye (OZD) plots of the inverse scattered irradiance versus the square of the wave vector are not linear, but curve towards the origin for small scattering angles.

Unlike the first two features mentioned above, lack of linearity at small angles has been experimentally observed.[7] Until now it was attributed solely to a deviation from the classical OZ theory near the critical point. Because gravity is responsible for similar behavior, it must be taken into account in any analysis of experimental data before one can infer a particular nonzero value for the critical exponent η.[8,9]

To illustrate these features we consider an approximate equation for the density profiles which has asymptotic validity and which Wilcox and Balzarini[2] have found suitable for xenon. We obtain analytic expressions for the forward scattering and angular distribution of the scattered irradiance for a fluid prepared such that the mean density is equal to its critical density. Using numbers typical of xenon with a 1-cm sample height, we give OZD plots of the inverse intensity versus the square of the wave vector, and compare these with the usual homogeneous predictions. We find marked differences in an experimentally accessible region of the critical point.

Recently Dobbs and Schmidt[10] have carried out similar calculations to determine the effect of gravity on small-angle x-ray scattering from argon. They have performed numerical computations for a particular collimation geometry using density profiles generated by the scaled equation of state of Sengers *et al.*[11] They have also considered the case where the mean density is not the critical density.[12] Their numerical computations indicate that the forward scattering does not diverge at the critical point and that the scattering at a particular angle is a maximum at a temperature different than T_c. Unlike the situation for light scattering, they conclude that deviations from the homogeneous assumption are not experimentally apparent for the shorter wavelength x-ray scattering, but may rapidly become so with a few technical improvements. They have not considered OZD plots which are usually used to see deviations from the ordinary OZ predictions.

II. MATHEMATICAL FORMULATION

In this section our aim is to find a general expression for the scattered irradiance in terms of the local density in the fluid. We shall draw three general conclusions before tackling a particular example in Sec. III.

The idealized experimental geometry considered here is the same as in our previous article. We define a Cartesian coordinate system (x, y, z) in the laboratory as follows: The origin is the centroid of the scattering volume V and the earth's gravitational force is in the $-\hat{z}$ direction. The incident beam is a plane monochromatic wave with incident intensity I_0, vacuum wave number κ_0, direction of polarization $\hat{x}$, and direction of propagation $\hat{z}$. The scattering volume has constant cross section in any horizontal plane and height $2L$. Within V the index of refraction n and the density ρ are monotone decreasing functions of z. To avoid discontinuities in n, index matching at the upper and lower boundaries ($z = \pm L$) is assumed. Moreover, ρ is taken to be equal to the critical density ρ_c for $z = 0$.

It is generally agreed[7] that the pair correlation function $G(r)$ in a three-dimensional homogeneous system close to its critical point may be expressed as

$$G(r) = (a/r^{1+\eta})e^{-\kappa r}, \tag{2.1}$$

for large values of r. In Eq. (2.1) κ and a are functions of ρ and T, and the critical exponent η is expected to be rather small: $0 \leq \eta \lesssim 0.1$. When η vanishes we have the usual OZ expression. Here we shall take η to be zero to demonstrate that gravity influences the scattered irradiance in the same manner as taking $\eta \neq 0$ and ignoring gravity altogether. From the assumption of local equilibrium we may regard κ and a as functions of the height z through their dependence on density ρ.

The irradiance I of the correlated scattering in the direction $\hat{n}$ is then given by[4]

$$I = Q \int_{-L}^{L} h(z) \frac{4\pi a}{\kappa^2 + q^2} dz, \tag{2.2}$$

where

$$Q = I_0 \frac{V}{2L} \left(\frac{n_0 k_0^2}{4\pi R}\right)^2 \cos\theta,$$

$$h(z) = \frac{1 - (n_0/n)^2(\hat{n}\cdot\hat{x})^2}{n(n^2-\gamma^2)^{1/2}} \rho^2 \left(\frac{\partial n^2}{\partial \rho}\right)^2,$$

$$q^2 = k_0^2\{n_0^2 \sin^2\theta + [n - (n^2-\gamma^2)^{1/2}]^2\}$$

$$n_0 = n(L), \quad \cos\theta = \hat{n}\cdot\hat{z}, \quad \gamma = n_0 \sin\theta,$$

R is the distance from the origin to the detector, and Eq. (2.2) is strictly valid only when $\cos\theta > 0$ (θ is the usual scattering angle). The incoherent scattering makes a negligible contribution in the critical region and we have ignored it.

The quantities in the integrand of Eq. (2.2) depend on z through their dependence on the density. The functions h and q^2 depend on the local refractive index n which in turn depends on the density through the Lorentz-Lorenz equation.[13,14] It is likely that a is roughly inversely proportional to the density, whereas κ^2 is inversely proportional to the isothermal compressibility and hence is a rapidly varying function of the density in the critical region.[6] Now, the total change in density for a typical macroscopic sample near the critical point is only on the order of a few percent. While κ^2 is very sensitive to this change through its dependence on $|\rho - \rho_c|$, the functions h, q^2, and a are not. Thus, to good approximation, we may evaluate h and q^2 by setting $n(z) = n_0 = n_C$ and evaluate a at $\rho = \rho_C$. Optically this means that the curvature of the light rays in the scattered field is neglected so that the restriction $\cos\theta > 0$ can be dropped. Moreover, because the compressibility is an even function of z, the corrections to this approximation are second order. Performing the above substitutions, we have

$$q^2 = 4k_0^2 n_C^2 \sin^2\tfrac{1}{2}\theta ,$$

$$n_0^2 h \cos\theta = \left(\rho\frac{\partial n^2}{\partial\rho}\right)_C^2 \sin^2\psi ,$$

$$I = Q'\mathscr{I} , \qquad (2.3)$$

$$Q' = I_0\left(\frac{V}{2L}\right)\frac{ak_0^{\,2}}{4\pi R^2}\left(\rho\frac{\partial n^2}{\partial\rho}\right)_c \sin^2\psi ,$$

$$\mathscr{I} = \int_{-L}^{L} dz\, \frac{1}{\kappa^2+q^2},$$

where $\cos\psi = \hat{n}\cdot\hat{x}$.

The only difference between the above and the homogeneous system with $\rho = \rho_C$ is that, in the latter case, $\mathscr{I}$ is replaced by

$$\mathscr{I}_h = \frac{2L}{\kappa_0^2+q^2}, \qquad (2.4)$$

where κ_0 is κ evaluated at $\rho = \rho_C$.

The remainder of the article is chiefly concerned with the behavior of $\mathscr{I}$ as compared with $\mathscr{I}_h$. In what follows the definitions

$$\beta = q^2/\kappa_0^2, \quad \kappa^2 = \kappa_0^2(1+U),$$

$$\epsilon = \frac{T-T_c}{T_c}, \quad \rho_r = \frac{\rho_c - \rho}{\rho_c},$$

will be convenient. For a given temperature κ^2 is a minimum at the critical density so U is an increasing function of $|\rho_r|$. Moreover, κ_0^2 vanishes at the critical point so U is a decreasing function of $|\epsilon|$ and diverges when $\epsilon = 0, \rho_r \neq 0$. In general, as $\kappa^2 \geq \kappa_0^2$, $U \geq 0$. All of the angular dependence appears in the dimensionless quantity β. From the compressibility equation[6]

$$c\kappa^2 = \left(\frac{\partial P}{\partial\rho}\right)_T ,$$

where P is the local pressure and c is essentially constant in the critical region. Thus the local value of κ^2 can be expressed in terms of the density profile, $z = z(\rho_r)$, by an application of the barometric equation:

$$\frac{\partial P}{\partial\rho} = g\frac{dz}{d\rho_r}, \quad \kappa^2 = \frac{g}{c}\frac{dz}{d\rho_r}, \qquad (2.5)$$

where g is the acceleration of gravity.

To obtain a useful expression for $\mathscr{I}$, write

$$\frac{1}{\kappa^2+q^2} = \frac{1}{\kappa^2}\left(1 - \frac{q^2}{q^2+\kappa^2}\right),$$

use Eq. (2.5) to express κ^2 in terms of $dz/d\rho_r$, and substitute the result into Eq. (2.5). The final expression may be conveniently expressed as

$$\mathscr{I} = (c/g)\Delta\rho_r J , \qquad (2.6)$$

where

$$J = 1 - [\beta/(1+\beta)]F, \qquad (2.7)$$

$$F = \frac{1}{\Delta\rho_r}\int_{\rho-L}^{\rho_L} d\rho_r\left[1 + \left(\frac{U}{1+\beta}\right)\right]^{-1}, \qquad (2.8)$$

$$\mathscr{I}_h = (2L/\kappa_0^2)J_h, \quad J_h = 1 - \beta/(1+\beta) . \qquad (2.9)$$

In the above, $\rho_L = \rho_r(L)$, $\rho_{-L} = \rho_r(-L)$, and $\Delta\rho_r = \rho_L - \rho_{-L}$. J is the angular distribution of the irradiance normalized such that $J(\theta = 0) = 1$ and J_h is simply the angular distribution so normalized for the homogeneous fluid.

At this point in the analysis it is immediately apparent that the scattering in the forward direction does not diverge at the critical point, but rather is proportional to the (bounded) change in density across the fluid. Clearly it is a maximum at the critical temperature. To observe that the scattered irradiance is a maximum for some $\epsilon > 0$ if $q^2 \neq 0$, we consider Eqs. (2.3) and (2.5). In particular consider the behavior of $d\rho_r/dz$ as a function of ϵ for fixed height. At $z = 0$, $d\rho_r/dz$ diverges as ϵ approaches zero from above. But $d\rho_r/dz > 0$ at all points in the sample and $\Delta\rho_r$ is bounded as ϵ vanishes. Thus, for any $z \neq 0$, there exists an ϵ_z such that $d\rho_r/dz$ decreases when ϵ decreases below ϵ_z. Conversely we can define a z_ϵ such that when $|z| > |z_\epsilon|$, $d\rho_r/dz$ increases with increasing ϵ, whereas for $|z| < |z_\epsilon|$, $d\rho_r/dz$ decreases with increasing ϵ. Now, for ϵ sufficiently small that $|z_\epsilon| < L$, the integral in Eq. (2.3) can be decomposed into two parts. For $|z| < |z_\epsilon|$ the contribution increases. But as ϵ approaches zero so does z_ϵ. Hence, for $q^2 \neq 0$, there is an $\epsilon > 0$ for which $\mathscr{I}$ is a maximum. For the case $q^2 = 0$ the singularity in the integrand of Eq. (2.3) at $\epsilon = 0$, $z = 0$ negates the argument and ensures that the forward scattering is a maximum at $\epsilon = 0$.

The quantity labeled F distinguishes J from J_h. Because $0 \leq F \leq 1$ it is apparent that $J_h \leq J \leq 1$. Like J_h, it is a simple matter to show that J is a monotone decreasing function of β, and hence of q^2. Qualitatively the angular distribution J is less peaked in the forward direction than J_h. In the limit of large ϵ, U vanishes and J approaches J_h. In fact U vanishes whenever there is a linear relationship between ρ_r and z because linearity implies that $\partial P/\partial\rho$, and hence κ^2 is independent of density. Thus gravity effects should not be regarded as due simply to changes in ρ_r with height, but rather to changes in $dz/d\rho_r$.

To demonstrate that OZD plots of $\mathscr{I}^{-1}$ vs q^2 are concave downwards for small scattering angles, we simply differentiate J^{-1} twice with respect to β and evaluate the derivatives at $\beta = 0$. The results may be expressed in the suggestive form

$$\left.\frac{dJ^{-1}}{d\beta}\right|_{\beta=0} = \langle H \rangle ,$$

$$\left.\frac{d^2J^{-1}}{d\beta^2}\right|_{\beta=0} = -2 \langle (H - \langle H \rangle)^2 \rangle ,$$

where $H = (1 + U)^{-1}$,

$$\langle H \rangle = \frac{1}{\Delta\rho_r} \int_{\rho_{-L}}^{\rho_L} d\rho_r H, \quad \text{etc.}$$

Thus the second derivative is negative and the concavity usually associated with a nonvanishing η is proved.

III. APROXIMATE DENSITY PROFILES AND COMPUTATIONS

Knowledge of the exact dependence of κ^2 on height and temperature requires a rigorous equation of state in the critical region. No such equation exists today. While a number of good candidates are available,[11,15] they are all very unwieldy for our purposes. We seek an analytically tractable representation of $\partial P/\partial\rho$ in the critical region from which rough quantitative predictions can be made regarding the behavior of $\mathcal{I}$. It must be exact on the critical isochore where the contributions to $\mathcal{I}$ are large and merely approximate in the remainder of the (ρ_r, ϵ) half-plane $(\epsilon > 0)$. Now, for small $|\rho_r|$, $\partial P/\partial\rho$ is dominated by its behavior on the critical isochore,

$$\left.\frac{\partial P}{\partial\rho}\right|_{\rho_r=0} = A\epsilon^{\gamma}, \tag{3.1}$$

whereas for large $|\rho_r|$, $\partial P/\partial\rho$ is dominated by its behavior on the critical isotherm,

$$\left.\frac{\partial P}{\partial\rho}\right|_{\epsilon=0} = \delta B |\rho_r|^{\delta-1} . \tag{3.2}$$

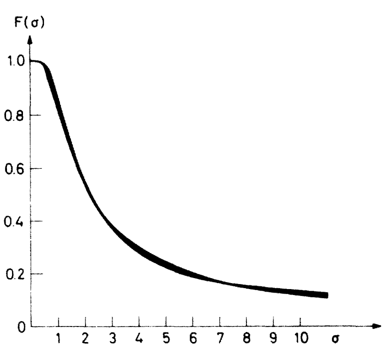

FIG. 1. Function $F(\sigma)$ as defined in Eq. (3.6). The width of the curve corresponds to $4 < \delta < 5$.

In Eqs. (3.1) and (3.2), A and B are essentially constant in the critical region and γ and δ are the usual critical exponents.[8] Simply combine Eqs. (3.1) and (3.2) to obtain the approximation

$$c\kappa^2 = \frac{\partial P}{\partial\rho} = A\epsilon^{\gamma} + \delta B |\rho_r|^{\delta-1} . \tag{3.3}$$

Integrate the barometric equation to construct the approximate density profiles

$$gz = \rho_r (A\epsilon^{\gamma} + B|\rho_r|^{\delta-1}) . \tag{3.4}$$

For large values of ϵ the density profiles generated by Eq. (3.4) are asymptotically correct for small values of $|z|$ and slowly deteriorate for large values. For small ϵ they are correct for large $|z|$ and deteriorate in a small neighborhood of $z = 0$. For intermediate values of ϵ there is an intermediate range of z for which Eq. (3.4) is a rough approximation. In the above, "small" and "large" cannot be rigorously defined without making direct reference to a particular equation of state. For example, for the scaled equation of state of Sengers *et al.*,[11] small ϵ means $0 < \epsilon \ll |\rho_r|^{1/\beta}$ and "large" ϵ means $\epsilon \gg |\rho_r|^{1/\beta}$. No other claims are made for Eqs. (3.3) and (3.4). In fact, as an equation of state, Eq. (3.3) does not satisfy the analyticity requirements in the (ρ_r, ϵ) plane set forth by Josephson[16] and Schofield.[17] However, the practicality of these profiles has been demonstrated by Wilcox and Balzarini,[2] who find good agreement with experimental observations on xenon.

Further analysis is expedited by introducing the

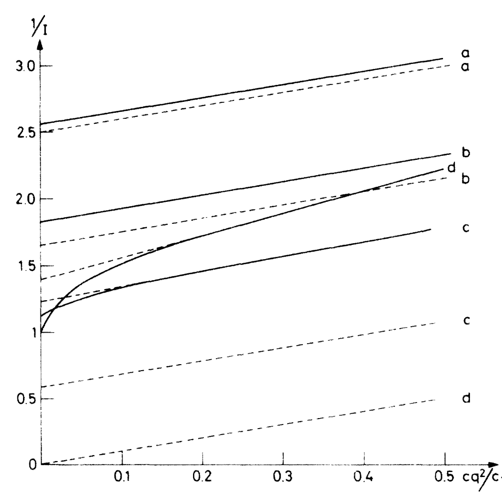

FIG. 2. OZD plots of inverse scattered irradiance vs cq^2/c_1 for different temperatures. The dotted curves are the homogeneous results. The values of $A\epsilon^{\gamma}/c_1$ are (a) 2.50, (b) 1.66, (c) 0.60, and (d) 0.001.

dimensionless variables u and v,

$$u = z/z_0, \quad v = \rho_r/\rho_0,$$

where

$$\rho_0 = (A\epsilon^\gamma/B)^{1/(\delta-1)}, \quad z_0 = (B/g)\rho_0^\delta .$$

In terms of these quantities Eq. (3.4) is simply

$$u = v(1 + |v|^{\delta-1}) . \qquad (3.5)$$

In view of the preceding discussion we expect good agreement with experiment for small and large $|v|$ and only approximate agreement when $|v|$ is on the order of unity. Now, the range of z over which u and v are linearly related is proportional to $\epsilon^{\gamma\delta/(\delta-1)}$. In this region κ^2 is indistinguishable from κ_0^2 which goes as ϵ^γ. Thus the contribution of the range of linear approximation to the forward scattering vanishes slowly as $\epsilon^{\gamma(\delta-1)}$ when T approaches T_c. Outside of this range, $1/\kappa^2$ does not diverge, confirming our earlier conclusion that the forward scattering is finite at the critical temperature.

To compute the angular distribution we need the functions U and F of Sec. II. For these profiles

$$U = \delta |v|^{\delta-1}, \quad F = \frac{1}{\sigma}\int_0^\sigma dx\, \frac{1}{1+x^{\delta-1}}, \qquad (3.6)$$

where

$$\sigma = \left(\frac{\delta}{1+\beta}\right)^{1/(\delta-1)} v_L ,$$

and we use u_L and v_L to indicate L/z_0 and ρ_L/ρ_0, respectively. Nearly all experiments on simple fluids yield $4 \leq \delta \leq 5$.[7] When δ is an integer, F can be expressed in terms of simple functions by the method of integration using partial fractions (see Appendix A) while, for nonintegral values of δ, F can be expressed in terms of the incomplete beta function. The behavior of F is roughly the same when δ is in this range (see Fig. 1).

Unfortunately, v_L and β are not the actual experimental variables. Rather, in a particular experiment, L is fixed and q^2 and ϵ are varied. Note that

$$\beta = \frac{cq^2}{A\epsilon^\gamma} = \left(\frac{cq^2}{c_1}\right) u_L^{(\delta-1)/\delta} ,$$

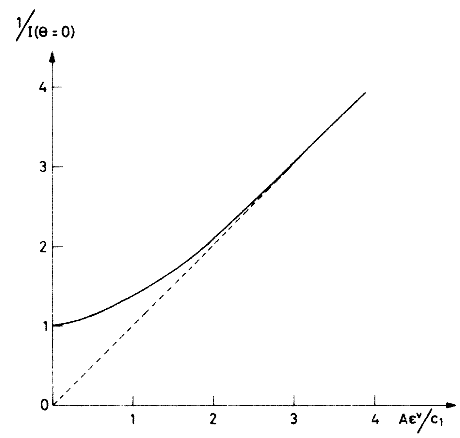

FIG. 3. Inverse forward scattering vs temperature.

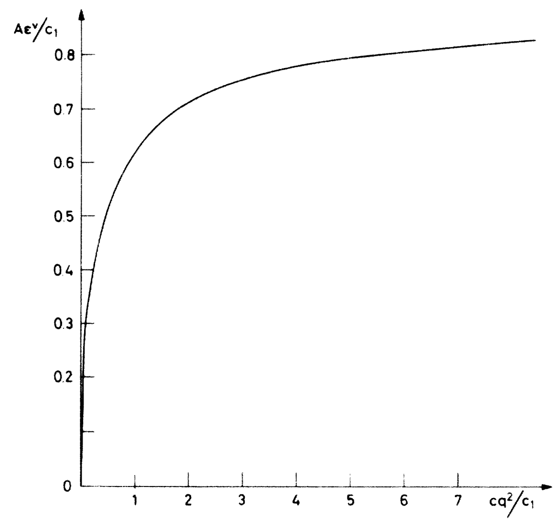

FIG. 4. Temperature corresponding to maximum scattered irradiance as a function of scattering angle.

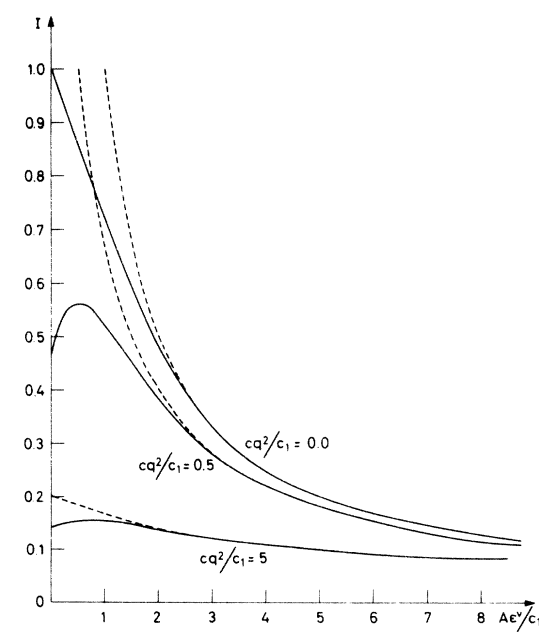

FIG. 5. Scattered irradiance as a function of temperature for fixed values of the scattering angle.

where

$$c_1=[B(gL)^{\delta-1}]1/\delta, \quad u_L^{(1-\delta)/\delta}=A\epsilon^\gamma/c_1.$$

When δ is specified, v_L and hence F can be expressed in terms of u_L by inverting Eq. (3.5) (see Appendix A). Thus the natural angular dependent variable is cq^2/c_1 and the natural temperature-dependent variable is $A\epsilon^\gamma/c_1$.

It is convenient for computational purposes to normalize the irradiance so that it is unity at $\theta=0$, $\epsilon=0$. Treating $\mathcal{I}_h$ in the same manner we find

$$\hat{I}=v_L u_L^{-1/\delta}J, \tag{3.7}$$

$$\hat{I}_h=u_L^{(\delta-1)/\delta}J_h \tag{3.8}$$

for the irradiance so normalized. In Fig. 2 we have constructed OZD plots of $1/\hat{I}$ and $1/\hat{I}_h$, vs cq^2/c_1 for different values of $A\epsilon^\gamma/c_1$. Here, and in the remaining computations, we have taken $\delta=4$. The range of cq^2/c_1 is taken as 0.5 corresponding to light scattering from xenon with a sample height of 1.0 cm (see Appendix B). For large values of ϵ there is a linear relation between density and height in the fluid and the results are very close to the homogeneous predictions. But as ϵ decreases, the curves calculated from Eq. (3.7) start to bend downwards for small scattering angles and the slope of the approximately straight part changes. Notice that, as expected, for some scattering angles the irradiance increases with temperature. We shall return to this point shortly. In Fig. 3 the inverse forward scattering is plotted versus $A\epsilon^\gamma/c_1$. For small ϵ the values slowly approach unity as anticipated. The dotted curve in Fig. 3 is the corresponding homogeneous result.

It is useful to compare the results illustrated in Figs. 2 and 3 with the experimental results. As stated by Fisher,[8] experimental results diverge in two ways from the prediction of the homogeneous treatment. First there is a tendency in OZD plots for the curves taken near T_c to be slightly curved and to dip downwards for the smallest accessible values of q^2. Second the intercepts obtained by extrapolating the best straight-line fits to the data do not approach zero as ϵ vanishes, but rather are concave upward when plotted versus ϵ along the critical isochore and extrapolate to a nonzero value for $\epsilon=0$. As can be seen in Fig. 2, both of these effects also result from taking gravity into account with $\eta=0$. It is suggestive that the curves calculated from Eq. (3.7) yield $\eta\cong 0.1$ when fitted to the homogeneous theory with $\eta\neq 0$. This is about the value usually expected for η. Thus, if η is not zero, gravity effects disguise its experimental manifestation. Of course, the homogeneous theory even with $\eta\neq 0$ still predicts a divergent forward scattering at the critical point and a straight line in Fig. 3. Thus, in principle, gravity effects can be experimentally distinguished from a nonvanishing η.

As indicated earlier, the irradiance in a particular direction achieves its maximum value at a temperature above T_c. The only exception is the forward direction. A somewhat lengthy but straightforward calculation shows that the value of ϵ for which the irradiance is a maximum can be computed from

$$\beta=\left(\frac{\delta-1}{\delta-2}\right)[F(\sigma)(\sigma^{\delta-1}+1)-1]^{-1}.$$

This curve is plotted in Fig. 4, and Fig. 5 demonstrates the temperature dependence of the irradiance for different scattering angles.

APPENDIX A

The indefinite integral

$$\int dx\,\frac{1}{1+x^{\delta-1}}$$

can be evaluated by the method of partial fractions. For $\delta=4$ we obtain

$$\frac{1}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}(x-\tfrac{1}{2})\right)+\tfrac{1}{3}\ln\left(\frac{1+x}{(1-x+x^2)^{1/2}}\right),$$

and for $\delta=5$,

$$\frac{1}{4\sqrt{2}}\left[\ln\left(\frac{x^2+(\sqrt{2})x+1}{x^2-(\sqrt{2})x+1}\right)+2\arctan[(\sqrt{2})x+1]+2\arctan[(\sqrt{2})x-1]\right].$$

For $\delta=4$, Eq. (3.5) can be inverted as follows. Let

$$\omega=s_++s_-,$$

where

$$s_\pm=\{\tfrac{1}{2}\pm[\tfrac{1}{4}+(\tfrac{4}{3}u)^3]^{1/2}\}^{1/3}.$$

Then the desired root of Eq. (3.5) is

$$v=\tfrac{1}{2}(-\omega^{1/2}+\{4[u+(\tfrac{1}{2}\omega)^2]^{1/2}-\omega\}^{1/2}).$$

APPENDIX B

Wilcox and Balzarini have measured the dependence of the refractive index of xenon on height in the critical region. Using the Lorentz-Lorenz relation they determined that the density profiles were in reasonable agreement with Eq. (3.4) with $\delta\simeq 4$, $A\simeq 8.7\times 10^7\ \mathrm{cm^2/sec^2}$, and $B\simeq 1.9\times 10^3$ $\mathrm{cm^2/sec^2}$.

The classical OZ theory gives[6]

$$c = l^2 k_B T/10m,$$

where m is the atomic mass, k_B is Boltzmann's constant, and l is on the order of the range of the interatomic force. For light scattering experiments we choose $l/\lambda = 10^{-3}$, where λ is the vacuum light wavelength, to find

$$\sup (cq^2/c_1) \simeq 0.5 .$$

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