Force-neutral beams and limiting currents*

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(Received 23 October 1973)

For intense relativistic electron-beam propagation into low-pressure neutral gas in a metallic drift tube, it is shown for typical experimental parameters that the beam leaves the anode only when it is almost space-charge neutral, and that a fully propagating beam occurs only if it is completely space-charge neutral. Ion ionization effects are shown to play a crucial role in the charge-neutralization process, and a new interpretation of existing experimental data is given.

Intense relativistic electron beams have been utilized in such diversified areas as simulation effects, collective ion acceleration, microwave generation, and controlled-thermonuclear-fusion research. Relevant to many of these areas (especially collective ion acceleration) is the process of intense beam transport in low-pressure neutral gases. New results reported here contradict previous interpretations of this process, and offer a new interpretation supported by analytical and numerical computations.

An intense relativistic electron beam is said to be radially force neutral if the fractional space-charge neutralization f_e equals γ^{-2} (where γ is the relativistic factor). This condition implies that the magnetic self-pinch force just balances the repulsive net electrostatic force for a beam electron with an axially directed velocity. In experiments where an intense beam is injected into a metallic drift tube filled with neutral gas at a low pressure, it has been observed[1-4] that the beam remains near the anode until a time about equal to τ^e_{FN}, after which it propagates downstream ($\tau^e_{FN} \equiv \tau^e_N \gamma^{-2}$, where τ^e_N is the time required for the beam electrons to collisionally ionize a volume of the neutral gas up to a density equal to the beam density). A resultant picture of beam propagation[3,4] is that (i) the beam waits near the anode until $f_e = \gamma^{-2}$, after which time, (ii) a propagating force-neutral beam can exist downstream. For typical experimental parameters[1,2] we show here analytically and numerically that (i) the bulk of the beam does not leave the anode region when $f_e = \gamma^{-2}$ and that (ii) a propagating intense beam with $f_e = \gamma^{-2}$ never occurs downstream. The reason that the beam does not leave the anode when $f_e = \gamma^{-2}$ is that the beam current is typically much larger than the space-charge limiting current. The reason the beam can leave the anode at the observed time τ^e_{FN} is that at this time, ionization processes by the *ions* can create enough ionization to make f_e of order unity.

The space-charge limiting-current problem may be stated simply as follows. Imagine a uniform, propagating, intense electron beam of radius r_b, current I_0, and kinetic energy $(\gamma - 1)mc^2$ [where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, $\beta = V/c$, V is the beam-electron velocity, m is the mass of an electron, and c is the speed of light] that is propagating inside a metallic drift tube of radius R. Then for distances sufficiently far away from the ends of the drift tube (i.e., $\gtrsim R$), the space-charge field is mainly radial, and the difference in electrostatic potential between the drift-tube wall (taken to be at zero potential) and the center of the beam is calculated to be

$$\varphi_0 = (I_0/\beta c)[1 + 2\ln(R/r_b)](1 - f_e) . \tag{1}$$

Here it is assumed that the average axial velocity $\beta_z c \approx V$, that there is no current neutralization, and that there is a fractional space-charge neutralization ($0 \leqslant f_e \leqslant 1$). For $0 \leqslant f_e \leqslant \gamma^{-2}$ the beam will blow up radially. For $\gamma^{-2} < f_e < 1$, the beam-particle trajectories will be oscillatory, β_z will be less than β, and the electrostatic potential will be slightly greater than that given by (1). For this case, and for $\nu/\gamma \ll 1$ (where ν is the number of electrons per classical electron radius), β_z is

$$\beta_z \approx \beta \{ 1 + (\nu/\gamma)[1 - (1 - f_e)\beta^{-2}] \}^{-1/2} .$$

For a force-neutral beam with axially directed trajectories, $\beta_z = \beta$ exactly. Thus, for rough estimates, it is useful to simply set $\beta_z = \beta$ for all cases, as was done in (1). Since the beam electrons enter the drift tube through the anode foil (which must be at the same potential as the drift tube, i.e., zero), it follows that for the beam to exist in a propagating state, the beam electrons must have kinetic energy greater than the electrostatic energy $e\varphi_0$, where e is the charge of an electron. Setting $e\varphi_0$ equal to $(\gamma - 1)mc^2$, and solving for I_0, gives for the limiting current

$$I_l = \beta(\gamma - 1)(mc^3/e)[1 + 2\ln(R/r_b)]^{-1}(1 - f_e)^{-1} . \tag{2}$$

In detailed studies[5-7] of the limiting-current problem for "uncompensated" beams ($f_e = 0$), an infinite axial magnetic field was assumed and the radial variation of the axial velocity of the beam particles was taken into account. In the nonrelativistic limit ($\beta \ll 1$), the simple result (2) differs from the numerical results of Smith and Hartman[5] and the results of Calbick[6] by at most a factor of about 2. In the ultrarelativistic limit ($\gamma \gg 1$), Eq. (2) agrees exactly with the results of Bogdankevich and Rukhadze.[7] The latter also present an interpolation formula, roughly valid for all cases,

$$I_l = (\gamma^{2/3} - 1)^{3/2}(mc^3/e)[1 + 2\ln(R/r_b)]^{-1}. \tag{3}$$

The result (2) agrees with (3) to within a factor of about 2, and also shows the correct functional dependence $I_l \sim \epsilon^{3/2}$ in the nonrelativistic limit, and $I_l \sim \epsilon$ in the ultrarelativistic limit [where the electron kinetic energy $\epsilon = (\gamma - 1)mc^2$].

Another view of the limiting-current problem is to consider the case of a beam that is stopped by its own space charge at the anode. If the beam penetration distance is small compared to r_b, then the one-dimensional (1D) results of Poukey and Rostoker[8] apply, which say the beam should penetrate into the drift tube a distance of order $2c/\omega_p$ [where $\omega_p^2 \equiv (4\pi ne^2)/(\gamma m)$ and n is the beam density in the laboratory frame]. Thus "1D stopping" should occur for $2c/\omega_p < r_b$, whereas 2D propagation may occur for $2c/\omega_p > r_b$. The transition occurs for $2c/\omega_p \approx r_b$, for which the current ($\pi r_b^2 ne\beta c$) is

$$I_T = \beta\gamma(mc^3/e). \tag{4}$$

Note that the limiting current [(2) or (3)] is always less than the transition current [(4)], so the former will determine whether or not the beam will propagate. Also note that all of the currents, (2)–(4), are of the order of the Alfvèn-Lawson current[9-11] for magnetic stopping of a charge-neutralized beam, $I_A \equiv \beta\gamma mc^3/e$, although all of the limiting-current effects discussed above are due to space-charge fields.

The limiting currents imposed by (2) or (3) have been seen experimentally.[12,13] In one case[13] the injected current was larger than (3), but much less than (4), and the propagating current was of the order of (3) for distances much larger than R from the anode. The experiments were performed with a large axial magnetic field in a vacuum. Here our concern is to establish how (2)–(4) affect beam propagation in low-pressure neutral gas with no external magnetic field. Specifically experiments have been performed in H_2 at pressures of 0.1–0.3 Torr.[1,2] For typical parameters[1] ($\gamma \approx 3$, $I_0 = 40$ kA, $R/r_b = 6$), if a force-neutral beam could propagate and keep the same radius at all distances (as assumed in Refs. 3 and 4), then the potential-energy drop to the center of the beam downstream would be $e\varphi_0 \approx 5$ MeV [using (1)], whereas the beam energy is only 1 MeV. Clearly such a beam could not exist. To verify this, and show that the beam cannot leave the anode region when $f_e = \gamma^{-2}$, we have performed numerical simulations using the following ionization processes.

If only collisional ionization by the beam electrons is considered, and instantaneous radial escape of the secondary electrons is assumed for $0 \leq f_e < 1$ (see, e.g., Ref. 8 or 14), then the background-ion density grows as

$$\frac{\partial n_i(t)}{\partial t} = \frac{n_b(t)}{\tau_e}, \tag{5}$$

where $n_b(t)$ is the beam electron density. For $\gamma \approx 3$, $\tau_e \approx 5/p$ nsec for H_2, where p is the pressure in Torr.[15] At low pressures ($p \sim 0.1$ Torr), even τ_{FN}^e is several nsec, and there is sufficient time for the background ions to move in the electron beam's space-charge field and create additional ionization. The ions are effectively much better ionizers than the fast beam electrons, and the process may be treated as an avalanchelike process with

$$\frac{\partial n_i(t)}{\partial t} = \frac{n_b(t)}{\tau_e} + \frac{n_i(t)}{\tau_i}, \tag{6}$$

where τ_i is an effective avalanche time.[16] A useful estimate is $\tau_i \approx 0.3/p$ nsec, where p is the pressure in Torr.[16] The ion ionization process in (6) results in a much more rapid growth of f_e than (5) produces.

A number of computer simulation runs were made to establish (i) if the beam leaves the anode when $f_e \approx \gamma^{-2}$ or when f_e is large enough so that $e\varphi_0$ is $\lesssim \epsilon$, (ii) if a "propagating" force-neutral beam can occur downstream, and (iii) which ionization processes [(5) or (6)] permit agreement with the data. The simulation runs were made with a 2D, finite-size-particle code adapted from one previously described.[17] For typical parameters[1,2] and using (5) with $p = 0.1$-Torr H_2, it is found that the beam remains near the anode for times much longer than τ_{FN}^e, and that the beam never really breaks away until $t \approx \tau_N^e$. This confirms our supposition that the beam should not propagate when $f_e = \gamma^{-2}$ because $e\varphi_0$ would be greater than ϵ. When the beam does propagate, it initially fills the tube owing to beam spreading, and the peak electrostatic potential energy is at most of order of the beam energy. Also it is clear from the computer studies that a propagating force-neutral beam in a drift tube is not a valid concept in that f_e depends on the axial distance and also must be of order unity (not γ^{-2}) for the beam to even propagate. With only electron ionization as in (5), the beam does not leave the anode until $t \approx \tau_N^e$, whereas it is observed experimentally[1-4] to leave at $t \approx \tau_{FN}^e$. However,

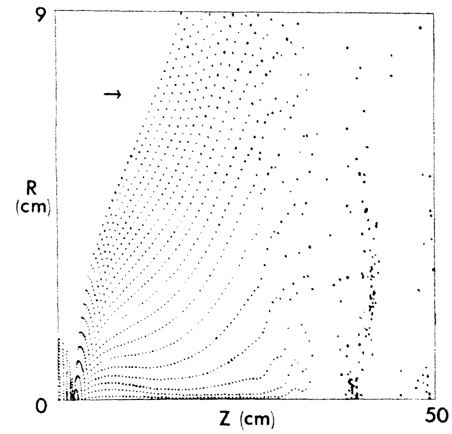

FIG. 1. Computer simulation of 3 MeV, 40-kA electron beam of radius 1.5 cm injected at $z=0$ into 0.3 Torr of initially neutral H_2 gas. Time is 3 nsec. Beam is being lost radially due to space-charge blowup.

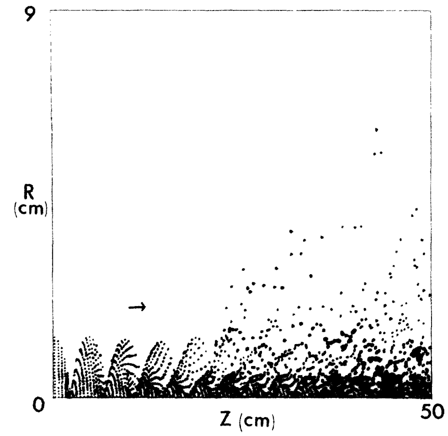

FIG. 2. Same as Fig. 1, but time is 9 nsec. Beam has just become completely space-charge neutral. The series of pinches is caused by the beam self-magnetic field.

including ion ionization as in (6), the true charge-neutralization time $\tau_N^{e,i}$ agrees with the experimentally observed time at which the beam leaves the anode. The time $\tau_N^{e,i}$ may be obtained by integrating (6) with appropriate initial conditions,[16] or by observing a simulation run which employs (6).

As a specific example, consider $\gamma=3$, $I_0=40$ kA, $r_b=1.5$ cm, $R=9$ cm, and a drift-tube length of 50 cm.[1] Then for $p=0.3$-Torr H_2, and assuming a linear-rise-time beam, the various times are $\tau_N^e \approx 33$ nsec, $\tau_{FN}^e \approx 3.7$ nsec, and $\tau_N^{e,i} \approx 3.3$ nsec. The simulation run using (6) shows that the beam leaves the anode at a time of about 3 to 4 nsec. Experimentally the beam leaves the anode at a time of about 4 nsec.[1] Thus in this and other cases, (2) and (6) and the simulation runs agree with the data. Using the same parameters but with $\gamma=7$ produces the simulation results shown in Figs. 1 and 2. In Fig. 1 the beam is just beginning to propagate. At $z=0$, $f_e \approx 1$. At $z=25$ cm, $f_e \approx 0.1$ which is still much larger than $\gamma^{-2} \approx 0.02$. The peak electrostatic potential energy is at $z \approx 8$ cm and is 1.8 MeV (which is less than the beam energy of 3 MeV). At later times, as in Fig. 2, the beam finally assumes the characteristics of a beam propagating with radius $\approx r_b$, but this does not occur until the beam has become completely space-charge neutral.

In summary, for typical experimental parameters, (i) the beam leaves the anode only when $f_e \approx 1$, (ii) ion ionization effects play a crucial role in establishing charge neutralization, (iii) a force-neutral-beam stage of propagation does not occur, and (iv) the entire beam propagates only when it is completely space-charge neutral. Result (iii) apparently contradicts the basic assumptions of some recent ion acceleration theories.[3,4] These theories are based on the assumption of a propagating force-neutral beam, which we have found does not occur for typical experimental parameters.[1,2]

It should be noted that for $I_0 \ll I_l$, the beam will leave the anode immediately, and the force-neutral condition will simply mark the transition from spreading trajectories to oscillatory trajectories. For example, the electron-ring accelerator concept is based on using a beam with $f_e \gtrsim \gamma^{-2}$, and this is a valid concept provided $e\varphi_0 \ll \epsilon$ at the stage when the beam is leaving the anode region and forming the ring.

*Work supported by the U. S. Atomic Energy Commission.

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