Production of fast highly excited atoms in proton collisions with atomic hydrogen and argon*

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The study of keV H^+-H(1s) collisions has been extended to the production by electron transfer of highly excited states with principal quantum numbers between 13 and 28. A fast-proton beam was passed through thermally dissociated hydrogen gas, and the resultant fast excited atoms were observed using electric field ionization and proton-counting techniques. The incident proton energy range was from 7 to 60 keV. The observed cross section for production of such excited states possesses a broad maximum near 30 keV. The sum of cross sections over the observed principal quantum numbers is 0.00089 $Å^2$ at 7 keV, 0.0070 $Å^2$ at 30 keV, and drops again with energy to 0.0023 $Å^2$ at 60 keV. Possible error in our cross section scale is $\pm$ 30%. Thus the fraction of electron-transfer collisions producing these highly excited states varies from a maximum value of about 0.03% in the range 30–60 keV to the lower value of 0.01% at 7 keV, values not markedly different from those found for inert-gas target atoms; this is contrary to the predictions of the Brinkman-Kramers-Oppenheimer approximation. The data at 60 keV agree with recent calculations by Band using the coupled Born approximation. The present results for H^+-Ar collisions and $13 \leq n \leq 28$ are in agreement with previous investigations for $9 \leq n \leq 15$, assuming that the Jackson-Schiff scaling rule is valid for this collision system.

I. ELECTRON TRANSFER COLLISIONS IN THE H^+ +H SYSTEM

The collision of protons with hydrogen atoms leading to electron transfer is the best-studied rearrangement-collision process involving a three-body system and electromagnetic forces. The study of this process at eV energies is fundamental to our understanding of molecular and chemical processes; at keV energies comparison of experiment with theoretical predictions is crucial to determining the best methods for calculating cross sections in this intermediate energy range; above perhaps several hundred keV, the Faadeev-Watson and other basic theories of general rearrangement collisions can be tested in practice, without uncertainties as to the nature of the forces between particles.

A long-standing theoretical interest in H^+-H collisions has stimulated a recent series of experiments on this collision system that in turn are now generating greatly increased theoretical activity in the field. The total cross section $\sigma_{10}(H)$ for production of fast atoms with all possible states of internal excitation has been measured by a number of investigators and is reasonably well known between 10 eV and 250 keV.[1-3] As the direct production of ground state H(1s) atoms is a resonant process requiring no change in internal electronic energy, $\sigma_{10}(H)$ is very nearly the cross section $\sigma_{1,1s}(H)$ for this process at energies below perhaps 500 eV, where electron transfer into excited states should be insignificant. As the energy increases toward 25 keV, the energy predicted by the adiabatic criterion at which production of the $n=2$ states might become important, $\sigma_{10}(H)$ becomes a nontrivial sum of cross sections for all excited states, and comparison of theory with experiment becomes more complicated. The total cross sections for 2s production and 2p production have been measured at keV energies.[2,4-7] In addition, some differential scattering measurements have been made for the $n=2$ states[8,9] as well as for all excited states summed.[10] These differential measurements show that excited states are important at surprisingly low energies in the case of small impact parameter collisions, although the latter contribute comparatively little to the total cross section. The present measurements on H^+-H collisions are the first for excited state formation beyond the second quantum level. A preliminary report of our first measurements on highly-excited-state production has been given.[11] Since then we have repeated the measurements, placed them on an absolute-cross-section basis, and continued the search for systematic errors in the data.

For energies above 1 keV a number of different approaches to theoretical calculations have appeared useful. Some are close-coupling calculations involving traveling atomic states,[12-15] whereas others involve molecular states.[16,17] High-energy type calculations using the Born (BA), distorted-wave and other more-refined approxima-

tions are often not unreasonable in the 10–100 keV range; some of these should, hopefully, be rather good as one goes upwards toward 1 MeV.[18–21] These highest energies still await experimental investigation. What seems clear now is that in the energy range above roughly 10 keV, production of all the excited states must be studied both experimentally and theoretically before careful tests of basic theory can be made. Experimentally, the problem is one of the magnitude of cascade effects arising from the optical decay of the excited states into other states in the time period between production and detection of the excited atoms. Theoretically, the problem is one of determining which states are so important that they may not be omitted in close-coupling expansions of wave functions.

Theoretical work on the production of highly excited states has been limited to the use of the high-energy type of approximation. The case of atomic hydrogen as a target for the incident proton has been especially neglected in the past, as no data have been available. Until very recently, the only specific calculations for this case were those of Schiff[21] and Butler, May, and Johnston.[22] The latter authors used the Brinkman-Kramers-Oppenheimer (OBK) first-order approximation together with some sum rules associated with closure to obtain their results for the cross section for any particular value of n. Now some additional calculations using a two-state high-energy approximation are being completed by Band.[23] Good close-coupling calculations of the highly-excited state production cross sections in our entire energy range may indeed turn out to be difficult to make, because of the very large number of states that may need to be coupled. This has not been investigated.

II. PRODUCTION AND DETECTION OF HIGHLY EXCITED HYDROGEN ATOMS

Considerable past experimental work has been done on proton collisions with various gas targets producing hydrogen atoms with principal quantum number varying between 6 and 26. The studies have concentrated on molecular gases,[24–30] inert gases[11,25,27,31] and metallic vapors.[24,26,31–33] The large cross sections found for this last class of target have been of special interest for the thermonuclear fusion program, since the production of highly excited states that can be electrically stripped inside a magnetic mirror confinement region is useful for particle injection into plasma devices.[34] Some calculations using the OBK approximation and simple atomic models have been made for many of the above mentioned targets.[35,36] Qualitative agreement between experiment and such theory is observed, in the sense that they both present reasonable results consistent with a general consideration of energy defects and the adiabatic criterion. A rough n^{-3} dependence of the cross section on principal quantum number is predicted by the OBK theory for S states, whereas for states of high angular momentum l the behavior might be more like $n^{-2.5}$.[33,37,41] A further prediction is that the cross section for $l > 1$ drops rapidly with increasing l.[22,23,38,39] Consideration of all these factors leads to the approximate Jackson and Schiff n^{-3} scaling rule for the cross section summed over all l.[40] Thus many experimentalists in the past have customarily described highly-excited-state production by a single reduced cross section defined by

$$\sigma_c \equiv n^3 \sigma_n \equiv n^3 \sum_l \sigma_{nl} . \tag{1}$$

The quantity σ_c is assumed n independent; then a measured sum of cross sections over a range of n again yields σ_c according to

$$\sigma_c = \sum_{n=n_1}^{n=n_2} \left(\sum_l \sigma_{nl} \right) \Big/ \sum_{n=n_1}^{n=n_2} \frac{1}{n^3} . \tag{2}$$

The validity of this procedure in describing all cross sections by one quantity σ_c has been only roughly verified experimentally,[41] as detection techniques have not completely resolved the signals arising from each separate principal quantum number, and optical decay corrections depending strongly on l and n are difficult to make under those conditions.

The method of detection of highly-excited-state atoms in both the past experimental work and the present one is electric field ionization of the atom with subsequent counting of the resultant fast protons. The calculation of the ionization probability of a hydrogen atom in a fairly strong external electric field is a classic quantum mechanics problem first pursued by Lanczos.[42] The superposition of the Stark-effect interaction with the field-free effective potential of the atomic electron results in the development of a potential barrier through which the electron may tunnel out and become free. This barrier height decreases with increasing electric field strength. As barrier penetration probability is associated with an exponentially varying wave function dependence on electron-proton separation, only a small change in field causes the ionization probability for a given atomic state to change from a very small number

to another close to unity. Thus theory predicts that it is possible to completely resolve proton signals arising from each separate principal quantum number n when n is less than 7. For higher values of n, the Stark-effect shifting of energy levels for different values of l is so large that some overlap of states with different n occurs. More accurate calculations than that of Lanczos have been made by several workers.[43-45] Experimentally only a few attempts to partially resolve different states by carefully sweeping the electric field strength have been very successful, the work of Riviere and Sweetman being most notable.[30] They found very good agreement with the theoretical predictions of Rice and Good[43] for values of n between 9 and 20. Such measurements involve problems of field uniformity and stability, focusing effects of the field on the protons produced, and optical decay of the atoms while in the field. Calculations of the probability for this last type of effect have been made by Hiskes, Tartar, and Moody.[46] The electric field ionization technique has been reviewed by Riviere.[47]

III. APPARATUS

Part of the apparatus needed for the present experiment has been fully described in earlier reports of measurements on fast 1s and 2s state atom production.[8,48] As seen in Fig. 1, the major alterations were associated with the new detection devices needed for the highly excited atoms. Thus only brief mention will be made here of the remainder of the apparatus, except where changes have been made or new information has been obtained.

The equipment used for ion beam formation, mass analysis, and intensity determination were the same as previously used. The collimation of the beam was tighter than before, being achieved by two 0.025-cm-diam holes a distance of 68-cm apart, yielding an angular spread of about 0.02 deg full width at half-maximum (FWHM). Some additional studies of the ion beam energy and energy spread have been made, using an electrostatic filter lens of the hyperbolic field type.[49] The energy spread of the collimated beam was determined to be 20± 10 eV. The energy shift of the beam relative to the sum of accelerator and ion source extraction voltages was measured to be 0(+0, −30) eV, an unexpected result. Thus the small correction made in previous work[8,48] for the often observed energy shift of 100±100 eV for rf discharge ion sources may not have been warranted. Since the operating conditions of such sources are generally not reproducible, nothing can be said about recorrecting the earlier data for this small energy shift effect.

The atomic hydrogen target has been described in detail.[50] Basically, this system was a 20-cm long, 2.5-cm-diam double-walled scattering cell assembly placed inside a horizontal open ended vacuum furnace. The furnace was heated by a 700-A pulsed current, with the cell mass being large enough to make the cell temperature not responsive to the furnace heater-current time dependence at the 10-Hz frequency used. The pulsed heater current feature was used to avoid magnetic field effects on the beam, the latter

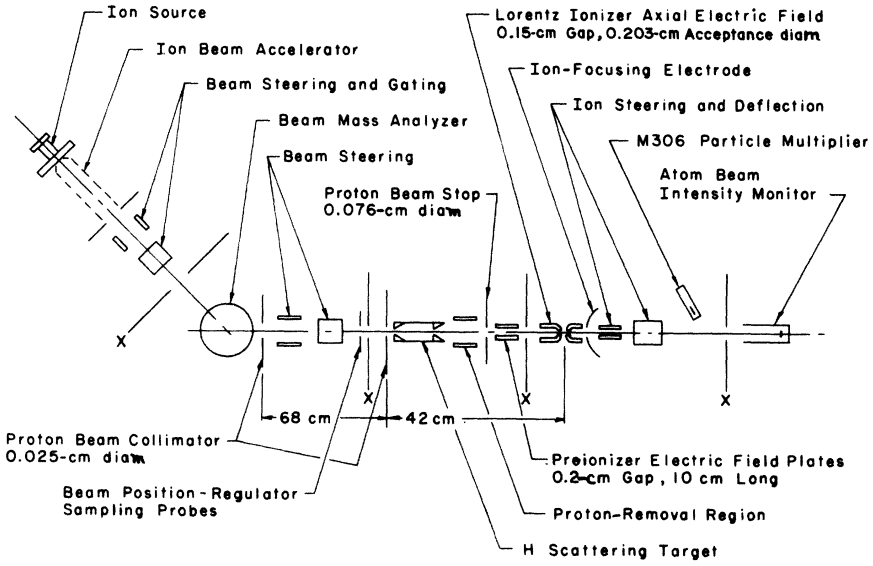

FIG. 1. Schematic diagram of the experimental apparatus, viewed from above.

having been pulsed out of phase with the current. The target pressures used in the present scattering experiments were very low, about 5×10^{-5} Torr. This made double-scattering effects negligible; equally important, the gas of hydrogen molecules inside the hot target region was highly dissociated. The dissociation fraction was measured using the double-electron-transfer proton-beam probe technique,[51] and was found to be above 0.95. Thus corrections for scattering from the small amount of molecular hydrogen were the order of a few percent; such corrections were made using the available data for $9 \leq n \leq 20$ (Refs. 25 and 27) and contribute only a small amount to the over-all uncertainty of our present results. We mention that a change in the design of the scattering cell itself has been made, successfully simplifying it considerably. Now the cell is made entirely of tungsten, with the outer cylinder one continuous seamless tube with a 0.127-cm thick wall. The slots at the cell ends used in the earlier designs are now omitted, without radial thermal expansion becoming a problem.

A drawing of the electric field ionizer used is given in Fig. 2. The design of the gap region where the field is localized is similar to that of Riviere and Sweetman.[30] The 1.5-mm gap originally used by us[8] has also been increased to 7.9-mm, following the warning of Riviere[47] that the smaller gap might have some field uniformity problems; the differences between the two gap separations produced changes in our results of only about 5%. The focusing electrode following the gap region was not found to be necessary in our work and was seldom used. A search for effects arising from a defocusing of the protons produced in the gap region was conducted by steering the protons across the cathode of our Bendix M-306 particle detector, using the electrostatic deflector field available. No noticeable defocusing was observed, the proton spot size being the same as that of an incident proton beam on the gap-assembly collimator aperture.

Following techniques previously introduced to define excited-state signals for the 2s atom case,[48] we defined the highly-excited-state signal not by turning the electric field ionizer on and off and observing a change in the particle detector counting rate, but rather by leaving the electric field ionizer on at a very high field value and turning on and off a parallel-plate preionizer voltage located before the main ionizer region (see Fig. 1). This transverse electric field removed from the beam those protons produced by the preionization process. Thus only highly-excited-state atoms converted to protons in the main ionizer region,

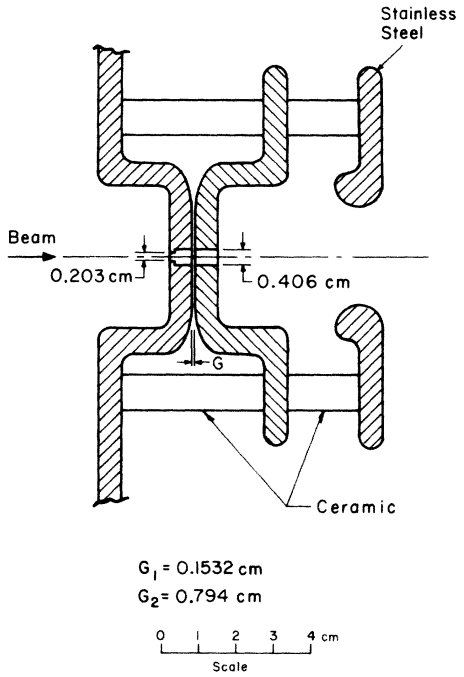

FIG. 2. Cross-sectional view of the ionizer. Two values for the gap G were used; $G_1 = 0.15$ cm, $G_2 = 0.79$ cm.

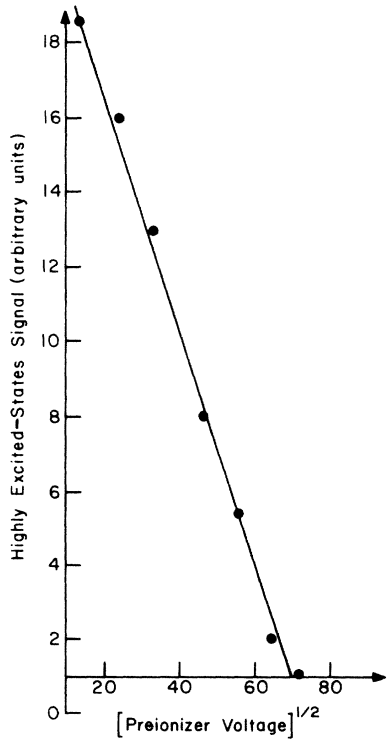

FIG. 3. Dependence of the highly-excited-state signal on the preionizer electric field at 52.5-keV incident proton energy. The ionizer field was set at 66 kV/cm.

but not previously preionized, contribute to the signal. The preionizer field was much more uniform than that of the ionizer, thereby improving the accuracy in definition of the field value as well as increasing the resolution of signals arising from atoms with different quantum levels. The signal defined this way is plotted versus the square root of the preionizer field strength in Fig. 3. The straight-line plot observed was characteristic of the polarity-independent over-all signal dependence of typical highly-excited-state signals; this has been discussed by Il'in *et al.* in theoretical terms[41] and has been observed by previous workers to be roughly true.[25,26,33] Of course, a more detailed study of such curves should reveal structure arising from the partial resolution of states of different n. A separate study made of such structure will now be described.

A schematic diagram of the electronic equipment used to study the structure in the preionization curves is given in Fig. 4. A Hamner model N-413 high-voltage power supply was modified to produce a relatively fast 1-kV sawtooth waveform to be applied to one prequench electrode. A base-value high voltage from a Fluke model 410B high-voltage power supply was applied to the other electrode. In this way a 1-kV section of the preionization curve could be repetitively scanned, accumulating good signal statistics in the presence of beam drifts and noise. The linearity of the voltage ramp was better than 2% over the entire sweep range. To obtain the signal dependence upon preionizer voltage, the detector output was synchronously scanned through 100 channels of a TMC model 402-6 pulse-height analyzer used in the multiscaler mode. The analyzer-stored signals were accumulated until 10^6 counts per channel were in the spectrum peak. Then the analyzer output was read onto paper tape, converted onto IBM cards, and numerically differentiated on an IBM 7090/7094 computer to obtain principal quantum-number spectra such as that shown in Fig. 5. The peaks labeled with n values between 13 and 20 are located at field values in close agreement with both the experimental observations of Riviere and Sweetman[30] and the theory of Rice and Good.[43] Our peaks corresponding to higher values of n are also in agreement with their theory (see Fig. 6). The absolute values of n assigned ultimately depend upon the unambiguous identification made in Ref. 30 for $n = 9$ and 10. It should be mentioned that the presently observed structure at high n appears so well-resolved only because the lower angular momentum states are preferred in the collision production process,[23,38-40] whereas the states with large l that contribute most to the overlapping of peaks of different n are not so strongly produced. A more quantitative study of

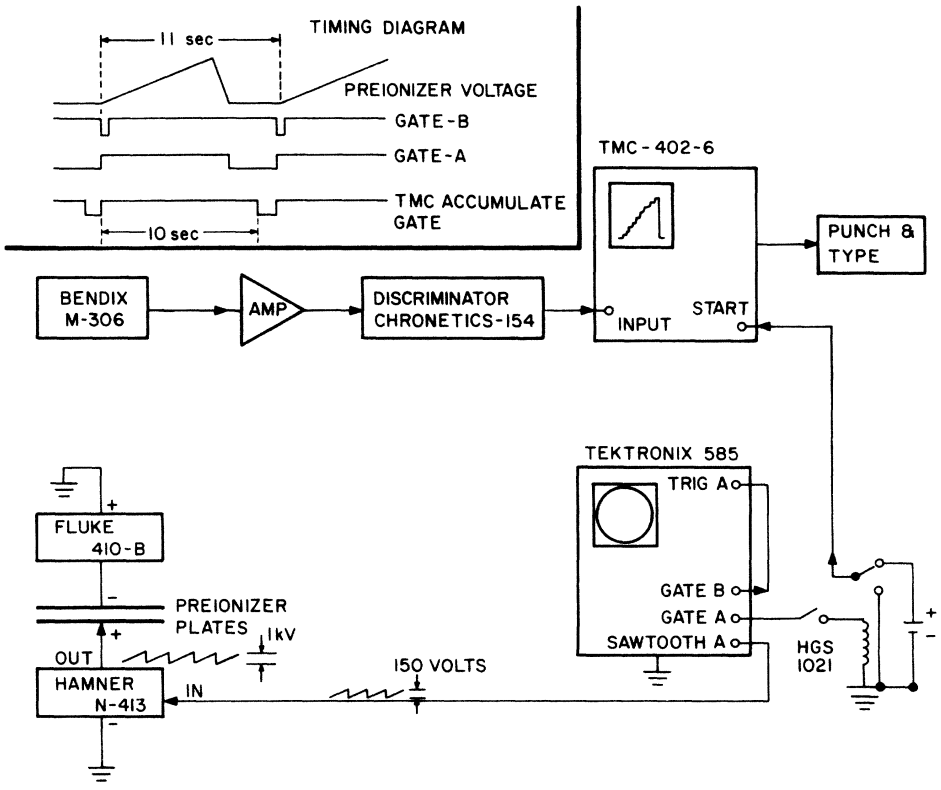

FIG. 4. Schematic diagram of the electronic equipment used to study the structure in the preionization curves.

the angular momentum dependence of the production cross section awaits the development of some new detection technique, perhaps involving radio frequency transitions between different states.

IV. ABSOLUTE MEASUREMENT OF THE CROSS SECTION FOR H^+ ON Ar

Following previously established techniques[2,48,52] the cross section for an argon gas target was first studied in detail as a function of energy. The needed measurements with the atomic hydrogen target at operating temperature were then reduced to ratio determinations of the scattering signals for hydrogen and agron alternatively in the target cell. In addition, comparison of the argon measurements with earlier results[25,26] for somewhat different ranges of n provided an important check of our techniques.

The absolute thickness of the argon gas target was determined from the total production of fast hydrogen atoms as in Ref. 48. This procedure depends on a knowledge of the total electron-transfer cross section for Ar, believed accurately determined in our energy range through a number of independent previous studies. The use of the standard cross-section curve of Fig. 4 in Ref. 48 has withstood numerous consistency checks with various independent cross-section measurements,[48,50,52,53] especially above 10 keV.

Considerable attention was given to the properties of the Bendix M-306 particle multiplier used to count individually the protons produced by electric field ionization of the highly excited atoms. Several reports have appeared concerning the spatial dependence of the detection efficiency over the multiplier cathode surface.[54,55] A study of such effects was made with our highly collimated incident proton beam for various multiplier operating voltages. Nonuniform spatial distribution effects were observed and found to be strongly dependent on operating voltages. The present data were taken under conditions of essentially uniform detector response over dimensions several times that of the observed size of the electric-field-produced proton beam.

Two different special calibrations of the particle-multiplier efficiency were performed. The first was a direct calibration using a series of dc measurements with various beam intensities and various multiplier dc gain values. First the low-gain dc multiplier efficiency was obtained absolutely with a 10^{-12}-A proton beam by measur-

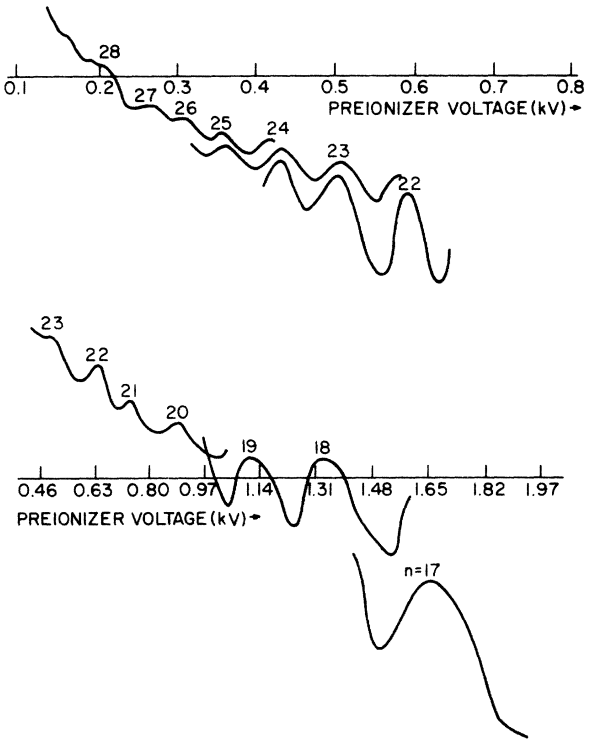

FIG. 5. Principal quantum number spectra as a function of the preionizer voltage. These curves are the results of digitally differentiating the multichannel analyzer output. Amplitudes of different scans have no significance since they were collected for different counting times.

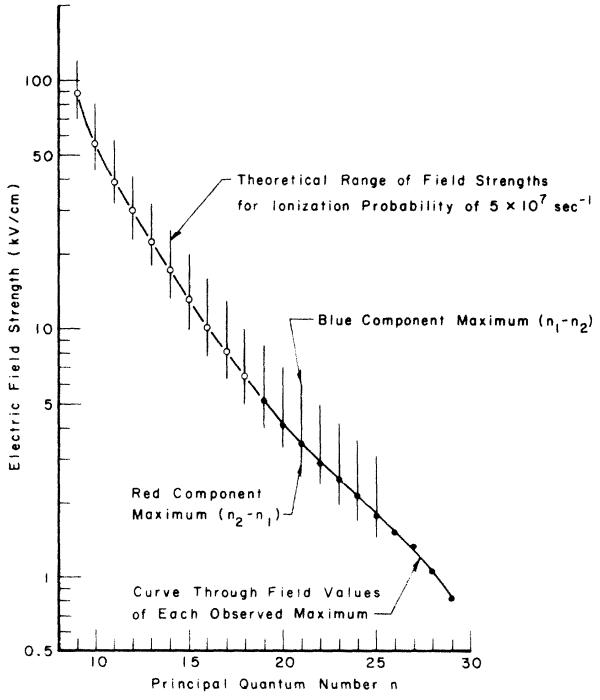

FIG. 6. Plot of the position of the peaks vs quantum number. ●, results of the present experiment; ○, from Ref. 30.

ing with an electrometer (Keithley-610) the ratio of multiplier output to the current obtained with the precision Faraday cup described in Ref. 48. A dc gain curve was then measured for gains up to 10^4 using this first standard beam. The beam intensity was then reduced to 10^{-16} A at constant, known, multiplier gain. Finally the multiplier was switched over to the pulse counting mode at the new, known, lower beam intensity to obtain a value for the over-all detection system efficiency that included electronics efficiency and the like. A value for this over-all proton detection efficiency η_B of 1.0 ± 0.35 was obtained. A value possibly larger than 1 was due to some afterpulsing of the multiplier, an effect easily observed using a time-to-amplitude converter and a pulse-height analyzer. Such effects did not enter into our earlier studies, in which slow (microsecond) electronics was used.[2,8,43] In the present work fast (nanosecond) electronics was employed. Such afterpulsing has been observed by others.[55,56]

Our second calibration involved the direct comparison of the Faraday cup reading of a 4.0×10^{-14}-A incident proton beam with a Bendix $2.7 \times 10^{+5}$-cps count rate while in the usual operating mode. A nominal value of $\eta_B = 1.1 \pm 0.25$ was obtained after studies of count-rate saturation with Bendix voltage and discriminator level were made.

As in the previous studies of other investigators, only highly excited atoms produced in scattering events at small scattering angles were accepted for detection. The earlier reports for H^+-Ar collisions contain insufficient information to ascertain what the acceptance angles were.[25,26] Our acceptance angle was determined by the collimator aperture within the electric-field-ionizer assembly (see Fig. 2) and corresponded to a maximum totally accepted scattering angle of 2.4 mrad. Such an angle was previously found sufficient for studies of total-electron transfer as well as transfer into the 2S state.[2,8,48] It is unlikely that electron momentum transfer during the collision would lead to significantly different scattering angles for the present highly-excited-atom case. Some evidence supporting this comes from the highly-excited-state production studies of Kingdon, Payne, and Riviere who present some data at low target pressures for the case of D^+-C_8F_{16}.[24] Their angular distribution of highly excited D^0 under thin-target conditions was not noticeably wider than the quoted angular size of their incident beam. As was the case in fast H(2S) production,[8] Kingdon *et al.* found excited state production to be *fractionally* more important as the scattering angle was increased, but still strongly peaked close to the forward direction. Thus we feel that corrections to our results arising from undetected large-angle scattering events should be small, amounting to perhaps 10% at 7 keV, where the effect is largest.

Our results for H^+-Ar collisions are shown in Fig. 7 along with those of previous workers. Our data are presented as the sum σ_* of cross sections σ_{nl} for $13 \leq n \leq 28$. As previous workers each studied a somewhat different range of n than ours, their data were converted using their values of σ_c as defined in Eq. (1). Very reasonable agreement is found between the results of different in-

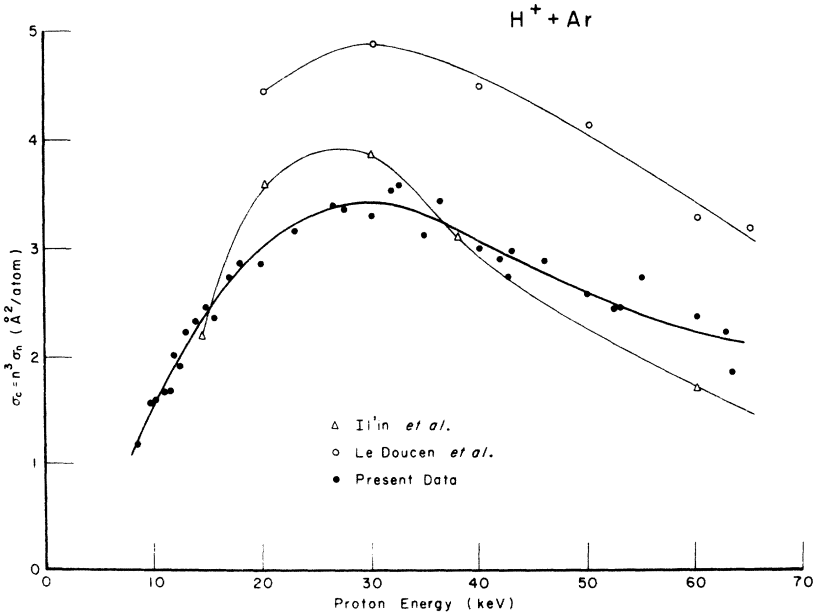

FIG. 7. A plot of σ_c(Å^2/atom) for the process $H^+ + Ar^+ \rightarrow H + Ar^+$. ●, present results; △, Ref. 26; ○, Ref. 25.

vestigators. It appears that this cross section for argon is established to an accuracy of about ±20% in the energy range 7–60 keV.

V. CROSS SECTION FOR H^+ -H COLLISIONS

The hydrogen data were obtained by experimentally measuring the ratio of cross sections for argon gas to hydrogen gas in the heated scattering target cell. That this procedure leads to relatively precise ratios has been established in earlier work.[2] As before no differences in cross sections for hot versus cold argon targets was observed. Each value of the ratio was then multiplied by a corresponding value of $\sigma_*(\mathrm{Ar})$, obtained by the methods described in Sec. IV, to yield an absolute value for $\sigma_*(\mathrm{H})$. Small corrections for the H_2 in the H-target were made (see Sec. III). The resulting data are shown in Fig. 8 and listed in Table I. The error bars indicated are an over-all 90% confidence measure accounting for all known sources of experimental error. The values given differ from the preliminary values previously reported[11] by an amount typically 20%. Note that the presentation of this numerical data as a directly measured sum over specified excited states does not assume the n^{-3} law, and relies only upon the proper identification of the contributing states (see Sec. II).

A first general remark on our results is that the cross section is not drastically different from that for argon, the H/Ar ratio being about 0.5 and having a broad maximum with incident proton energy at a value of 22 keV. This is at variance with previous indications based upon the theoretical results of Butler, May, and Johnston[22] that highly-excited-state production for an H-target should be an order of magnitude larger than that for any of the nonmetallic gases; this possibility had been first brought out in the review article by Fedorenko *et al.*[41] We also note that previously obtained values for the H_2/Ar cross-section ratio at a somewhat lower range of n values are about 0.5–1.0,[25,26] and thus in our energy range H is not a more efficient producer of highly excited states than the molecular form H_2. Another remark is that the energy dependence of $\sigma_*(\mathrm{H})$ is consistent with the Massey adiabatic criterion, with the position of the maximum at 30 keV not very different from the value of 23 keV obtained for electron transfer into the 2s state.[2]

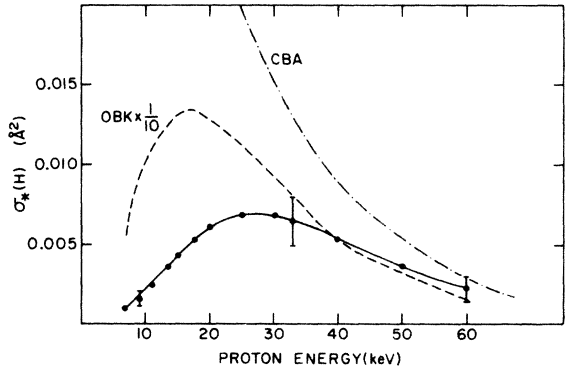

FIG. 8. The summed cross section σ_* for formation of hydrogen in states with $13 \leq n \leq 28$ with an estimated total uncertainty of ±30%. The circles with error bars are the results of the present measurements. The dashed curve is the result of the OBK approximation divided by 10 (Refs. 21 and 22). The dash-dot curve is the CBA result (Ref. 23).

Two theoretical curves are shown in Fig. 8. The curve labeled OBK is from the work of Schiff [21,57] who used the Brinkman-Kramers-Oppenheimer approximation to the first Born cross section to calculate the cross section for total electron transfer into the state with principle quantum number n:

$$\sigma_n(\mathrm{OBK}) = \pi a_0^2 (2^{18} S^8 / 5 n^3 \alpha^5), \tag{3}$$

where

$$\alpha \equiv [S^4 + 2S^2(1 + 1/n^2) + (1 - 1/n^2)^2] \tag{4}$$

and

$$S \equiv v/v_0, \tag{5}$$

where v_0 is one atomic unit of velocity.

The curve labeled OBK is obtained from this expression; we calculate the cross section σ_* (OBK), the sum of individual cross sections σ_n for capture into states whose principle quantum numbers lie between 13 and 28:

$$\sigma_*(\mathrm{OBK}) = \sum_{n=13}^{28} \sigma_n(\mathrm{OBK}). \tag{6}$$

TABLE I. Total cross section for $H^+ + H(1s) \rightarrow H(13 \leq n \leq 28) + H^+$. Over-all value of the uncertainty is ±30%.

Energy (keV)	σ^* (H) (10^{-3} Å^2/atom)
7.0	0.89
9.0	1.62
11.0	2.43
13.5	3.73
15	4.37
17.5	5.26
20	6.16
25	6.80
30	6.97
33	6.48
40	5.27
50	3.65
60	2.35

It is interesting to note that the theoretical curve peaks at a lower energy than the experimental one; this is the same situation as that found when one compares either the BA or OBK results with the experimental data in the case of electron transfer into $H(2s)$.[2]

Quite recently a series of H^+-H electron-transfer calculations have been carried out by Band[19,23] using a two state high-energy type of approximation which we call the coupled Born approximation (CBA). The basic improvement of this approximation over the usual Born approximation (BA) is its taking account of the nonorthogonality of initial and final atomic states. This leads to expressions for cross sections having the same dependence on direct, exchange, and overlap integrals as those obtained in the two-state, close-coupled, impact parameter method[58] as well as the energy expressions found in the LCAO theory of molecules.[59] Thus the CBA theory should be a definite improvement over BA as well as OBK, since the unusually strong exchange coupling is now taken into account. Indeed, significantly improved agreement with experiment is found for energies up to the experimental limit of 250 keV; the exchange coupling effect (i.e., the difference between BA and CBA results) is greater than 10% up to collision energies of at least 125 keV.[20]

Shown in Fig. 8 are some CBA results for $\sigma_*(H)$ recently obtained by Band.[23] The improvement over the OBK theoretical curve is remarkable. An important point underlying the CBA theoretical curve is that it depends to some extent upon the Jackson-Schiff n^{-3} scaling rule, whose validity Band has studied through a comparison with CBA results for different n values. It is interesting to note that the OBK result is just 5.5 times the CBA result over the energy range of Ref. 23.

We finally mention how the present results affect previous estimates of the cascade contributions to the experimental cross section $\sigma_{10}(H)$ for total electron transfer.[2] Such contributions are comparatively small in H^+-$H(1s)$ collisions, in which transfer into the ground state is resonant. However, cascade effects might be much more important in other keV energy collision processes such as electron transfer in the He^{++}-$H(1s)$ system, in which there is only quasiresonance with the $n=2$ states of He^+. Previous cascade contributions have been estimated using experimental $n=2$ cross sections, BA cross sections for $n=3$ and 4, and a decay-chain calculation accounting for all electric-dipole decays.[2] No consideration of higher states was made. At present, still no data exist for $3 \leq n \leq 12$, and no BA or CBA values have been computed for $5 \leq n \leq 12$. We do, however, now know that CBA values are not far from experimental values for $13 \leq n \leq 28$ as well as for $n=1$ and 2, whenever the energy is larger than about 50 keV. Furthermore, the Jackson-Schiff scaling rules appear to agree with CBA scaling predictions to about 30% accuracy.[19,23] A new better-established upper bound for the quantity η_{10} defined by $\sigma_{10}=\eta_{10}\sigma_{1s}$ is therefore calculated beginning with CBA values in a cascade calculation that includes states with $n \leq 4$. The Jackson-Schiff scaling rule is then used in conjunction with the present data in a computation of the entire production of states with $n \geq 5$, which we finally add to obtain the quantity η_{10}. Our result is that $\eta_{10} \simeq 1.30$ at 50 keV with the $n \geq 5$ part contributing 60% of the correction. If instead the $n=4$ CBA values are used to estimate the contribution for $n \geq 5$, then we obtain $\eta_{10} \simeq 1.21$. Thus the continued direct comparison of theory with experiment assuming $\eta_{10}=1$ still seems not a very bad approximation for the case of H^+-$H(1s)$ collisions. Nevertheless, recent improvements in close-coupling calculations[13,14] are reducing theoretical uncertainties to the point where 20% cascade effects are becoming a significant question. Further experimental studies of the l dependence of the cross section for $n>2$ will be required to completely resolve this problem.

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