Comment on the pressure dependence of phonon-phonon scattering rates in He II

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The pressure dependence of the parallel three-phonon process is investigated and the expected pressure dependence of the wide-angle phonon scattering rate is discussed.

Recently, Dynes, Narayanamurti, and Andres[1] studied the transition from ballistic phonon flow to second sound in He II at different pressures. They found that the relevant relaxation time for phonon pulse propagation below 0.7 K is strongly pressure dependent. While a marked effect of phonon-phonon collisions is observed at saturated vapor pressure, no such effect is found for higher pressures between 10 and 24 atm. The authors suggest that the apparent reduction of phonon-phonon collision rates results from the increase of normal dispersion in the phonon spectrum at higher pressures, which affects the three-phonon process (3PP).

Very useful information about the variation of the phonon dispersion curve with pressure has been obtained[2] from an analysis of ultrasonic attenuation[3] and specific-heat[4] measurements. Roach, Ketterson, and Kuchnir[3] found that the ultrasonic attenuation at pressures between 14 and 19 atm is reduced in an intermediate temperature range around 0.5 K. These results were interpreted as an indication of a strong pressure dependence of the phonon dispersion curve in He II. It was inferred from these and specific-heat data that the region of anomalous dispersion[5] shrinks with pressure and that normal dispersion becomes large at higher pressures. The prediction of large (normal) dispersion under higher pressure was confirmed by inelastic neutron scattering.[6] It is the purpose of this note to investigate the pressure dependence of phonon-phonon scattering rates resulting from the pressure dependence of the phonon dispersion curve.

A reduction of the ultrasonic attenuation is most clearly seen at pressures above 14 atm, whereas Dynes *et al.* report that their phonon collision rate is already strongly reduced at $P = 10$ atm. The ultrasonic attenuation is determined by the "parallel" 3PP. On the other hand, the phonon scattering rate τ_{pp}^{-1} determining the transition from ballistic phonon flow to second sound is certainly related to wide-angle scattering. It is not known how to calculate this scattering rate, but we assume that it can be estimated by the wide-angle scattering rate τ_{η}^{-1} measured in heat-flow experiments.[7] This assumption is supported by the fact that the experimental values of τ_{pp}^{-1} and τ_{η}^{-1} differ by less than a factor of 2 between 0.4 and 0.6 K at saturated vapor pressure (SVP). The relaxation rate τ_{η}^{-1}, which is given by the viscosity of the phonons, has been calculated recently by Maris[8] at SVP. Starting from the basic assumption of anomalous dispersion, which allows angular spread in the 3PP, he derived this rate from repeated small-angle processes. His result contains two factors, one of which is the small-angle scattering rate between thermal phonons, $\tau_{\parallel}^{-1}$, while the other can be interpreted as the mean value of the fourth power of the scattering angle α. Both factors are strongly pressure dependent. We discuss them separately in what follows.

First we show that the small-angle scattering rate between thermal phonons $\tau_{\parallel}^{-1}$ depends on pressure more strongly than the ultrasonic attenuation. In ultrasonic attenuation a phonon of small wave number is absorbed by a phonon of thermal wave number (of order $k_BT/\hbar c$), whereas in the processes determining $\tau_{\parallel}^{-1}$ both phonons are of thermal wave number. We find that the average cutoff wave number[2] Q_c for the allowed 3PP is reduced by a factor of $1/\sqrt{3}$ in the thermal process as compared to the ultrasonic process. Thus the increase of normal dispersion with increasing pressure has a stronger effect on the thermal 3PP which determines $\tau_{\parallel}^{-1}$ than on the ultrasonic attenuation.

In order to investigate dispersion effects on the 3PP between thermal phonons quantitatively, we have calculated the thermal mean value $\tau_{\parallel}^{-1}$ of the sum of the three-phonon emission and absorption rates, taking energy and momentum conservation into account. The phonon energy curve was parametrized by $\omega(Q) = cQ(1 - \Gamma Q^2 - \Delta Q^4)$, where c is the sound velocity and Γ, Δ the disper-

TABLE I. Choice of cutoff wave number and dispersion parameters.

	P (atm)			
	SVP	4.8	10	14
Q_c (Å^{-1})	0.43	0.24	0.13	0.083
Γ(Å^2)	−0.46	−0.46	−0.4	−0.4
Δ(Å^4)	1.51	4.79	16.3	34.9

sion parameters. The nontrivial problem is the choice of the dispersion parameters. We have interpolated the cutoff wave number Q_c ($Q_c^2 = -3\Gamma/5\Delta$) between SVP (Q_c determined from a fit of neutron data below $Q = 0.6$ Å^{-1} with Γ from specific-heat data) and $P = 14$ atm (Q_c determined from ultrasonic data) and have chosen Γ to be almost pressure independent (see Table I). Similar data for Q_c have been given in Ref. 9. The fit of the heat-conduction experiments by the calculated[8] wide-angle rate τ_η^{-1} seems to require $Q_c = 0.38$ Å^{-1} and $\Gamma = -1.1$ Å^2 at SVP.[10] The values of the parameters Γ and Δ given in Table I should be considered only as rough estimates which are used to calculate the approximate pressure dependence of the phonon process rates $\tau_\parallel^{-1}$ and τ_η^{-1}. Our results for $\tau_\parallel^{-1}$ are shown in Fig. 1. Deviations from the T^5 behavior of the fully allowed 3PP already appear at $P = 4.8$ atm at temperatures below 1 K. At higher pressures $\tau_\parallel^{-1}$ is already strongly reduced at intermediate temperatures.

According to Maris, the second factor determining the wide-angle scattering rate at SVP is a mean value of the fourth power of the scattering angle α. The fourth power is a consequence of angle correlations introduced by momentum conservation. $\langle\alpha^4\rangle$ is strongly influenced by effects of normal dispersion. α^4, which is proportional to $\Gamma^2 Q^4$, has a thermal mean value $\propto \Gamma^2 T^4$ only at the lowest temperatures; at higher temperatures and intermediate pressures Q^4 has to be replaced by the cutoff wave number $Q_c{}^4 \propto \Gamma^2/\Delta^2$ such that $\langle\alpha^4\rangle$ becomes almost temperature independent. We have estimated $\langle\alpha^4\rangle$ for different pressures using the dispersion parameters listed in Table I. Our results are plotted in Fig. 2. The fact that $\langle\alpha^4\rangle$ is practically constant and $\tau_\parallel^{-1}$ rises less steeply than T^5 at higher temperatures explains qualitatively why the wide-angle scattering rate τ_η^{-1} at SVP can have a temperature dependence proportional to $T^{4.3}$. Of course one has to solve the Boltzmann equation for the phonons[8] or make a variational calculation in order to get quantitative results.

The pressure dependence of the wide-angle scattering rate τ_η^{-1} is mainly determined by the

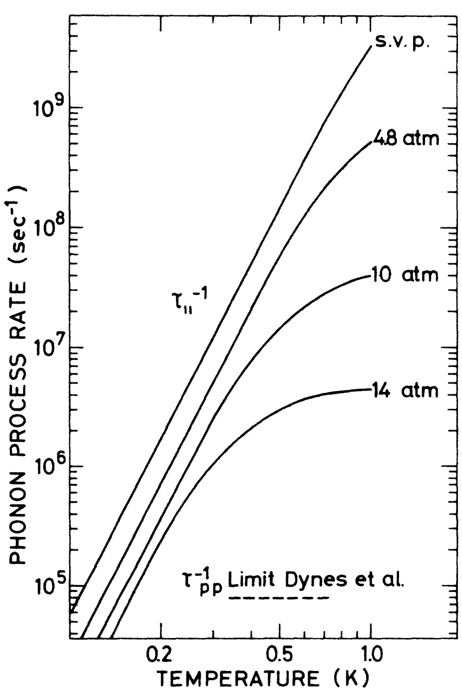

FIG. 1. Thermal mean value $\tau_\parallel^{-1}$ of the sum of three-phonon emission and absorption rates. Dashed line: limiting value for τ_{pp}^{-1} of Dynes *et al*. for $P \geq 10$ atm.

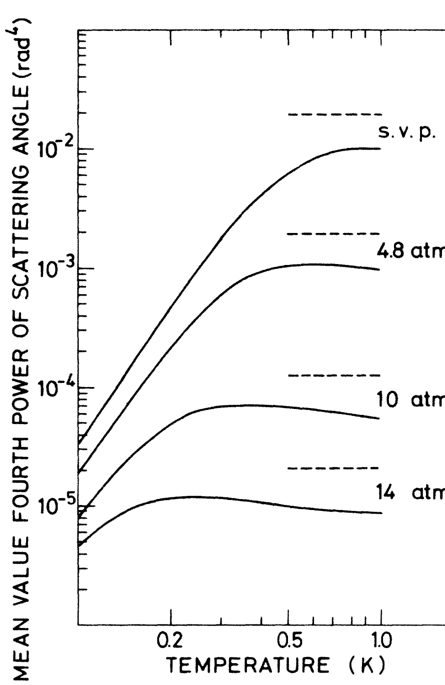

FIG. 2. Thermal mean value $\langle\alpha^4\rangle$ of the fourth power of the scattering angle of three-phonon emission and absorption processes. Dashed lines: $\Gamma^2 Q_c^4$ at the corresponding pressures.

pressure dependence of the phonon dispersion. Qualitatively, its behavior is found from the pressure dependence of $\tau_{\parallel}^{-1}(P)$, which has been given in Fig. 1, and from the pressure dependence of $\langle\alpha^4\rangle$. $\langle\alpha^4\rangle$ varies with pressure, for not too low temperatures, essentially as $\Gamma^2(P) \times Q_c^4(P)$, cf. Fig. 2. Thus at 10 atm and $T=0.6$ K we expect a typical wide-angle scattering rate for heat conduction of the order of only 2×10^2 $\sec^{-1}$. This rate is considerably lower than the upper limit for τ_{pp}^{-1} of 8×10^4 $\sec^{-1}$ given by Dynes *et al.*[1] for pressures $P \geq 10$ atm. Even though the nature of the scattering rate τ_{pp}^{-1} entering the transition from ballistic phonon flow to second sound is not understood in detail, and τ_{pp}^{-1} is probably somewhat different from the viscous relaxation rate τ_{η}^{-1}, the above considerations give a plausible explanation of the result of Dynes *et al.* An investigation of the wide-angle scattering rates by heat conduction[7] or heat-pulse experiments[11] under pressure (below 10 atm) would supply valuable additional information on the pressure dependence of the phonon energy curve. Detailed calculations of the pressure dependence of the wide-angle scattering rate τ_{η}^{-1} will be published elsewhere. In conclusion, we want to emphasize that the description of the phonon energy curve with a region of anomalous dispersion is consistent with at least three independent experiments.

Very recently Narayanamurti, Andres, and Dynes[12] measured the dispersion and the mean free path of superthermal phonons in He II using a pulse technique. They found no indication of anomalous dispersion and concluded that anomalous dispersion at SVP must be much smaller than the estimates based on specific heat and ultrasonic attenuation. We believe there are several open questions concerning the validity of these new results. It is very surprising that such high-frequency phonons can propagate over a distance of several millimeters, since one would expect a very short mean free path (2×10^{-5} cm for $\nu = 0.91\times 10^{11}$ $\sec^{-1}$) for splitting of such phonons by the 3PP. On the other hand, the results of the same authors for the pressure dependence of the mean free path of the superthermal phonons can be interpreted as an indication of the occurrence of the 3PP, in a similar way as the results for the pressure dependence of the thermal phonon mean path $\tau_{\parallel}^{-1}$ discussed above. The observed mean free path increases strongly around a pressure of 14 atm, which can be attributed to the suppression of the 3PP. One has to wait for further clarification before definite conclusions can be drawn from these new results.

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[1]R. C. Dynes, V. Narayanamurti, and K. Andres, Phys. Rev. Lett. 30, 1129 (1973).

[2]J. Jäckle and K. W. Kehr, Phys. Rev. Lett. 27, 654 (1971).

[3]P. Roach, J. B. Ketterson, and M. Kuchnir, Phys. Rev. Lett. 25, 1002 (1970); Phys. Rev. A 5, 2205 (1972).

[4]N. E. Phillips, C. G. Waterfield, and J. K. Hoffer, Phys. Rev. Lett. 25, 1260 (1970).

[5]H. J. Maris and W. E. Massey, Phys. Rev. Lett. 25, 220 (1970).

[6]E. C. Svensson, A. D. B. Woods, and P. Martel, Phys. Rev. Lett. 29, 1148 (1972).

[7]R. W. Whithworth, Proc. R. Soc. A 246, 390 (1958).

[8]H. J. Maris, Phys. Rev. A 8, 1280 (1973).

[9]A. D. B. Woods and R. A. Cowley, Rep. Prog. Phys. 36, 1135 (1973).

[10]Using the variational principle for the viscosity, we could fit the experimental data[8] for τ_{η}^{-1} at SVP with the parameters $\Gamma = -0.68$ Å^2 and $Q_c = 0.35$ Å^{-1}. Given the value of Q_c, the value of $|\Gamma|$ represents a lower limit. These results are compatible with the Maris results[8]. However, it is not clear whether the four-phonon process can be really neglected in the calculation of the viscosity. In principle the four-phonon processes are necessary in order to obtain a finite viscosity, since the three-phonon process is forbidden for phonons with high momentum. This can be seen from the variational principle.

[11]R. W. Guernsey and K. Luszczynski, Phys. Rev. A 3, 1052 (1971).

[12]V. Narayanamurti, K. Andres, and R. C. Dynes, Phys. Rev. Lett. 31, 687 (1973).