Anisotropic susceptibilities and NMR shifts in superfluid ^{3}He †

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The effect of terms describing the anisotropic susceptibilities on the NMR shifts observed in superfluid ^{3}He is discussed. It is shown that they are such that the isotropic susceptibility in the expressions of Leggett is replaced by the correct temperature-dependent susceptibilities. The anisotropy energies of the Balian-Werthamer (BW) state are calculated and shown to be of order $(\gamma H/T_c)^2$ times those pinning the spin and orbit coordinates in either the BW or Anderson-Brinkman-Morel state.

I. INTRODUCTION

Since the discovery of the new phases in ^{3}He by Osheroff, Richardson, and Lee,[1,2] considerable interest has been focused on the shifts observed in the NMR experiments.[2] That such shifts could occur in an anisotropic superfluid phase was first pointed out by Leggett[3] and by Anderson and Varma.[4] An attempt to give a more complete account of the resonances was published by Anderson,[5] where although the dipole interaction was used, the anisotropy in the susceptibility played a central role. Recently, Leggett[6] has calculated the resonance shifts and in his calculations the shifts arise from quadratic dependences of the dipolar energy on the angles between the coordinate systems specifying the spin and orbital components of the condensed pairs. Leggett showed that the spin axes are forced to rotate in a resonance experiment since the spin operators are generators of rotations in spin space. The nonexistence of shifts in the Balian-Werthamer (BW)[7] state is due to the fact that there is no quadratic dependence on the angles involved in transverse resonance.

In his calculations, Leggett did not take into account the anisotropic nature of the susceptibility. It is the purpose of this paper to introduce into the phenomenological Hamiltonian used by Leggett additional terms which describe the anisotropy in the susceptibility and to estimate the importance of these terms on the resonance frequencies.[8] We show that the corrections are equivalent to introducing the proper anisotropic susceptibilities into the expressions for the frequency shifts.

The terms describing the anisotropic susceptibilities can also be used to calculate the anisotropy energies, briefly mentioned by Leggett, which tend to orient the various axes with respect to the external field in the BW state. The energies for orienting the axes relative to the external field in such a way that there are no transverse resonance shifts is shown to be equal to $(\gamma H/T_c)^2$ times the anisotropy energies involved in the orientation of the spin and orbital systems with respect to one another. Therefore, in a 1-kG field, these energies are 10^{-2} times the spin-orbit interaction energy, and one might suppose that if one can break the orientation between the field and the spin-orbit axes, one could also break the relation between the spin and orbit coordinates. Resonance under these circumstances should indeed be unusual.

II. CORRECTIONS TO RESONANCE FREQUENCIES

Our general approach to the nuclear magnetic resonance problem in ^{3}He will parallel that used by Leggett.[6] The starting phenomenological Hamiltonian is

$$H = \frac{\gamma^2}{2}\,\frac{1}{\chi_0} M^2 - \gamma \vec{\mathcal{H}} \cdot \vec{\mathrm{M}} + H_D = H_1 + H_D \,, \tag{1}$$

where γ is the nuclear magnetic moment of a ^{3}He atom, χ_0 is the normal isotropic susceptibility of liquid ^{3}He, $\vec{\mathcal{H}}$ is the external magnetic field, $\vec{\mathrm{M}}$ the total magnetization and H_D is the dipolar interaction which breaks the invariance of the Hamiltonian under separate space and spin rotations,

$$H_D = \frac{\gamma^2}{2} \int d^3r\, d^3r'\, \mathfrak{D}(\rho)[\vec{\mathrm{S}}(r)\cdot\vec{\mathrm{S}}(\vec{r}') - 3\hat{\rho}\cdot\vec{\mathrm{S}}(r)\hat{\rho}\cdot\vec{\mathrm{S}}(r')] \,, \tag{2}$$

$$\vec{\rho} = |\vec{r} - \vec{r}'| \,, \quad \mathfrak{D}(\rho) = 1/\rho^3 \,.$$

We will work with the spin densities $\vec{\mathrm{S}}(\vec{r})$, rather than with the gap operators $T(n)$ used by Leggett. It is only at the last stage when one evaluates the resonance matrix that the nature of the superfluid state is taken into account.

In addition to the above Hamiltonian, we will con-

sider additional terms which must enter a phenomenological treatment of the superfluid state. These may be written in terms of the gap matrix operators $d_{\alpha i}$ used by Brinkman and Anderson[9]:

$$d_{\alpha i} = \int \frac{d\Omega_k}{4\pi} Y_\alpha(\hat{k}) a_{-k\mu}(\sigma_y \sigma_i)_{\mu\nu} a_{k\nu} , \tag{3}$$

where $Y_\alpha(\hat{k}) = Y_\alpha(-\hat{k})$ is the αth component of the real spherical harmonics corresponding to a gap function having unit orbital angular momentum; σ_i represents the ith Pauli matrix.

The gap operator introduced in Eq. (3) and its Hermitian adjoint have nonvanishing expection values in the superfluid state.

The additional terms in the Hamiltonian are written as

$$\Delta H = c_1 M_i d^\dagger_{\alpha i} d_{\alpha j} M_j + i c_2 \epsilon_{ijk} d^\dagger_{\alpha i} d_{\alpha j} M_k$$
$$\equiv c_1 |\vec{M}\cdot\vec{d}_\alpha|^2 + i c_2 \vec{M}\cdot(\vec{d}^\dagger_\alpha \times \vec{d}_\alpha) . \tag{4}$$

That both these types of terms may enter on symmetry grounds is fairly obvious. That a term of the form $|d_{\alpha i}|^2 M_j^2$ does not occur is a result of microscopic theory and not true in general. One of the most important effects of the first term (with coefficient c_1) is that it leads to the anisotropic susceptibility in the A phase of superfluid ^{3}He, and the reduction of the isotropic susceptibility in the B phase. Since we are expanding the energy in powers of both the magnetization and the order parameter, the χ_0 in Eq. (1) is the normal susceptibility. This approach differs from Leggett in that he introduces a "thermal dynamic susceptibility" as χ_0 and does not take into account the higher-order terms. The second term (with coefficient ic_2) leads to the description of the linear dependence of the gap function on the magnetic field. The order of magnitude of c_2 may be obtained using a calculation of Brinkman,[10] or from the linear splitting of the transition in a magnetic field.[11] As we will show this second term does not affect the resonance frequency and so we will not review the calculation of c_2.

To obtain the coefficient c_1 we calculate the susceptibility $\chi_{\alpha\beta}$ using the total Hamiltonian $H + \Delta H$:

$$\chi_{\alpha\beta} = -\frac{\partial^2\langle H + \Delta H\rangle}{\partial\mathcal{H}_\alpha \partial\mathcal{H}_\beta} \tag{5}$$

subject to the condition

$$\frac{\partial\langle H + \Delta H\rangle}{\partial M_i} = 0 .$$

The expectation values we will consider are those appropriate to the Anderson-Brinkman-Morel (ABM) state[12] corresponding to the A phase and the BW[7] state corresponding to the B phase.

In the ABM state the only nonvanishing expectation values of d are

$$\langle d_{xy}\rangle_{\rm ABM} = \Delta/\sqrt{2} , \quad \langle d_{zy}\rangle_{\rm ABM} = i\Delta/\sqrt{2} . \tag{6}$$

For the BW state we will only use the unitarity condition on the expectation value

$$\langle d^\dagger_{\alpha i} d_{\alpha j}\rangle_{\rm BW} = \tfrac{1}{3}\Delta^2 \delta_{ij} . \tag{7}$$

We obtain in the ABM state

$$\chi_{xx}^{-1} = \chi_{zz}^{-1} = \chi_0^{-1} , \quad \chi_{yy}^{-1} = \chi_0^{-1} + 2c_1\Delta^2/\gamma^2 . \tag{8}$$

Whereas in the isotropic BW state

$$\chi_{ii}^{-1} = \chi_0^{-1} + \tfrac{2}{3}(c_1/\gamma^2)\Delta^2 . \tag{9}$$

Thus as mentioned previously c_1 determines the anisotropy of the susceptibility in the A phase and the reduction in the susceptibility in the B phase.

To determine the resonance frequencies, we will need the equation of motion for the magnetization

$$i\hbar\frac{\partial\vec{M}}{dt} = [\vec{M}, H + \Delta H] . \tag{10}$$

The terms in ΔH will not contribute to this equation of motion because of their invariance under rotations in spin space. This follows mathematically making use of the commutation properties

$$[M_i, d_{\gamma j}] = i\hbar\epsilon_{ijk} d_{\gamma k} , \quad [M_i, d^\dagger_{\gamma j}] = i\hbar\epsilon_{ijk} d^\dagger_{\gamma k} , \tag{11}$$

which is easily derived using (3). To find the effect of the dipolar term, we need the commutator

$$[S_i(r), M_j] = ih\epsilon_{ijk} S_k(\vec{r}) \tag{12}$$

which is also easily derived. The resulting equation of motion is

$$\frac{dM_i}{dt} = \gamma(\vec{M}\times\mathcal{H})_i$$
$$-3\gamma^2\int dr\, dr'\, \mathfrak{D}(\rho)[\hat{\rho}\times\vec{S}(\vec{r})]_i[\hat{\rho}\cdot\vec{S}(\vec{r}')] . \tag{13}$$

To obtain the resonance frequencies, we go to the second-order differential equation:

$$\frac{d^2M_i}{dt^2} = \gamma\left(\frac{d\vec{M}}{dt}\times\vec{\mathcal{H}}\right)_i - 3\gamma^2\int dr\,dr'\,\mathfrak{D}(\rho) \times\left[\left(\frac{\hat{\rho}\times d\vec{S}(r)}{dt}\right)_i [\hat{\rho}\cdot\vec{S}(r')] + [\hat{\rho}\times\vec{S}(r)]_i\left(\frac{\hat{\rho}\cdot d\vec{S}(r')}{dt}\right)\right] . \quad (14)$$

The shifts of the resonance frequencies from the Larmor frequency are determined by the equation of motion for the spin density $\vec{S}(r)$ which appears in the dipolar interaction. We assume that the time-dependent part of $S_i(\vec{r})$ is slowly varying on the scale of a coherence length so that we can use the phenomenological Hamiltonian to describe its motion. The additional terms of the Hamiltonian (4) require us to introduce a new operator which results from the commutator of the spin density with the gap operator

$$[S_i(r), d_{\alpha j}] = ih\epsilon_{ijl}\int\frac{d\Omega_k}{4\pi}Y_\alpha(\hat{k}) \times a_{-k\mu}(\sigma_y\sigma_l)_{\mu\nu}e^{-i\vec{k}\cdot\vec{r}}\psi_\nu(r) \equiv ih\epsilon_{ijl}d_{\alpha l}(\vec{r}) \quad (15)$$

or using the Fourier transform of the spin density operator

$$[S_i(q), d_{\alpha j}] = ih\epsilon_{ijl}\int\frac{d\Omega_k}{4\pi}Y_\alpha(\hat{k}) \times a_{-k,\mu}(\sigma_y\sigma_l)_{\mu\nu}a_{k+q,\nu} \equiv ih\epsilon_{ijl}d_{\alpha l}(\vec{q}) . \quad (16)$$

As long as we neglect the dispersion in the resonance frequency, corresponding to $q \to 0$, we may approximate $d_{\alpha l}(r) \cong d_{\alpha l}$.

We work to first order in the dipolar interaction and so may neglect H_D when calculating $\dot{\vec{S}}(r)$

$$\frac{dS_i(r)}{dt} = \gamma\left|\left(\frac{\gamma}{\chi_0}\vec{M}-\vec{\mathcal{H}}\right)\times\vec{S}(\vec{r})\right|_i + c_1\{[\vec{d}_\beta^{\,\dagger}\times\vec{S}(r)]_i(\vec{d}_\beta\cdot\vec{M}) + (\vec{M}\cdot\vec{d}_\beta^{\,\dagger})[\vec{d}_\beta\times\vec{S}(r)]_i + [\vec{M}\times\vec{d}_\beta^{\,\dagger}(r)]_i(\vec{d}_\beta\cdot\vec{M}) - (\vec{M}\cdot\vec{d}_\beta^{\,\dagger})[\vec{d}_\beta(r)\times\vec{M}]_i\} + ic_2\{[(\vec{d}_\beta^{\,\dagger}\times\vec{d}_\beta)\times\vec{S}(r)]_i - [\vec{d}_\beta^{\,\dagger}(r)\times(\vec{d}_\beta\times\vec{M})]_i - [(\vec{d}_\beta^{\,\dagger}\times\vec{M})\times\vec{d}_\beta(r)]_i\} . \quad (17)$$

Now if we use only the first term of (17)(with coefficient γ) in the dipolar part of (14), we obtain the results of Leggett[1] for the matrix giving the shift in resonance frequency squared

$$(\Omega^2)_{ij} = \frac{3\gamma^4}{\chi_0}\int dr\,dr'\,\mathfrak{D}(\rho)\langle\{\delta_{ij}\hat{\rho}\cdot S(r)\hat{\rho}\cdot\vec{S}(r') - \hat{\rho}\cdot\vec{S}(r')\hat{\rho}_j S_i(r) - [\hat{\rho}\times\vec{S}(r)]_i[\hat{\rho}\times\vec{S}(r')]_j\}\rangle . \quad (18)$$

The normal-state terms in this expectation value cancel to give zero shift of the resonance frequency due to the dipolar interactions. To obtain the anomalous contribution in the superfluid states, we may replace the $\vec{S}(r)$ by the expectation values of the gap function moments $\vec{T}(n)$ as in the final results of Leggett. The results predicted by Leggett are that in the ABM state there is a longitudinal resonance at

$$\omega_l^2 = \tfrac{1}{5}\Omega_0^2 \quad (19)$$

in addition to a single transverse resonance shifted from the Larmor frequency ω_L:

$$\omega_t^2 = \omega_L^2 + \tfrac{1}{5}\Omega_0^2 . \quad (20)$$

In the BW state, which is most favorable for the dipolar interaction, the transverse resonance is unshifted from the Larmor frequency

$$\omega_t^2 = \omega_L^2 \quad (21)$$

and there is still a longitudinal resonance at a frequency

$$\omega^2 = \tfrac{1}{2}\Omega_0^2 . \quad (22)$$

To see the effects of the extra terms in (17), we first concentrate on the terms with coefficient c_2. We first note that for both the ABM and the BW states $\vec{d}_\beta^{\,\dagger}\times\vec{d}_\beta = 0$, when we take the expectation value. If as we mentioned previously, we replace $\vec{d}_\beta(r)$ by d corresponding to the $q = 0$ limit, the combination

$$[\vec{d}_\beta^{\,\dagger}(r)\times(\vec{d}_\beta\times\vec{M})]_i + [(\vec{d}_\beta^{\,\dagger}\times\vec{M})\times\vec{d}_\beta(r)]_i \simeq [(\vec{d}_\beta^{\,\dagger}\times\vec{d}_\beta)\times\vec{M}]_i$$

so that the expectation values of all of the terms with coefficient c_2 vanish in both the states of interest.

The only extra terms which give a nonvanishing shift to the resonance frequency are the first two terms with coefficient c_1 in (17).

We write this out explicitly as

$$\Delta S_i(r) = c_1\{[\vec{d}_\beta^{\,\dagger}\times\vec{S}(r)]_i(\vec{d}_\beta\cdot\vec{M}) + (\vec{M}\cdot\vec{d}_\beta^{\,\dagger})[\vec{d}_\beta\times\vec{S}(r)]_i\}\ . \tag{23}$$

Substituting this in the dipolar term of (14), we have

$$\begin{aligned}\Delta\ddot{M}_i = 6\gamma^2 c_1 \int dr\,dr'\,\mathfrak{D}(\rho) \times\{&d_{\beta i}^{\dagger}[\hat{\rho}\cdot\vec{S}(r)](\vec{d}_\beta\cdot\vec{M})\hat{\rho}\cdot\vec{S}(r') \\ &-S_i(r)(\hat{\rho}\cdot\vec{d}_\beta^{\,\dagger})(\vec{d}_\beta\cdot\vec{M})\hat{\rho}\cdot\vec{S}(r') \\ &-[\hat{\rho}\times\vec{S}(r)]_i(\vec{d}_\beta^{\,\dagger}\cdot[\hat{\rho}\times\vec{S}(r')])(\vec{d}_\beta\cdot\vec{M})\}\ .\end{aligned} \tag{24}$$

We take the expectation value in the BW state using

$$d_{\beta i}^{\dagger} d_{\beta j} = \tfrac{1}{3}\Delta^2\delta_{ij}\ ,$$

We then find we have added to the resonance matrix $(\Omega^2)_{ij}$ a shift due to the extra terms of identical form to $(\Omega^2)_{ij}$ of (18):

$$\begin{aligned}(\Omega^2+\Delta\Omega^2)_{ij}^{\mathrm{BW}} = 3\gamma^2\left(\frac{\gamma^2}{\chi_0}+\frac{2}{3}c_1\Delta^2\right)\int dr\,dr'\,\mathfrak{D}(\rho) \times\{&\delta_{ij}[\hat{\rho}\cdot\vec{S}(r)][\hat{\rho}\cdot\vec{S}(r')] \\ &-\rho_i S_j(r)[\rho\cdot\vec{S}(r')] \\ &-[\hat{\rho}\times\vec{S}(r)]_i[\hat{\rho}\times\vec{S}(r')]_j\}\ .\end{aligned} \tag{25}$$

Going back to our evaluation of c_1 for the BW state (9), we see that we may identify the coefficient

$$3\gamma^2(\gamma^2/\chi_0+\tfrac{2}{3}c_1\Delta^2) = 3\gamma^4/\chi\ .$$

If we look at the shift in (24) for the ABM state, $d_{\beta i}^{\dagger} d_{\beta j}$ is nonzero only for $i=j=y$ [Eq. (6)]. Thus the only correction to $(\Omega^2)_{ij}$ is for $i=j=y$ and proportional to the matrix $(\Omega^2)_{ij}$. However, as calculated by Leggett, $(\Omega^2)_{yy}=0$ for the ABM state. Thus there is no shift of the resonance frequency due to the extra terms in the ABM state. For the last two terms of (17) with coefficient c_1 which would lead to a field-dependent shift in resonance frequency if they were nonzero, we obtain zero directly in the BW state using (7) after these terms are substituted in (14). These terms give zero contribution in the ABM state on the basis of symmetry considerations. The only possible nonvanishing component of $(\Delta\Omega^2)_{ij}$ is

$$(\Delta\Omega^2)_{yy} \propto \int d\Omega_q\,\mathfrak{D}(q)[q_z^2 S_z(q) - q_x^2 S_z(q)] = 0\ .$$

Thus the effects of these somewhat complex terms appears in the physically simple effect of using the true susceptibility in the expression for the shift. This agrees with Leggett's work in that his thermodynamic susceptibility is the true susceptibility in the B phase.

III. ESTIMATES OF ANISOTROPY ENERGIES

The terms coupling the magnetization and the superconducting order parameter are useful in estimating the anisotropy energies discussed by Leggett for the BW state. In the BW state the dipole interaction gives rise to a unique axis in that the minimum dipolar energy is obtained when the spin and space coordinate systems describing the order parameter are rotated relative to one another by $\cos^{-1}(-1/4)$. The axis of rotation then is a unique direction in the BW state. This axis which we will call $\vec{n}$ was assumed to be along the external field in the calculations by Leggett because with that alignment the combination of the field dependence of the gap and the dipolar energy can be minimized. Leggett argued that this orientational energy should be quite small and the $\vec{n}$ axis could easily become disoriented with respect to the field in which case the resonance modes transverse to the field would be shifted in frequency. In order to have some idea when one can expect $\vec{n}$ to be out of equilibrium with the field, it is useful to have a quantitative number for the anisotropy energy. We can calculate the anisotropy energy by calculating the change in the gap function in the presence of a field and then calculate the field dependence of the dipolar energy. In order to do this we must have explicit values of c_1 and c_2. The value of c_1 can be obtained from the expressions for the susceptibility and the microscopic results of BW and Leggett including Fermi-liquid effects:

$$c_1 = \frac{\gamma^2}{\chi_0}\left(\frac{7}{4}\zeta(3)\right)\frac{1}{(\pi T)^2}\,\frac{1}{1+z_0/4} \tag{26}$$

The value of c_2 is more difficult to estimate. It describes the change in T_c for spin pairs aligned parallel and antiparallel to the external field, that is near T_c we substitute $\vec{M}=\chi_0\vec{H}$ and find

$$ic_2\epsilon_{ijk}d_{\alpha i}^{\dagger}d_{\alpha j}M_k = \tfrac{3}{2}c_2\chi_0\langle|\Delta_{\uparrow\uparrow}(\vec{k})|^2 - |\Delta_{\downarrow\downarrow}(\vec{k})|^2\rangle H\ , \tag{27}$$

where the braces mean angular averages over $\vec{k}$. This form was used by Ambegaokar and Mermin[11] to predict the linear field splitting of T_c. The difficulty in estimating c_2 theoretically is that it describes the change in the effective coupling constant for up- and down-spin pairs in the presence of a field. In the work of Ambegaokar

and Mermin the only change with field was assumed to be the change in the density of states at the up- and down-spin Fermi surfaces. In spin-fluctuation theory the part of the effective interaction due to spin fluctuations is expected to have an almost cancelling dependence on field. In any case c_2 can eventually be obtained from the experimental splitting. If we use the Ambegaokar-Mermin result we find that

$$c_2 = \frac{\gamma}{\chi_0} N(\epsilon_F) \frac{1}{2\epsilon_F \lambda_{\text{eff}}} . \tag{28}$$

Here $N(\epsilon_F)$ is the density of states at the Fermi energy ϵ_F and λ_{eff} is the effective coupling constant $\sim\frac{1}{5}$. The c_2 term being linear in field is larger than c_1 in fields of less than 500 G. However, in calculating the anisotropy energy the c_2 term enters quadratically so that its contribution is always of the order of $(T_c/\epsilon_F)^2$ compared to c_1. [The exception to this statement being when T is very close to T_c ($T - T_c \sim 10^{-6}$ °K) where the c_2 term causes the splitting of the transition.] Therefore, we only take into account the c_1 term in the free energy. In order to calculate the change in the order parameter, we consider the free-energy expansion used in Ref. 9 in the absence of a field

$$\begin{aligned} F = {} & a_0 |d_{\alpha i}|^2 + \tfrac{1}{2} a_1 \left| \sum_{\alpha i} d_{\alpha i}^2 \right|^2 \\ & + \tfrac{1}{2} a_2 d_{\beta i}^* d_{\beta j}^* d_{\alpha i} d_{\alpha j} + \tfrac{1}{2} a_3 d_{\alpha i}^* d_{\beta i}^* d_{\alpha j} d_{\beta j} \\ & + \tfrac{1}{2} a_4 \left(\sum_{\alpha i} |d_{\alpha i}|^2 \right)^2 + \tfrac{1}{2} a_5 d_{\alpha i}^* d_{\beta j}^* d_{\alpha j} d_{\beta i} \end{aligned} \tag{29}$$

and add to it the term involving c_1 with $M = \chi_0 H$. If we then assume that $d_{\alpha i} = (\Delta/\sqrt{3})\delta_{\alpha i} + d'_{\alpha i}$ where $d'_{\alpha i} \propto H^2$ and minimize the free energy with respect to $d_{\alpha i}$, we find that with H along the z axis

$$\begin{aligned} d'_{xx} &= d'_{yy} = -\frac{B}{(A+2B)} d'_{zz} , \\ d'_{zz} &= -\frac{c_1 \chi_0^2 H^2 \Delta}{\sqrt{3}[A + B - 2B^2/(A+2B)]} , \end{aligned} \tag{30}$$

where

$$\begin{aligned} A &= a_0 + \Delta^2(a_1 + a_2 + a_3 + a_4 + a_5) , \\ B &= \tfrac{2}{3}\Delta^2(a_1 + a_4) . \end{aligned}$$

In weak-coupling theory with Δ^2 chosen to minimize the zero-field energy $A = -4a_0/5$ and $B = -2a_0/5$ where $a_0 = N(0)[(T - T_c)/T_c]$ so that

$$d'_{zz} = +c_1 \chi_0^2 H^2 \Delta/\sqrt{3} a_0 . \tag{31}$$

The dipolar energy can be written to order $(d_{\alpha i})^2$ as

$$F_{\text{dipole}} = \Gamma(\vec{d}_{ii} \vec{d}^*_{jj} + \vec{d}^*_{\alpha i} \vec{d}_{i\alpha}) , \tag{32}$$

where previously we had not specified the relationship between the spin and space coordinates but in Eq. (32) the coordinate system specifying $\vec{d}$ must be the same. The value of Γ can be obtained from the shifted resonance frequency. Since we have assumed that the dipole energy has been minimized in the absence of the field, the two coordinate systems can differ only by a rotation through $\cos^{-1}(-1/4)$ about some axis $\vec{n}$. If we assume $\vec{n}$ is along z then

$$\vec{d} = \frac{\Delta}{\sqrt{3}} \begin{pmatrix} -\frac{1}{4}(1 - \frac{1}{4}\beta) & \frac{1}{4}\sqrt{15}(1 - \frac{1}{4}\beta) & 0 \\ -\frac{1}{4}\sqrt{15}(1 - \frac{1}{4}\beta) & -\frac{1}{4}(1 - \frac{1}{4}\beta) & 0 \\ 0 & 0 & 1 + \beta \end{pmatrix} \tag{33}$$

Here $\beta \equiv \sqrt{3} d'_{zz}/\Delta$. Substituting into Eq. (32), we find

$$F_{\text{dipole}} = -\tfrac{1}{3}\Gamma\Delta^2(\tfrac{1}{2} - 4\beta) . \tag{34}$$

If $\vec{n}$ is along the x axis then performing the same calculation gives that

$$F_{\text{dipole}} = -\tfrac{1}{3}\Gamma\Delta^2(\tfrac{1}{2} + \tfrac{9}{4}\beta). \tag{35}$$

The total anistropy energy can be written as

$$F_A \equiv +\frac{25}{12} \frac{\Gamma\Delta^2 c_1 \chi_0^2}{a_0} (\vec{n} \cdot \vec{H})^2 . \tag{36}$$

Using the result that

$$5\Gamma\Delta^2/\chi = \hbar^2\omega_i^2 \equiv \hbar^2\omega_0^2(T_c - T)/T_c$$

is the square of the longitudinal frequency shift in the BW state, we find

$$F_A = -\frac{35}{192} \frac{\zeta(3)}{\pi^2} \frac{1}{(1 + z_0/4)^2} \chi_0 (\hbar\omega_0)^2 \left(\frac{\gamma \vec{H} \cdot \vec{n}}{k_B T_c} \right)^2 . \tag{37}$$

The approximate experimental value[2] of $(\omega_0)^2 \approx 2 \times 10^{11}$ (Hz)^2

$$F_A \approx 0.5 \left(\frac{(\hbar\omega_0)^2}{\epsilon_F} \right) \left(\frac{\gamma \vec{H} \cdot \vec{n}}{k_B T_c} \right)^2 /\text{atom} . \tag{38}$$

Therefore in a 1-kG field the anisotropy energy is

$$F \approx 1 \times 10^{-12} \text{°K/atom} . \tag{39}$$

This means that domains in He must be large compared to 10^{-14} cm^3 in order for thermal fluctuations not to act to randomize the spin axes with respect to field. The anisotropy energy in a 1-kG field is

only 10^{-2} times smaller than the anisotropy energies which keep the spin and space axes aligned. Therefore, if the heat-pulse experiments of Osheroff[2] are to be explained as being due to the breaking away of $\vec{n}$ from $\vec{H}$ and as a consequence the resonance shifts, it should also be possible to break the correlation between the spin and space coordinates. One would get very unusual resonance behaviors under these circumstances.

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