Radiation from a moving planar dipole layer: Patch potentials versus dynamical Casimir effect

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We study the classical electromagnetic radiation due to the presence of a dipole layer on a plane that performs a bounded motion along its normal direction, to the first nontrivial order in the amplitude of that motion. We show that the total emitted power may be written in terms of the dipole layer autocorrelation function. We then apply the general expression for the emitted power to cases where the dipole layer models the presence of patch potentials, comparing the magnitude of the emitted radiation with that coming from the quantum vacuum in the presence of a moving perfect conductor (dynamical Casimir effect).

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I. INTRODUCTION

Due to the unavoidable presence of imperfections, impurities, and even spatial variations in its chemical composition, the surface of a real metal is not a perfect equipotential. In other words, those effects manifest themselves in the existence of an electrostatic potential, with a nontrivial space dependence, on the metallic surface [1]. These residual "patch potentials" may even produce forces between different metallic surfaces, which become relevant for high sensitivity experiments, in many areas of physics [2]. In particular, they can be crucial when determining the vacuum forces of quantum electromagnetic (EM) origin between neutral objects (static Casimir effect) and in experiments looking for modifications to the gravitational inverse-square law [2–5]. In those experimental setups, the existence of patch potentials results in the presence of a force, electrostatic in origin, and the theoretical goal is then to have a proper, and hopefully simple, way to quantify it. Ideally one should be able to disentangle its contribution in any experimental attempt to determine forces which are of a different nature. For static situations, one is usually able to derive general expressions of the effects due to the patch potentials in terms of just their autocorrelation functions.

In this paper we quantify an effect due to the presence of patch potentials in a rather different, complementary situation: namely, we consider a metallic object that undergoes accelerated motion and study the resulting EM radiation. The physical reason to expect such radiation becomes clear when one recalls that patch potentials can be thought of as due to the existence of a (space-dependent) dipole layer on the surface of an otherwise neutral body [3]. Therefore, when accelerated, the moving dipole layer shall emit radiation. Our main goal here is to evaluate the power emitted by a flat surface containing patch potentials, in terms of the acceleration of the plane and the characteristics of the patch potentials.

One of the motivations that lead us to this calculation is to make a quantitative comparison between the magnitudes of this classical effect with the power emitted by an accelerated ideal, perfectly conducting mirror. This "motion-induced radiation" or "dynamical Casimir effect" (DCE) [6] is a purely quantum effect, that, as we will see, could also be interpreted as coming from a moving dipole layer with an *ad hoc* autocorrelation function.

The radiation field of a single time-dependent dipole at rest is a classical problem, described in almost all texts on classical electrodynamics. The radiation field of a moving dipole, however, is not so widely known, although it has already been investigated in the sixties [7]. To our knowledge, the spectrum of classical EM radiation due to the presence of a *dipole layer* on a plane that moves rigidly has not been computed before. Therefore, since it is a crucial ingredient for our study, we present this calculation in Sec. II. As we shall show, there is a rather simple formula for the spectral density associated to the radiated energy, in terms of a two-point function that describes the correlation of the dipoles at different points of the surface. Using the model of Ref. [3], this leads immediately to an expression for the emitted power by moving patch potentials. In this section, we also comment on the case of time dependent dipole layers. In Sec. III we compute the spectral density for the particular autocorrelations previously used in the literature to describe patch potentials, and make a comparison between the classical emitted power and the DCE. Section IV contains the conclusions of our work.

II. RADIATION

In this section we evaluate the classical EM radiation due to the presence of a dipole layer on a plane that moves rigidly along the direction defined by its normal (we use CGS-Gaussian units throughout). The instantaneous position of the plane may be defined in terms of a single function $q(t)$ such that $x_3 = q(t)$. Here x_3 is one of the three Cartesian coordinates (x_1,x_2,x_3), for which we shall also use the notation $\mathbf{x}_\parallel \equiv (x_1,x_2)$. The dipole layer density D shall then be a function of the two coordinates parallel to the plane, namely, with the notation just introduced, $D = D(\mathbf{x}_\parallel)$.

To proceed to the calculation of the emitted radiation, we need the charge and current densities ρ and $\mathbf{j}$, which for the system we are considering are given by

$$\begin{aligned} \rho(\mathbf{x},t) &= -D(\mathbf{x}_\parallel)\, \delta'(x_3 - q(t)), \\ \mathbf{j}(\mathbf{x},t) &= -D(\mathbf{x}_\parallel)\, \delta'(x_3 - q(t))\, \dot{q}(t)\, \hat{\mathbf{e}}_3, \end{aligned} \tag{1}$$

where $\hat{\mathbf{e}}_3$ is the unit vector along the direction of motion.

Finally, we shall assume that the motion is bounded, namely, that there is a length l such that $|q(t)| \leqslant l$, $\forall t$.

To determine the radiated power, we introduce the retarded potentials, and use the Lorentz gauge fixing condition, obtaining for the potentials the inhomogeneous wave equations

$$\phi(\mathbf{x},t) = \int d^3x'\, dt'\, G(\mathbf{x},t;\mathbf{x}',t')\,\rho(\mathbf{x}',t'),$$
$$\mathbf{A}(\mathbf{x},t) = \frac{1}{c}\int d^3x'\, dt' G(\mathbf{x},t;\mathbf{x}',t')\mathbf{j}(\mathbf{x}',t'), \tag{2}$$

where G denotes the retarded Green's function for the wave equation, which satisfies

$$\left(c^{-2}\partial_t^2 - \nabla_{\mathbf{x}}^2\right)G(\mathbf{x},t;\mathbf{x}',t') \;=\; 4\pi\,\delta(\mathbf{x}-\mathbf{x}')\,\delta(t-t'). \tag{3}$$

A more explicit expression may be obtained by introducing the Fourier transformation:

$$G(\mathbf{x},t;\mathbf{x}',t') = \int \frac{d\omega}{2\pi}\frac{d^3\mathbf{k}}{(2\pi)^3}\, e^{-i\omega(t-t')+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\widetilde{G}(\mathbf{k}_\parallel,k_3,\omega), \tag{4}$$

with

$$\widetilde{G}(\mathbf{k}_\parallel,k_3,\omega) = \frac{4\pi}{\mathbf{k}_\parallel^2 + k_3^2 - \left(\frac{\omega}{c}+i\eta\right)^2}. \tag{5}$$

The next step in the derivation of the emitted power is the introduction of the Poynting vector, $\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B}$, where

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A},$$
$$\mathbf{B} = \nabla\times\mathbf{A}. \tag{6}$$

Of course, for an arbitrary dipole layer, the radiation flux may have a rather cumbersome spatial dependence on the details of the layer. On the other hand, one is presumably more interested in the global effect, namely, the average flux of energy, since the spatial dependence of that flux is hardly detectable. The geometry of the system suggests to evaluate the total flux of radiated energy due to the moving plane. It is then convenient to evaluate the third component of $\mathbf{S}$, on a constant-x_3 plane, far from the region where the plane moves ($|x_3| > l$).

We see that the third component of $\mathbf{S}$ may be written as follows:

$$S_3 \;=\; \frac{c}{4\pi}\,\epsilon_{ij}\,E_i B_j, \tag{7}$$

where the indices i, j shall be assumed, from now on, to run from 1 to 2. Since $\mathbf{A}$ points in the $\hat{\mathbf{e}}_3$ direction, we see that the components of the electric and magnetic field relevant to the calculation of (7) are given by

$$E_i \;=\; -\partial_i\phi,\;\; B_i \;=\; \epsilon_{ij}\partial_j A_3. \tag{8}$$

Thus S_3 becomes

$$S_3 \;=\; \frac{c}{4\pi}\,\partial_j\phi\,\partial_j A_3. \tag{9}$$

The total flux of energy through one such plane shall be, in general, a divergent quantity, something that may be dealt with by dividing it by the total area. Besides, it is also convenient to calculate the total radiated energy since, when written in terms of the Fourier transforms of the time-dependent functions, it will allow us to extract the spectral density of radiation.

Thus, the (average) radiated energy per unit area through a constant-x_3 plane becomes

$$U_{\rm rad}(x_3) \;=\; \frac{1}{L^2}\int dt \int d^2\mathbf{x}_\parallel\, S_3(\mathbf{x}_\parallel,x_3,t), \tag{10}$$

where L^2 is the area of the plane, assumed temporarily large and finite, but an $L\to\infty$ limit at the end is assumed. We expect it to be independent of x_3 far from the planes.

After some algebra, we see that, to second order in $q(t)$, $U_{\rm rad}(x_3)$ may be written as follows:

$$U_{\rm rad}(x_3) = -4\pi\int \frac{d^2k_\parallel}{4\pi^2}\frac{d\omega}{2\pi}\frac{dk_3}{2\pi}\frac{dp_3}{2\pi} \times \left\{ \frac{|\mathbf{k}_\parallel|^2 k_3^2 p_3\omega}{\left[\mathbf{k}_\parallel^2 + k_3^2 - \left(\frac{\omega}{c}+i\eta\right)^2\right]\left[\mathbf{k}_\parallel^2 + p_3^2 - \left(\frac{\omega}{c}-i\eta\right)^2\right]} \times\; \widetilde{\Omega}(\mathbf{k}_\parallel)\, |\tilde{q}(\omega)|^2 e^{ix_3(k_3+p_3)} \right\}, \tag{11}$$

where we have introduced the Fourier transform of the dipole layer autocorrelation function

$$\Omega(\mathbf{x}_\parallel) \;=\; \frac{1}{L^2}\int d^2y_\parallel D(\mathbf{y}_\parallel)D(\mathbf{x}_\parallel+\mathbf{y}_\parallel). \tag{12}$$

It is worth noting that, in natural ($\hbar = 1$ and $c = 1$) units, $\widetilde{\Omega}$ is a dimensionless quantity. We mention at this point that a similar expression to the one above could have been obtained if one had a random patch potential distribution, with a translation invariant stochastic correlation. Namely, even without evaluating the average over a constant-x_3 plane, the translation invariance of the system does produce an entirely analogous expression to the one above, now interpreting Ω as the result of an average with a statistical weight.

We then evaluate the integrals over k_3 and p_3, which can be performed, for example, by using Cauchy's theorem in a rather straightforward way, obtaining a result that contains both convection and radiation terms. The latter are, for $x_3 > 0$, independent of x_3. On the other hand, an evaluation of $U_{\rm rad}(-x_3)$, the average energy flux through a plane symmetrically located with respect to $x_3 = 0$, yields the same result as $U_{\rm rad}(x_3)$.

Next we introduce $U_{\rm rad}$, the total radiated energy per unit area:

$$U_{\rm rad} \;=\; U_{\rm rad}(x_3) \;+\; U_{\rm rad}(-x_3) \;=\; 2U_{\rm rad}(x_3), \tag{13}$$

which, moreover, may be conveniently written as follows:

$$U_{\rm rad} \;=\; \int_0^\infty \frac{d\omega}{2\pi}\,\mathcal{P}(\omega), \tag{14}$$

where the spectral density $\mathcal{P}(\omega)$ is

$$\mathcal{P}(\omega) \;=\; |\omega||\tilde{q}(\omega)|^2\int_0^{\omega/c} dk_\parallel\, k_\parallel^3\,\widetilde{\Omega}(k_\parallel)\sqrt{\left(\frac{\omega}{c}\right)^2 - k_\parallel^2}, \tag{15}$$

where we have assumed the autocorrelation function to be isotropic.

Equations (14) and (15) constitute the main result of this section, namely, a rather general and compact expression for the spectral density of emitted energy in terms of the two main ingredients that characterize the system: its motion $\tilde{q}(\omega)$ and the autocorrelation function of the dipole layer density.

Assuming that the correlation length is much smaller than c/ω, we can approximate $\widetilde{\Omega}(k_\parallel) \simeq \widetilde{\Omega}(0)$. With this approximation

$$U_{\rm rad} \simeq \frac{2}{15}\widetilde{\Omega}(0)\int dt\, \dot{a}^2, \tag{16}$$

where a is the acceleration. From this equation, one can show that the radiation reaction force per unit area on the dipole layer is

$$f_{\rm rad} \simeq \tfrac{2}{15}\widetilde{\Omega}(0)\ddot{a}. \tag{17}$$

Notably, in this approximation the results coincide with those of a single moving dipole d with $d^2 \equiv \widetilde{\Omega}(0)c^5$ [7,8].

Until now we considered a dipole layer. In order to describe an imperfect conductor with patch potentials, following Ref. [3] we can consider a dipole layer close to a grounded perfect conductor. For this configuration, and for nonrelativistic motion of the mirror, the radiated power can be computed using the method of images: in the presence of the grounded conductor, the dipole density is increased by a factor 2, and therefore the spectral density by a factor 4.

Finally, we consider the case of time dependent dipole layers, namely, $D = D(\mathbf{x}_\parallel,t)$. This time dependence may be produced by an external agent or may describe intrinsic fluctuations of the material. The generalization of (1) to this situation is

$$\begin{aligned}\rho(\mathbf{x},t) &= -D(\mathbf{x}_\parallel,t)\,\delta'(x_3 - q(t))\\ \mathbf{j}(\mathbf{x},t) &= \Big[-D(\mathbf{x}_\parallel,t)\,\delta'(x_3 - q(t))\,\dot{q}(t)\\ &\quad + \frac{\partial D(\mathbf{x}_\parallel,t)}{\partial t}\,\delta(x_3 - q(t))\Big]\hat{\mathbf{e}}_3,\end{aligned} \tag{18}$$

where the presence of the last term is required by current conservation.

A lengthier but otherwise entirely analogous calculation allows one to obtain the spectral density for this case:

$$\begin{aligned}\mathcal{P}(\omega) = \frac{1}{L^2}\,|\omega| \int_0^{\omega/c} dk_\parallel\, k_\parallel^3 \sqrt{\left(\frac{\omega}{c}\right)^2 - k_\parallel^2} \int \frac{d\nu}{2\pi}\frac{d\nu'}{2\pi}\\ \times \tilde{q}(\nu)\,\widetilde{D}(\mathbf{k}_\parallel,\omega-\nu)\widetilde{D}(-\mathbf{k}_\parallel,-\omega-\nu')\,\tilde{q}(\nu'),\end{aligned} \tag{19}$$

in terms of the space and time Fourier transforms on the patch potentials [9].

The derivation was performed without any assumption about the origin of the time dependence. Let us now focus on the case in which the time dependences are correlated. In the absence of external agents producing that time dependence, it is reasonable to assume that they depend only on the time difference between the two potentials. In Fourier space, that amounts to

$$\frac{1}{L^2}\,\widetilde{D}(\mathbf{k}_\parallel,\omega)\widetilde{D}(-\mathbf{k}_\parallel,\omega') \to \widetilde{\Omega}(\mathbf{k}_\parallel,\omega)\,(2\pi)\delta(\omega+\omega'), \tag{20}$$

which inserted into (19) yields

$$\begin{aligned}\mathcal{P}(\omega) = \frac{1}{L^2}\,|\omega| \int_0^{\omega/c} dk_\parallel\, k_\parallel^3 \sqrt{\left(\frac{\omega}{c}\right)^2 - k_\parallel^2}\\ \times \int \frac{d\nu}{2\pi}\tilde{q}(-\nu)\,\widetilde{\Omega}(\mathbf{k}_\parallel,\omega-\nu)\,\tilde{q}(\nu).\end{aligned} \tag{21}$$

The last equation reduces to the static one for instantaneous correlation, namely, when

$$\widetilde{\Omega}(\mathbf{k}_\parallel,\omega) = \widetilde{\Omega}(\mathbf{k}_\parallel)(2\pi)\delta(\omega). \tag{22}$$

The calculation for time dependent dipole layers could be a useful starting point to develop a microscopic approach of the DCE. One should consider both electric and magnetic dipoles as sources, with a particular time-dependent correlation function to describe the quantum fluctuations.

III. EXAMPLES AND COMPARISON WITH THE DCE

As a first example of a patch potential autocorrelation, we first consider the Gaussian approximation to the quasilocal correlation function proposed in Ref. [10]:

$$\widetilde{\Omega}(\mathbf{k}_\parallel) = \frac{\pi}{8}V_{\rm rms}^2\ell^2 \exp\left[-\frac{1}{16}|\mathbf{k}_\parallel|^2\ell^2\right], \tag{23}$$

where $V_{\rm rms}$ is the variance of the potential and ℓ a characteristic length. For this particular correlation, the spectral density reads

$$\mathcal{P}(\omega) = V_{\rm rms}^2|\tilde{q}(\omega)|^2\frac{\omega^4}{c^3}f\,(\ell\omega/4c)\,, \tag{24}$$

with

$$f(x) = \frac{2\pi}{x^3}[3x - (3+2x^2)D_+(x)], \tag{25}$$

where D_+ denotes the Dawson function (note that in the equations above we have included the factor 4 coming from the image dipole layer). For a fixed frequency, the spectral density is a nonmonotonous function of the correlation length ℓ. Indeed, $\mathcal{P}$ vanishes for $\ell \to 0$ (no patch potentials) and also vanishes in the opposite limit $\ell \to \infty$, since by a simple application of symmetry arguments and Gauss's law one sees that a uniform density of charges or dipoles on a plane cannot radiate. Therefore, it must have a maximum at an intermediate value. A plot of the function $f(x)$ shows that the maximum is located at $x \sim 1.5$. As a consequence, if the plane moves with a definite frequency ω_0, the radiation emitted is maximum when the characteristic size of the patches is $\ell \sim 6c/\omega_0$.

As a second example, we will consider the sharp-cutoff model proposed in Ref. [3]:

$$\widetilde{\Omega}(\mathbf{k}_\parallel) = \frac{4\pi V_{\rm rms}^2}{k_{\rm max}^2 - k_{\rm min}^2}\theta(|\mathbf{k}_\parallel| - k_{\rm min})\theta(k_{\rm max} - |\mathbf{k}_\parallel|), \tag{26}$$

which yields for the spectral density

$$\begin{aligned}\mathcal{P}(\omega) = \frac{64\pi V_{\rm rms}^2}{15c^5\left(k_{\rm max}^2 - k_{\rm min}^2\right)}\omega^6|\tilde{q}(\omega)|^2\\ \times \left[1 - \left(\frac{k_{\rm min}c}{\omega}\right)^2\right]^{3/2}\left[2 + 3\left(\frac{k_{\rm min}c}{\omega}\right)^2\right],\end{aligned} \tag{27}$$

where we assumed that $\frac{k_{\rm min}c}{\omega} < 1$ and $\frac{k_{\rm max}c}{\omega} > 1$. Note that $\mathcal{P}(\omega)$ vanishes for $\frac{k_{\rm min}c}{\omega} > 1$. As a consequence, for the particular case in which the plane moves with a definite frequency ω_0, there is a threshold to have a nonvanishing emitted radiation $k_{\rm min} < \omega_0/c$.

In both examples one can consider the limiting case of small correlation length $\ell\omega/c \ll 1$, which is physically the more relevant limit. For the quasilocal correlations we obtain

$$\mathcal{P}(\omega) \simeq \frac{\pi}{60} V_{\rm rms}^2 \ell^2 \frac{\omega^6}{c^5} |\tilde{q}(\omega)|^2 . \quad (28)$$

A similar expression can be obtained for the sharp-cutoff model.

We will now compare the last result with that coming form the DCE. A single accelerated perfect mirror produces photons due to the interaction with the quantum fluctuations of the electromagnetic field. On dimensional grounds, in the nonrelativistic limit we expect the dissipative force per unit length on the mirror ($f_{\rm DCE}$) to be proportional to $\hbar\dddot{a}/c^4$. This corresponds to a spectral density proportional to $\hbar\omega^6|\tilde{q}(\omega)|^2/c^4$. An explicit calculation yields [6]

$$\mathcal{P}_{\rm DCE}(\omega) = \frac{\hbar}{30\pi^2} \frac{\omega^6}{c^4} |\tilde{q}(\omega)|^2 . \quad (29)$$

Defining $\xi = \mathcal{P}/\mathcal{P}_{\rm DCE} = f_{\rm rad}/f_{\rm DCE}$, we obtain

$$\xi = \frac{\pi^3 V_{\rm rms}^2 \ell^2}{2\hbar c} \simeq \frac{V_{\rm rms}^2}{(40\,{\rm mV})^2} \frac{\ell^2}{(100\,{\rm nm})^2} , \quad (30)$$

where we have written the result in terms of typical values that characterize the patch potentials. This shows that the reaction force due to the classical radiation could be comparable or even larger than the one in the DCE.

IV. CONCLUSIONS

We have shown that a real, accelerated metallic surface produces classical EM radiation due to the unavoidable presence of patch potentials. The calculation of the total emission spectrum for a flat surface undergoing bounded motion is a rather straightforward exercise in classical electrodynamics, and the result depends only on the most relevant physical quantity characterizing the patch potentials: their autocorrelation function.

Remarkably, when the correlation length of the patch potentials is sufficiently small, the emitted radiation coincides with that of a single moving dipole and has the same frequency dependence than the radiation induced by the motion of a perfect conductor through the quantum vacuum. Although these facts could have been anticipated by dimensional analysis, the explicit calculations in this paper allowed us to compare the classical radiation of the moving patches with the quantum radiation associated to the DCE. Depending on the characteristics of the patches, the dissipative effects associated to the classical radiation could be comparable with those coming from the quantum vacuum for perfect conductors.

We have considered the simplest situation, corresponding to a single accelerated mirror. It is well known that the dynamical Casimir effect for this configuration is far from being optimal regarding the possibility of its experimental detection. It would be interesting to assess whether classical radiation from patch potential masks the dynamical Casimir effect or not in more realistic experimental settings, such as a closed cavity with variable length, in a regime of parametric resonance [6].

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