Density operator of unpolarized radiation

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A direct derivation of the forms of density operator and phase-space distribution functions for unpolarized radiation is given.

The authors recently defined[1] the unpolarized state of radiation as one which does not change on introducing any phase shift in any polarized component, and showed[1,2] that its density operator has the form,

$$\rho = \sum_n B_n \sum_r |r, n-r\rangle\langle r, n-r| \,, \tag{1}$$

where $|r,s\rangle = (r!\,s!)^{-1/2}\hat{a}_1^{\dagger r}\hat{a}_2^{\dagger s}|0,0\rangle$ is the state having r and s photons in the transverse modes 1 and 2, respectively. Agarwal[3] recently gave another derivation of the authors' result[1,2] and studied the phase-space distribution functions.[4] In the present note, we give a simple and direct derivation of Eq. (1) and the functional forms of phase-space distribution functions.

The above-mentioned definition of the unpolarized state is equivalent[1] to the relation,

$$\exp[i\theta\hat{a}_{\vec{\epsilon}}^{\dagger}\hat{a}_{\vec{\epsilon}}]\hat{\rho}\exp[-i\theta\hat{a}_{\vec{\epsilon}}^{\dagger}\hat{a}_{\vec{\epsilon}}] = \hat{\rho} \,, \tag{2}$$

where $\hat{a}_{\vec{\epsilon}} = \epsilon_1\hat{a}_1 + \epsilon_2\hat{a}_2$ is the annihilation operator for radiation polarized in the mode characterized by the complex unit vector $\vec{\epsilon} = (\epsilon_1, \epsilon_2)$ and θ is the arbitrary phase shift introduced in this mode. Equation (2) is identical with

$$[\hat{a}_{\vec{\epsilon}}^{\dagger}\hat{a}_{\vec{\epsilon}}, \hat{\rho}] = 0 \,. \tag{3}$$

Putting $\vec{\epsilon} = (1,0)$ and $(0,1)$ in Eq. (3), we see that $\hat{\rho}$ has a diagonal form. If we write

$$\hat{\rho} = \sum_{r,s} C_{rs}|r,s\rangle\langle r,s| \,, \tag{4}$$

substitution in Eq. (3) gives $C_{rs} = C_{r-1\,s+1}$, the repeated use of which leads to

$$C_{rs} = C_{r-1\,s+1} = \cdots = C_{0\,r+s} = B_{r+s} \,. \tag{5}$$

This reduces Eq. (4) to Eq. (1).

Using Eq. (5), it is very easily seen that $\rho_A(\alpha,\beta) \equiv \langle\alpha,\beta|\hat{\rho}|\alpha,\beta\rangle$ is a function of $|\alpha|^2 + |\beta|^2$ only. In the diagonal representation,[5,6]

$$\hat{\rho} = \int d^2\alpha\, d^2\beta\, \rho_N(\alpha,\beta)|\alpha,\beta\rangle\langle\alpha,\beta| \,, \tag{6}$$

the extension of Mehta's result[7] to the case of two modes, leads to

$$\rho_N(\alpha,\beta) = \pi^{-4}\int d^2\xi\, d^2\eta\, \langle -\xi, -\eta|\hat{\rho}|\xi,\eta\rangle \times \exp(|\alpha|^2 + |\beta|^2 + |\xi|^2 + |\eta|^2 + \xi^*\alpha + \eta^*\beta - \xi\alpha^* - \eta\beta^*) \,. \tag{7}$$

If we substitute $\xi = R\cos\theta e^{i\phi_\xi}$, $\eta = R\sin\theta e^{i\phi_\eta}$, and use Eq. (1) for $\hat{\rho}$, direct integration over θ, ϕ_ξ, and ϕ_η shows that $\rho_N(\alpha,\beta)$ is a function of $|\alpha|^2 + |\beta|^2$ only.

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