Comments and Addenda

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Influence of phonon dispersion on the velocity of second sound in He II*

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(Received 13 September 1973)

We present calculations of the velocity of second sound u_2 below 1 K as a function of pressure. These calculations show that u_2 rises above the Landau limit in the presence of positive phonon dispersion. The effect should be experimentally observable in a suitably designed apparatus.

In a recent article Saslow observed that phonon dispersion should affect the second-sound velocity u_2 at low temperatures.[1] In this Comment we present the results of calculations which explore this observation, and which describe the pressure-dependent deviation of u_2 from the Landau limit $u_1/\sqrt{3}$ (where u_1 is the first-sound velocity) as $T \to 0$. We find that phonon dispersion may affect u_2 as much as 3%, whereas a direct determination of first-sound dispersion yields only a 0.01% effect in the frequency range 10^6-10^9 MHz.[2] This pronounced enhancement of $\Delta u_2/u_2$ over $\Delta u_1/u_1$ should prove valuable in further experimental investigations of the first nonlinear terms in the excitation spectrum.

To facilitate our calculations, we have utilized a series representation of the He II excitation curve in momentum p which includes the variation of the spectrum parameters with temperature and pressure, and is given by

$$\epsilon(p) = u_1 p + a_3 p^3 + a_4 p^4 + a_5 p^5 + a_6 p^6 + a_7 p^7 + a_8 p^8 , \tag{1}$$

where the coefficients $a_3 - a_8$ are determined from constraints imposed by the results of neutron scattering data.[3] In particular, Eq. (1) has dispersion in the phonon region which changes from positive ($a_3 > 0$) to negative ($a_3 < 0$) at a pressure of about 11 atm.

The second-sound velocity can be calculated from Eq. (1) in several ways, assuming thermal equilibrium is maintained among excitations (the hydrodynamic regime). Saslow[1] provides an expression for the deviation of u_2 from the zero-temperature value $u_1/\sqrt{3}$, which is given as

$$\frac{\Delta u_2}{u_2} = -\frac{15}{7}\gamma\left(\frac{2\pi kT}{u_1}\right)^2 - \frac{945}{8} r\gamma'\left(\frac{2\pi kT}{u_1}\right)^3 , \tag{2}$$

where $r = 3.0047\times 10^{-2}$, $\gamma = -a_3/u_1$, and $\gamma' = -a_4/u_1$. Alternatively, u_2 can be calculated directly from $\epsilon(p)$ by an expression described by Kwok[4] which assumes energy- and momentum-conserving three-phonon processes:

$$u_2^2 = \frac{\frac{1}{3}(\sum_p \epsilon_p S_p^0 \vec{p}\cdot\vec{\nabla}_p \epsilon_p)^2}{(\sum_p S_p^0 \epsilon_p^2)(\sum_p S_p^0 p^2)} . \tag{3}$$

Here $S_p^0 = n(p)[n(p)+1]$; $n(p) = e^{\epsilon(p)/kT} - 1$. A third method is to compute u_2 from the relation

$$u_2^2 = \rho_s T S^2/\rho_n C , \tag{4}$$

where the normal and superfluid densities, ρ_n and ρ_s, and the entropy S and specific heat C can be calculated by integration over $\epsilon(p)$.

In the temperature range $0 < T < 0.5$ K, all three expressions predict a rise in u_2 for $a_3 > 0$. For $a_3 \leq 0$, no rise in u_2 appears, and the normal asymptotic behavior ($u_2 \to u_1/\sqrt{3}$, as $T \to 0$) is evident. Figure 1 shows results from Eq. (4) at the vapor pressure. The lowest curve is based on the Landau approximation to the dispersion

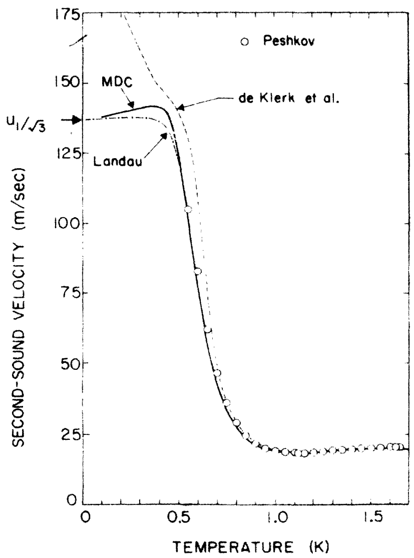

FIG. 1. Temperature dependence of second sound as calculated with no phonon dispersion (Landau), and with Eq. (4) (model dispersion curve or MDC). Also plotted are the data of Peshkov (Ref. 5) and de Klerk *et al.* (Ref. 6). The limit $u_2 = u_1/\sqrt{3}$ is indicated.

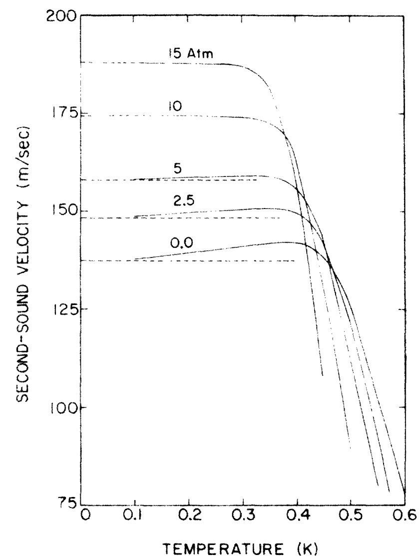

FIG. 2. Velocity of second sound at several pressures below 0.6 K as calculated with Eq. (4). The dotted lines indicate u_2 calculated with no phonon dispersion.

curve, $\epsilon = u_1 p$ in the phonon region, and $\epsilon = \Delta + (p - p_0)^2/2\mu$ in the roton region; u_2 approaches the Landau limit as shown. The curve above uses Eq. (1) and shows that in the region of 0.5 K, u_2 exceeds $u_1/\sqrt{3}$ by about 5 m/s. Peshkov's measurements[5] agree within a few percent with our calculations down to 0.55 K. Data of de Klerk, Hudson, and Pellam[6] are shown in the upper curve. While there is a discrepancy in absolute magnitude, the data show a pronounced tendency to follow the calculated curve with phonon dispersion. At the lowest temperatures, in a limited geometry, heat pulses will consist of ballistic phonons of velocity u_1, and the data clearly approaches this limit. Figure 2 shows the pressure dependence of u_2 from Eq. (4). The rise in u_2 disappears above 10 atm.

The effects of an increasing phonon mean-free path at low temperatures can be avoided by using large second-sound chambers. (Saslow[1] suggests dimensions $\gtrsim 10$ cm.) With modern cryogenics, such an apparatus is feasible. We propose that a careful search be made under pressure to ascertain the disappearance of the effect of phonon dispersion in the manner predicted by the pressure dependece of u_2 shown in Fig. 2.

We wish to thank Professor W. M. Saslow for his comments.

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*Research supported by the Air Force Office of Scientific Research under Grant No. AF-AFOSR-71-1999, and by the National Science Foundation under Grant No. NSF-GP-26361.

[1]W. M. Saslow, Phys. Rev. A 5, 1491 (1972). See also S. Ma, J. Math. Phys. 12, 2157 (1971).

[2]B. M. Abraham, Y. Eckstein, J. B. Ketterson, M. Kuchnir, and J. B. Vignos, Phys. Rev. 181, 347 (1969).

[3]J. S. Brooks, Ph.D. thesis (University of Oregon) (unpublished); J. S. Brooks and R. J. Donnelly, Phys. Lett. A (to be published). Note that H. Gould and V. K. Wong, Phys. Rev. Lett. 27, 301 (1971) have proposed a non-analytic excitation spectrum. Equation (1) is not proposed as a unique representation of the neutron scattering data, but it is an accurate representation over the range $0 \leq p/\hbar \leq 2.15$ Å^{-1}.

[4]P. C. Kwok, Physics (N. Y.) 3, 221 (1967); see also R. C. Dynes, V. Narayanamurti, and K. Andres, Phys. Rev. Lett. 30, 1129 (1973).

[5]V. P. Peshkov, Zh. Eksp. Theor. Fiz. 38, 799 (1960) [Soviet Phys.—JETP 11, 580 (1960)].

[6]D. de Klerk, R. P. Hudson, and J. R. Pellam, Phys. Rev. 93, 28 (1953).