Intensity-dependent ionization potentials for H and He in intense laser beams*

Chan K. Choi, Walter C. Henneberger, and Frank C. Sanders
Department of Physics, Southern Illinois University, Carbondale, Illinois 62901
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Intensity-dependent ionization potentials for atomic hydrogen and helium in intense laser beams are computed using a theory published earlier. The transformation that yields intensity-dependent potentials is identical to one proposed by Kramers in 1948. Results are given in terms of a parameter that depends on both intensity and frequency of the incident radiation.

I. INTRODUCTION

Over the past decade there has been considerable theoretical effort devoted to the problem of ionization of gases by the simultaneous absorption of several photons of a given frequency.[1-11] Almost all of this work is based on time-dependent perturbation theory which neglects the effect of the intense radiation on the initial state of the atom. Moreover, the validity of perturbation theory at the intensities under discussion is highly questionable and agreement between theory and experiment has been poor, with experimental ionization probabilities sometimes exceeding the calculated ones by a factor of about 10^3.

One of the authors (WCH) has introduced a perturbation method which yields an effective potential determined by the time average of the combined effects of the Coulomb and electromagnetic fields.[5] It is the authors' contention that the ground state corresponding to this potential represents an improvement over the unperturbed ground state, which is taken as the initial state by the other investigators. The transformation used here turns out to be identical with a transformation proposed by Kramers[12] in order to avoid divergences in nonrelativistic quantum electrodynamics in the dipole approximation. Kramers' derivation differs considerably from that of Ref. 5. The method was again rediscovered by Faisal,[9] who completely neglected the extremely large effects due to the effective potential.

This paper reports on new ionization potentials obtained for atomic hydrogen and helium in the framework of the theory of Ref. 5. These depend on the frequency and intensity of the incident light. A group of investigators in Saclay[13] have reported that at very high intensities of the incident light the degree of nonlinearity of the ionization probability differs from that given by the number of photons required to ionize an atom in its ground state. The present theory will also yield a degree of nonlinearity that depends upon the frequency and intensity of the incident light. There is, therefore, some hope that the theory will have a range of validity extending to intensities beyond those for which standard perturbation theory is valid.

II. CALCULATIONS

The theory of Ref. 5 yields an effective-potential function for an electron in a Coulomb field and an intense plane-polarized wave (the theory assumes the validity of dipole approximation) given by

$$V(\vec{r}) = -\frac{e^2}{2\pi^2}\int_{-\infty}^{+\infty}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}J_0(\vec{\alpha}_0\cdot\vec{q})d^3q\,, \tag{1}$$

where $\vec{\alpha}_0$ is the amplitude of the classical displacement of a free electron from its center of oscillation in a radiation field $\vec{E}(t)$. Thus $\vec{\alpha}_0$ depends upon both the intensity and the frequency of the incident light.

The approximate eigenstates and eigenvalues of the Hamiltonian corresponding to an electron in the potential of Eq. (1) have been found by a variational method. The exact wave function is approximated by a trial function φ so that

$$\frac{\int\varphi^* H_0(\vec{r})\varphi\, d\tau}{\int\varphi^*\varphi\, d\tau} \geq E_0\,, \tag{2}$$

where

$$H_0(\vec{r}) = \frac{\vec{p}^2}{2m} + V(\vec{r}) \tag{3}$$

and

$$\varphi = \sum_{n=0}^{N}\sum_{l=0}^{n} C_{nl}\, r^n e^{-\beta r}\cos^l\theta\, e^{im\phi}$$

$$= \sum_{i}^{N} C_i\chi_i\,, \tag{4}$$

where $i \equiv (n, l)$. Corresponding symmetrized trial functions have been used for the helium calculations. The variational method leads to the secular

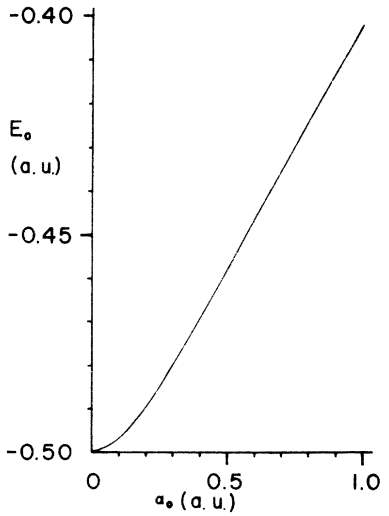

FIG. 1. Ground-state energy E_0 of atomic hydrogen vs the classical displacements α_0 (in atomic units).

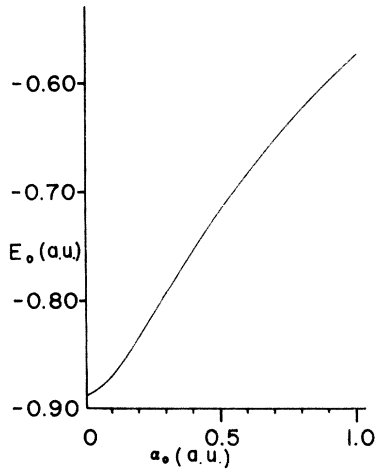

FIG. 3. One-electron ionization potential E_0 of atomic helium vs the classical displacements α_0 (in atomic units).

equation

$$\det(H_0 - E_0 S) = 0 ,$$

where

$$S_{ij} = \int \chi_i^* \chi_j d\tau . \tag{5}$$

The matrix elements H_{0ij} and S_{ij} have been computed analytically. The Coulomb repulsion term is, of course, included in the helium calculation. In the hydrogen case, twelve basis functions were used corresponding to $N \leq 5$ in Eq. (4). In the helium case, 36 basis functions were used. In all cases it was determined that the ground-state energy was sufficiently insensitive to the use of a larger number of basis terms. For example, it was found that the ground-state energy of helium changes by less than 0.1% when one increases the number of basis functions from 12 to 36.

The results are best given in graphical form. Figure 1 shows the ground-state energy of atomic hydrogen as a function of α_0. Figures 2 and 3 illustrate corresponding data for helium. Computations for hydrogen and for helium were carried out up to quite large values of α_0. Since the range of validity of the present theory remains a question for further investigation, we give the results for the one-electron ionization potential of atomic helium for larger values of α_0 in Fig. 4.

In Table I we list values of α_0 corresponding to frequencies and intensities for which experimental

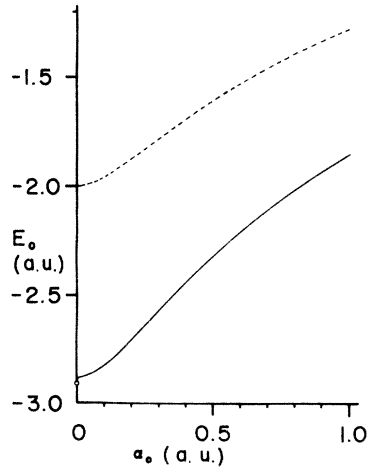

FIG. 2. Ground-state energies of both atomic helium and helium ion vs the classical displacements α_0 (in atomic units). (Solid line for atomic helium and dashed line for helium ion; open circle at -2.903 a.u. is the best value of the ground-state energy for atomic helium.)

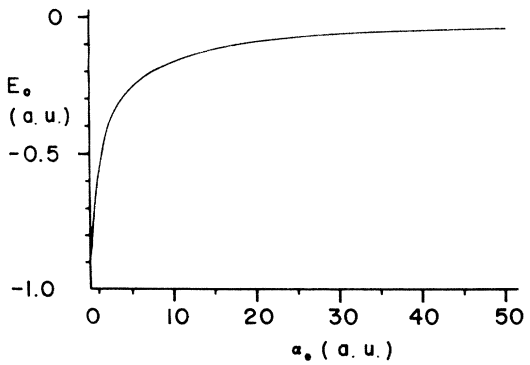

FIG. 4. One-electron ionization potential E_0 of atomic helium vs the classical displacements α_0 (in atomic units).

TABLE I. Values of the parameter α_0 corresponding to values of photon energy and beam intensity for existing experimental data.[13]

E_{rms}(V/cm)	I (W/cm^2)	α_0(a.u.)	Photon wavelength, λ (μ)	$\hbar\omega$ (eV)
3.0×10^7	2.4×10^{12}	1.1153	0.5293	2.3422
2.3×10^7	1.4×10^{12}	3.4126	1.0586	1.1710

ionization probabilities are available. The reader is cautioned, however, that the ionization potentials of the present theory are not the degrees of nonlinearity discussed in Ref. 13. The purpose of this paper is merely to demonstrate that the ionization potentials of the present theory differ strongly from the ground-state energies. Comparison with experiment will first be possible when ionization probabilities have been computed.

III. CONCLUSION

The motivation behind this work was the hope that a reduction in the ionization potentials of atoms due to the presence of the intense radiation would serve to close the gap between theoretical and experimental ionization probabilities. The results indicate that in the case of infrared radiation the effect may be much larger than one should wish. However, one may argue that the perturbation here [Eq. (24) of Ref. 5] is not as strong as that of the usual theory, since the electric field strength enters only in the arguments of the Bessel functions. The authors plan to compute ionization probabilities using the ground-state wave functions obtained here, enabling a comparison of theory with experiment.

Keldysh[1] has shown that the low-frequency limit of a more standard perturbation calculation is the case of tunneling through a barrier (corresponding to an atom in a static field). The corresponding phenomenon in the present case may be the decreasing importance of the time-dependent terms in the infrared limit (large α_0 for a given intensity), since the rapidly oscillating arguments of the Bessel functions might lead to comparatively small matrix elements between the ground state and the continuum states (approximated by plane waves). Thus in the infrared limit, it is possible that the principal effect would be a shift in the ionization potential of the atom. In any case, such an effect is necessary in order to make the present theory plausible in the infrared region. It is noteworthy that the only assumption made was the validity of the nonrelativistic dipole approximation. This approximation should improve as one goes toward the infrared region of the spectrum.

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