Testing the a_μ anomaly in the electron sector through a precise measurement of h/M

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The persistent $a_\mu \equiv (g-2)/2$ anomaly in the muon sector could be due to new physics visible in the electron sector through a sub-ppb (parts per 10^9) measurement of the anomalous magnetic moment of the electron a_e. Driven by recent results on the electron mass [S. Sturm *et al.*, Nature **506**, 467 (2014)], we reconsider the sources of uncertainties that limit our knowledge of a_e including current advances in atom interferometry. We demonstrate that it is possible to attain the level of precision needed to test a_μ in the naive scaling hypothesis on a time scale similar to next-generation $g-2$ muon experiments at Fermilab and JPARC. In order to achieve this level of precision, knowledge of the quotient h/M, i.e., the ratio between the Planck constant and the mass of the atom employed in the interferometer, will play a crucial role. We identify the most favorable isotopes to achieve an overall relative precision below 10^{-10}.

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I. INTRODUCTION

In the last 40 years, the experimental accuracy of the anomalous magnetic moment of the muon $a_\mu = (g-2)_\mu/2$ has been improved by more than five orders of magnitude [1]. The final results of the Fermilab E821 experiment [2] shows a clear discrepancy with respect to the Standard Model (SM) prediction, corresponding to an $\sim 3.5\sigma$ deviation. This puzzling outcome has boosted a vigorous experimental program, and new results from the E989 Fermilab [3] and g-2 JPARC [4] experiments are expected in a few years. If the origin of the muon discrepancy is due to physics beyond the SM, similar effects are expected in the electron sector too. In particular, corrections due to new physics [(NP); i.e., physics beyond the SM] should appear in the electron magnetic moment $a_e = (g-2)_e/2$. In general, these corrections will be suppressed by an $O((m_e/m_\mu)^2)$ factor with respect to muons ["naive scaling" (NS)], m_e and m_μ being the mass of the electron and muon, respectively.

In fact, progress in the measurement and theoretical understanding of a_e is so impressive that a_e could be implemented as an observable to test the NP in the electroweak sector of the SM [5]. Driven by recent results on the electron mass [6], in this paper we reconsider the sources of uncertainties that limit our knowledge of a_e including current advances in atom interferometry. We demonstrate that it is possible to attain the level of precision needed to test a_μ in the NS hypothesis on a time scale similar to next-generation $g-2$ muon experiments and we identify the best experimental strategy to reach this goal.

In particular, in Sec. II we discuss the electron counterpart of NP effects that can generate the muon discrepancy and we set the scale for the experimental precision that has to be attained. The observables that must be determined with a high precision are discussed in Sec. III: they are $a_e^{\rm expt}$ (Sec. III A), the fine-structure constant α (Sec. III B), and four ancillary observables—the Rydberg constant R_∞ (Sec. III B), the electron mass in atomic mass units (Sec. III C), the mass of the atom employed in the atomic interferometer (Sec. III D), and the ratio between the Planck constant and the atom mass (h/M; Sec. III E). For each of these observables we determine the best current accuracy and the improvements that are needed to reach the goal sensitivity. The sensitivity to the NP of a_e and the comparison with NP effects in the muon sector are discussed in Sec. IV.

II. THE a_μ ANOMALY AND ITS ELECTRON COUNTERPART

Precise measurement of the anomalous magnetic moment of the electron $a_e = (g-2)_e/2$ is one of the most brilliant tests of QED and a key metrological observable in fundamental physics. In the last 20 years the relative experimental precision of a_e has reached sub-ppb (parts per 10^9) (see Sec. III A). Progress in theoretical predictions matches the improved precision of the measurements, and given an independent determination of the fine-structure constant α, a_e provides a clean test of perturbative QED at the five-loop level. Conversely, if we assume QED to be valid, a_e offers the most precise measurement of α available to date and drives the overall CODATA fits both for α and for several correlated quantities such as the molar Planck constant [7]. In high-energy-physics, a_e plays a role both as a constraint for $\alpha(q^2 \to 0)$ [8] and for determination of the QED contributions to the muon anomaly a_μ. In fact, progress in the measurement and theoretical understanding of a_e are so impressive that a_e could be implemented as an observable to test the NP in the electroweak sector of the SM [5]. Up to now, this role has pertained solely to a_μ since NP effects in a_μ and a_e (loop effects due to an NP scale Λ_μ and Λ_e) decouple as $(m_\mu/\Lambda_\mu)^2$ and $(m_e/\Lambda_e)^2$, respectively. The case where $\Lambda_\mu \equiv \Lambda_e$ is referred to as (NS), and when NS is at work, we thus expect a_μ to be $(m_\mu/m_e)^2$ more sensitive to NP than its electron counterpart. On the other hand, a_e is currently measured $\sim$2300 times more accurately than a_μ and further improvements are expected in the years to come. These considerations have led the authors of Ref. [5] to evaluate the physics potential of a_e as a probe of NP both in the NS approximation and in specific models where NS is violated.

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The main motivation to promote a_e to a probe for NP is the above-mentioned discrepancy in the measurement of a_μ. The final results of the Fermilab E821 experiment sets the scale of the a_μ discrepancy. It amounts to a shift with respect to the SM prediction of [2]

$$\Delta a_\mu = a_\mu^{\rm expt} - a_\mu^{\rm SM} = 2.90\,(90) \times 10^{-9}, \tag{1}$$

corresponding to a 3.5σ discrepancy. If the discrepancy is due to NP, we can always parametrize such NP effects as

$$|\Delta a_\mu| = \frac{m_\mu^2}{\Lambda_\mu^2}, \tag{2}$$

where the NP scale Λ_μ encodes possible loop factors, loop functions, and couplings of new particles to the muon. As a result, the central value of Eq. (1) can be accommodated for $\Lambda_\mu \approx 2$ TeV. Defining the NP effects for the electron $g-2$ analogously to Eq. (2), it turns out that

$$\left|\frac{\Delta a_e}{\Delta a_\mu}\right| = \frac{m_e^2}{m_\mu^2}\frac{\Lambda_\mu^2}{\Lambda_e^2}. \tag{3}$$

Assuming NS, the a_μ discrepancy could be tested in the electron sector once the experiments reach a precision of

$$\sigma_{a_e} = 2.9 \times 10^{-9} \times \left(\frac{m_e}{m_\mu}\right)^2 = 6.8 \times 10^{-14}\ (0.06\ \text{ppb}). \tag{4}$$

However, in concrete examples of NP theories, NS could be violated and larger effects in a_e might be expected. For instance, in supersymmetric theories [5,9] with nondegenerate slepton masses $m_{\tilde{e}} \neq m_{\tilde{\mu}}$, we can identify $\Lambda_\mu \equiv m_{\tilde{\mu}}$ and $\Lambda_e \equiv m_{\tilde{e}}$, and Δa_e can even saturate the current experimental bound $\Delta a_e \approx 10^{-12}$.

In spite of the progress in the experimental determination of a_e, it is generally believed that NP effects that could be responsible for the a_μ anomaly will be observed in the electron sector only with the occurrence of a strong violation of NS ($\Lambda_\mu \gg \Lambda_e$). This is due to the fact that a_e is deeply entangled with most of the fundamental constants in physics and a direct observation of NP in the lepton sector requires an independent determination of such constants. As discussed in the following, the latest advances in metrology remove most of these obstacles and make it possible to attain a level of precision close to the target of Eq. (4).

III. EXPERIMENTAL OBSERVABLES

A. The experimental determination of a_e

In the last 20 years the experimental precision achieved on a_e by cylindrical Penning traps has improved by more than one order of magnitude the precision of hyperbolic traps, and the opportunities offered by these techniques have not been fully exploited yet [10].

The best world measurement of a_e, i.e., the 2010 measurement with a cylindrical Penning trap [11], achieved a relative accuracy of 0.24 ppb. This uncertainty is four times larger than the precision needed to observe NS effects in the electron sector. Still, cylindrical Penning traps have not saturated their systematics and major improvements can be envisaged. The 2006 measurement [12] was mostly dominated by cavity shift modeling. In cylindrical Penning traps the interaction of the trapped electron and the cavity modes shifts the cyclotron frequency and the shift must be properly modeled to extract a_e. This drawback is unavoidable if spontaneous emission of radiation has to be inhibited. Note, however, that the current measurement of a_e [13] is not dominated by the cavity shift yet; systematics arise from an anomalous broadening of the spectroscopy line shapes (probably due to fluctuations in the magnetic field [11]) and due to statistics. In particular, line-shape modeling accounts for most of the systematic budget of a_e. A breakdown of the systematics of the 2008 analysis is available in Table 6.6 of Ref. [14], where the (run-to-run correlated) line-shape model uncertainty accounts for a relative uncertainty of 0.21 ppb in a_e. The overall uncertainty in a_e reported in Ref. [11] is 0.24 ppb [0.28 parts per 10^{12} (ppt) in $g/2$].

Most likely, experiments based on cylindrical Penning traps will ultimately be limited by cavity shift uncertainties, which are a source of systematics intrinsic to this technology. Results from Refs. [11,14] indicate that this contribution to the relative uncertainty in a_e can be reduced below 0.08 ppb (<0.1 ppt for $g/2$).

B. The fine-structure constant and the link to h/M

The measurement of $a_e^{\rm expt}$ from Ref. [11] would already be able to constrain specific models that break NS and enhance NP contributions in a_e with respect to a_μ. However, the measurement becomes quite marginal once we diagonalize the correlation matrix between $a_e^{\rm expt}$ (which is commonly used to extract α) and its theory expectation $a_e^{\rm SM}$. This is due to the fact that $a_e^{\rm SM}$ is $\alpha/2\pi$ at leading order, and hence, it is highly dependent on α. If we resort to a fully independent, albeit less precise, determination of α, the accuracy in the theory prediction for a_e ($a_e^{\rm SM}$) is worsened to 0.66 ppb [5].

The possibility of having a ppb measurement of α independent of a_e became viable with the measurement of the narrow (1.3-Hz) 1*S*-2*S* two-photon resonant line of the hydrogen atom with a relative precision of 3.4×10^{-13} [15] and with the precise measurement of the h/M ratio by atom interferometry [16,17]. The new data on hydrogen spectroscopy resulted in a measurement of the Rydberg constant with a precision better than 0.01 ppb (7×10^{-12} [7]). Since $R_\infty = m_e\alpha^2 c/2h$, the outstanding precision in R_∞ links α to the evaluation of the quotient h/m_e. In fact, for a given atom X whose mass is M_X,

$$\alpha^2 = \frac{2R_\infty}{c}\frac{M_X}{m_e}\frac{h}{M_X} = 2\frac{R_\infty}{c}\frac{M_X}{m_u}\frac{m_u}{m_e}\frac{h}{M_X}, \tag{5}$$

m_u being the atomic mass units, i.e., the mass of 1/12 the mass of ^{12}C. Equation (5) paved the way for an independent determination of α based on cold-atom interferometry (see Sec. III E) since atom interferometers are well suited for measurement of the quotient h/M_X. This quotient is also of great metrological interest for the redefinition of the kilogram [18]. On the other hand, exploitation of the Rydberg relationship to extract α causes two additional sources of uncertainties: the systematics due to the knowledge of the mass (M_X/m_u) of the atom employed to measure h/M_X and the uncertainty in m_e in atomic mass units (m_e/m_u).

C. The electron mass

The most straightforward way to employ Eq. (5) with minimum penalty from the knowledge of masses would be to design an ancillary experiment aimed at determining the mass ratio between the electron and the isotope employed in the atomic interferometer. The expected uncertainty for such a dedicated experiment would be of the order of the *direct* measurement of $A_r(m_e) \equiv m_e/m_u$ [7]. In fact, while the most precise evaluation of $A_r(m_e)$ can be obtained from bound-state electrons [6] (see below), the best direct measurement of $A_r(m_e)$ remains the one of the Farnham *et al.* experiment [19]. Here, the cyclotron frequencies of an electron and a $^{12}C^{6+}$ ion were compared in a Penning trap. The measurement determined $A_r(m_e)$ with just a 2.1-ppb relative accuracy, and at present, this technique does not seem appropriate for reaching the target of Eq. (4).

Since the 2002 CODATA adjustment, $A_r(m_e)$ is determined from the measurement of a_e in bound-state QED. For a bound electron in hydrogen-like systems of nuclear charge Z, the electron anomalous magnetic moment g_b is perturbed with respect to the free-particle value. The leading-order (pure Dirac) contribution is $g = 2$ for a free electron and $g_b^{\rm Breit} = 2/3(1 + 2\sqrt{1 - Z^2\alpha^2})$ [20] for a bound electron in a field generated by an atom with nuclear charge Z. In the past, the best measurements of g_b for C^{5+} and O^{7+} ions have been performed at the GSI using a Penning trap to measure the ratio between the Larmor ω_L and the cyclotron ω_c frequencies of the stored ions [21]. The relationship between g_b and these frequencies,

$$g_b = 2\frac{\omega_L}{\omega_c}\frac{m_e}{M_{C^{5+}/O^{7+}}} = 2\frac{\omega_L}{\omega_c}A_r(m_e)\frac{m_u}{M_{C^{5+}/O^{7+}}}, \qquad (6)$$

links g_b with the electron mass. Very recently [6], Sturm *et al.* have improved by a factor of 13 the CODATA value by performing a measurement of the frequency ratio for the hydrogen-like atom $^{12}C^{5+}$. The corresponding estimate for m_e relies on bound-QED calculations [22], which can be checked by ancillary g_b measurements on $^{28}Si^{13+}$ [23,24].

The GSI evaluation of m_e in atomic mass units is

$$\frac{m_e}{m_u} = 0.000\,548\,579\,909\,067(14)(9)(2), \qquad (7)$$

where the errors in parentheses are the statistical error, the experimental systematic uncertainty, and the theoretical (bound-QED) error, respectively. This measurement provides m_e/m_u with a relative precision of 0.03 ppb, which scales to an uncertainty in α of 0.015 ppb. This error is well below the systematic budget for observation of the NS muon anomaly in the electron sector.

D. M/m_u

The only remaining source of uncertainty due to the Rydberg relationship, (5), is the knowledge of the M_X/m_u ratio. It is therefore interesting to estimate current uncertainties in M_X/m_u for viable cold-atom candidates and evaluate whether the M_X/m_u error can be improved with an appropriate choice of the isotope X. As concerns alkali-metal atoms, the atomic masses of the isotopes relevant for the determination of α have been measured with a high precision employing orthogonally compensated Penning traps. Recent results [25,26] determine the atomic masses of ^{23}Na, ^{39,41}K, ^{85,87}Rb, and ^{133}Cs with a precision of $\simeq$0.1 ppb. In particular, the error associated with the current best atom for the measurement of h/M (^{87}Rb) is 0.115 ppb [26]. This corresponds to a relative uncertainty in α of 0.06 ppb. Hence, performing experiments with isotopes different from ^{87}Rb but chosen among the standard alkali-metal candidates does not cause significant improvements in α. A notable exception is ^{4}He, which is a good candidate for atom interferometry (see Sec. III E) whose mass is known with outstanding accuracy (0.015 ppb [27]). The use of ^{4}He would allow exploitation of the Rydberg α–to–h/M link without any significant penalty since the overall relative error due to m_e and $M_{\rm He}$ impacts on α at the $\frac{1}{2}(3 \oplus 1.5) \times 10^{-11} = 0.017$ ppb level.

E. The h/M quotient

In order to test the muon anomaly in the electron sector, measurement of the quotient h/M_X remains the main source of uncertainty and a remarkable experimental challenge. The most precise value obtained so far by atom interferometers is for ^{87}Rb [28] and it corresponds to $h/M_{\rm Rb} = 4.591\,359\,272\,9(57) \times 10^{-9}$ m^2 s^{-1} (1.24 ppb). The error budget of [28] updated with the latest determination of m_e/m_u [6] and $M_{\rm Rb}/m_u$ [26] is thus

$$\frac{\sigma_\alpha}{\alpha} \simeq \frac{1}{2}\,[1.24 \oplus 0.03 \oplus 0.115]\ \text{ppb} = 0.62\ \text{ppb}, \qquad (8)$$

well above the scale needed to test the a_μ discrepancy in the NS framework. However, there is plenty of scope for improvement in the measurement of h/M_X and the potential of the experimental technique is still to be fully exploited.

The very first measurement of α employing atom interferometry was carried out in Stanford [29] using cesium atoms. The value of α was measured with a relative precision $\sigma_\alpha/\alpha = 7.4 \times 10^{-9}$, mainly limited by possible index-of-refraction effects in the cold-atom sample. More recently, in an experiment at Berkeley, Cs is used in an interferometer based on a Ramsey-Bordé scheme. At present, the achieved relative uncertainty for α is $\sim$2 ppb [30]. The error here is mostly statistical (1.7 ppb); the next largest error is a parasitic phase shift caused by the beam splitters in the simultaneous conjugate interferometers. Recently, using Bloch oscillations to increase the separation of the interferometers, the signal-to-noise ratio was improved by about one order of magnitude and the parasitic phase shift reduced, so that a precision below ppb should be within reach [31].

As already mentioned, the most accurate determination of h/M_X has been obtained with rubidium atoms. Several experiments were performed [28,32,33] in Paris with Rb using an atom interferometer based on a combination of a Ramsey-Bordé interferometer [34] with Bloch oscillations. The precision in [28] is mostly limited by laser-beam alignment, wave-front curvature, and Gouy phase effects. A new project is ongoing, which is aimed at improving the accuracy of $h/M_{\rm Rb}$ and therefore of α by increasing the sensitivity of the atom interferometer and reducing the systematic effect due to the Gouy phase and the wave-front curvature [35]. Key elements of the new experiment will be the use of evaporatively cooled atoms and an atom interferometer based on large-momentum beam splitters [18].

Future prospects are the use of other atoms such as helium and strontium. An experiment on He was started in Amsterdam [36]. It is based on metastable ^{4}He in a one-dimensional-lattice setup to perform Bloch oscillations and velocity measurement with an atom interferometer. Metastable ^{4}He has some advantages compared to Rb and other atoms. These relate to the low mass, the lower sensitivity to magnetic fields, and the availability of high-power infrared fiber lasers at the relevant wavelength of 1083 nm. The use of a metastable state enables an alternative detection on a microchannel plate detector but also causes Penning ionization losses at high densities. Therefore helium has the potential to become at least as accurate as Rb using the same method, and as noted above, the helium mass is known with a relative uncertainty of 0.015 ppb [25,27]. The potential of Sr for high-precision atom interferometry was demonstrated in experiments based on Bloch oscillations [37,38]. Because of the specific characteristics of this atom, experiments with Sr using atom interferometry schemes such as the ones already demonstrated for different atoms promise to reach a very high precision [39]. The mass ratio for Sr isotopes was measured in [40] with a relative precision of 0.11 ppb.

These experimental programs are aimed at a precision in the determination of h/M_X of <0.1 ppb. On a much longer time scale, a final precision of 10^{-11} could be achieved in a space experiment where microgravity would allow full exploitation of the potential sensitivity of atom interferometers [41].

IV. SENSITIVITY TO NEW PHYSICS IN THE ELECTRON SECTOR

The relevance of the contributions discussed in previous sections can be expressed in terms of constraints to the NP scale Λ_e, as depicted in Fig. 1. The (red) horizontal line indicates the best fit of Λ_μ from the muon anomaly: $\Lambda_\mu = \sqrt{m_\mu^2/2.90 \times 10^{-9}} \simeq 2$ TeV. The horizontal band is the corresponding 1σ uncertainty. The three vertical lines represent the constraints from the electron sector computed under different assumptions on the systematics. The total uncertainty in a_e (leftmost thick line) is computed using the 2010 measurement [11] and taking α from the best measurement of the h/M ratio ($h/M_{\rm Rb}$ [28]). Assuming the NS expectation from the muon anomaly as the central value [thin (red) vertical line in Fig. 1], this accuracy sets a limit of $\Lambda_e \gtrsim 0.6$ TeV (thick black line in Fig. 1). With the occurrence of NS, $\Lambda_e = \Lambda_\mu$ holds (diagonal line) and a deviation of a_e from its SM prediction is expected for $|\Delta a_e| \simeq \sigma_{a_e} \simeq 6.8 \times 10^{-14}$. The tighter constraints on Λ_e are computed removing the systematics from α (dashed vertical line) and envisaging a reduction in the experimental uncertainty in the cavity shift systematics for Penning traps (dotted vertical line).

If α can be disentangled from $a_e^{\rm expt}$ at the appropriate level of precision, perspectives to test the NP in the a_e sector are very encouraging. In fact, the uncertainty in the theoretical determination of $a_e^{\rm SM}$ is appropriate for testing the a_μ anomaly at the NS level. Such uncertainty mostly resides in the numerical approximation employed to evaluate four- and five-loop QED contributions (0.06 ppb) [42] and in the hadronic term (0.02 ppb) [43]. The overall amount is within the error budget for NS (0.06 ppb) and further improvements are within reach.

The experimental systematics budget is summarized in Fig. 2. As mentioned above, the technology of the cylindrical

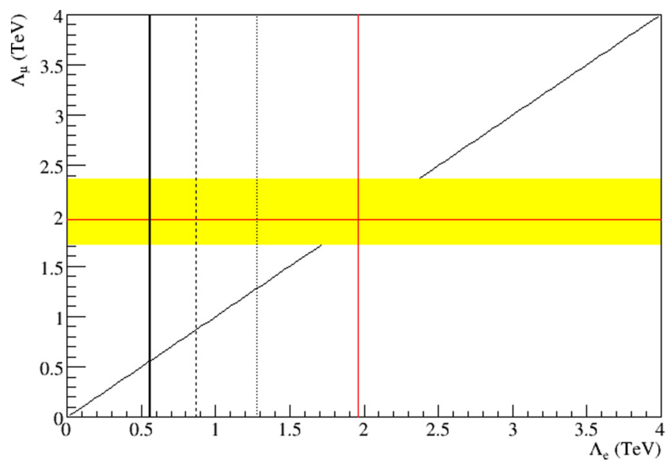

FIG. 1. (Color online) Λ_μ versus Λ_e (in TeV). The (red) horizontal line indicates the best fit of Λ_μ from the muon anomaly. The shaded horizontal band is the corresponding 1σ uncertainty. The area to the right of the thick vertical line shows the allowed values of Λ_e from the the current accuracy of a_e and assuming the NS expectation from the muon anomaly [thin (red) vertical line] as the central value. The diagonal line corresponds to the NS expectation $\Lambda_\mu = \Lambda_e$. The tighter constraints on Λ_e are computed removing the systematics from α (dashed line) and envisaging a reduction in the experimental uncertainty of the cavity shift systematics for Penning traps (dotted line).

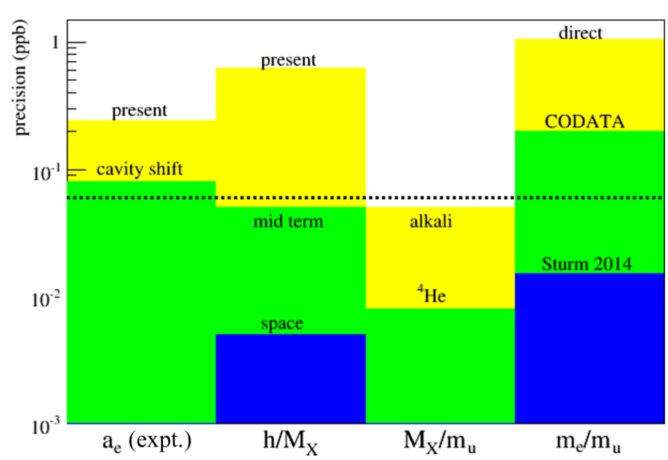

FIG. 2. (Color online) Summary of the contributions to the relative precision in a_e (in ppb). a_e (expt) is the experimental contribution from the measurement of a_e with Penning traps. h/M_X is the contribution from the quotient h/M measured with atom interferometers for an isotope X. The contribution due to the knowledge of the isotope mass in atomic mass units is labeled M_X/m_u. The uncertainty in $A_r(m_e) \equiv m_e/m_u$ is shown in the m_e/m_u column. Here, "direct" refers to the direct measurement from [19]; the CODATA 2010 value is labeled "CODATA" and the recent GSI measurement [6] is labeled "Sturm 2014." The horizontal line corresponds to the NS size of the expected anomaly (0.06 ppb).

Penning trap (first column in Fig. 2) can be pushed below the current cavity shift limit (0.08 ppb) to reach the NS precision range (0.06 ppb; horizontal line in Fig. 2). However, such a measurement can be effective only if an independent measurement of α is available with a precision of <0.1 ppb. The outstanding accuracy reached for the Rydberg constant allows us to obtain such a measurement from atom interferometers through precision measurement of the h/M quotient (second column). This experimental approach profits greatly from recent advances in the measurement of the electron mass, which was considered a possible limiting factor in the past (fourth column). Atom interferometers based on alkali atoms are able to reach the required accuracy, although they introduce an additional source of uncertainty due to the error in M_X/m_u (third column). This systematics is negligible in ^{4}He-based interferometers.

V. CONCLUSIONS

The long-standing anomaly of the muon $g-2$ could be due to systematics in previous measurements or could signal a departure from the SM caused by NP in loop contributions. Most likely (NS), such NP will manifest in the electron sector with a $(a_e - a_e^{\rm SM})/(a_\mu - a_\mu^{\rm SM}) \simeq (m_e/m_\mu)^2$ suppression due to the different lepton masses. A major experimental effort is ongoing to clarify this issue and we expect new data in the muon sector to be available in a few years. In this paper, we have shown that—on a similar time scale—the experimental measurement of a_e can provide key information since the precision that is attainable is comparable with NS expectations. From the experimental point of view, the most critical challenge is a sub-ppb determination of the h/M quotient. Atom interferometry can provide this measurement and, through the Rydberg relationship, measure the fine-structure constant independently of a_e. Recent advances in metrology and, in particular, the revised measurement of the electron mass have reduced the systematics due to the m_e/m_u ratio to a level appropriate for this goal. Among the atom candidates, ^{4}He is particularly appealing due to the outstanding accuracy (0.015 ppb) obtained for $M_{\rm He}/m_u$.

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[1] F. Jegerlehner and A. Nyffeler, Phys. Rep. **477**, 1 (2009).
[2] G. W. Bennett *et al.*, Phys. Rev. D **73**, 072003 (2006); Phys. Rev. Lett. **92**, 161802 (2004); **89**, 101804 (2002); **89**, 129903(E) (2002); H. N. Brown *et al.*, *ibid.* **86**, 2227 (2001).
[3] R. M. Carey, K. R. Lynch, J. P. Miller, B. L. Roberts, W. M. Morse, Y. K. Semertzides, V. P. Druzhinin, B. I. Khazin *et al.*, FERMILAB-PROPOSAL-0989, 2009, http://inspirehep.net/record/818866/files/fermilab-proposal-0989.PDF.
[4] M. Aoki *et al.*, KEK-JPARC Proposal P34 (2009). T. Mibe, presented at International Workshop on Neutrino Factories, Super Beams and Beta Beams (NUFACT2013), Beijing, China, August 19–24 (2013).
[5] G. F. Giudice, P. Paradisi, and M. Passera, J. High Energy Phys. 11 (2012) 113.
[6] S. Sturm *et al.*, Nature **506**, 467 (2014).
[7] P. J. Mohr, B. N. Taylor, and D. B. Newell (CODATA 2010), Rev. Mod. Phys. **84**, 1527 (2012).
[8] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[9] A. Aboubrahim, T. Ibrahim, and P. Nath, arXiv:1403.6448 [hep-ph].
[10] G. Gabrielse, in *Lepton Dipole Moments*, edited by B. Lee Roberts and W. J. Marciano (World Scientific, Singapore, 2009).
[11] D. Hanneke, S. Fogwell Hoogerheide and G. Gabrielse, Phys. Rev. A **83**, 052122 (2011).
[12] B. C. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse, Phys. Rev. Lett. **97**, 030801 (2006)**99**, 039902 (2007).
[13] D. Hanneke, S. Fogwell, and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008).
[14] D. A. Hanneke, Ph.D. thesis, Harvard University, 2007; available at: https://www3.amherst.edu/~dhanneke/.
[15] T. Udem, A. Huber, B. Gross, J. Reichert, M. Prevedelli, M. Weitz, and T. W. Hansch, Phys. Rev. Lett. **79**, 2646 (1997).
[16] A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, Rev. Mod. Phys. **81**, 1051 (2009).
[17] G. M. Tino and M. A. Kasevich (eds.), *Atom Interferometry, Proceedings of the International School of Physics "Enrico Fermi," Course CLXXXVIII, Varenna 2013* (Societá Italiana di Fisica and IOS Press, Amsterdam, 2014).
[18] R. Bouchendira, P. Cladé, S. Guellati-Khélifa, F. Nez, and F. Biraben, Ann. Physik **525**, 484 (2013).
[19] D. L. Farnham, R. S. Van Dyck, and P. B. Schwinberg, Phys. Rev. Lett. **75**, 3598 (1995).
[20] G. Breit, Nature **122**, 649 (1922).
[21] G. Werth *et al.*, Int. J. Mass Spectrom. **251**, 152 (2006).
[22] K. Pachucki, A. Czarnecki, U. D. Jentschura, and V. A. Yerokhin, Phys. Rev. A **72**, 022108 (2005); T. Beier, Phys. Rep. **339**, 79 (2000).
[23] S. Sturm, A. Wagner, B. Schabinger, J. Zatorski, Z. Harman, W. Quint, G. Werth, C. H. Keitel, and K. Blaum, Phys. Rev. Lett. **107**, 023002 (2011).
[24] S. Sturm, A. Wagner, M. Kretzschmar, W. Quint, G. Werth, and K. Blaum, Phys. Rev. A **87**, 030501(R) (2013).
[25] E. G. Myers, Int. J. Mass Spectrom. **349**, 107 (2013).
[26] B. J. Mount, M. Redshaw, and E. G. Myers, Phys. Rev. A **82**, 042513 (2010).
[27] R. S. Van Dyck, S. L. Zafonte, S. Van Liew, D. B. Pinegar, and P. B. Schwinberg, Phys. Rev. Lett. **92**, 220802 (2004); R. S. Van Dyck, D. B. Pinegar, S. Van Liew, and S. L. Zafonte, Int. J. Mass Spectrom. **251**, 231 (2006).

[28] R. Bouchendira, P. Cladé, S. Guellati-Khélifa, F. Nez, and F. Biraben, Phys. Rev. Lett. **106**, 080801 (2011).
[29] A. Wicht, J. M. Hensley, E. Sarajlic, and S. Chu, Phys. Scr. **102**, 82 (2002).
[30] S.-Y. Lan, P.-C. Kuan, B. Estey, D. English, J. M. Brown, M. A. Hohensee, and H. Muller, Science **339**, 554 (2013).
[31] H. Müller, private communication, University of California, Berkeley.
[32] P. Cladé, E. de Mirandes, M. Cadoret, S. Guellati-Khélifa, C. Schwob, F. Nez, L. Julien, and F. Biraben, Phys. Rev. Lett. **96**, 033001 (2006).
[33] M. Cadoret, E. de Mirandes, P. Cladé, S. Guellati-Khélifa, C. Schwob, F. Nez, L. Julien, and F. Biraben, Phys. Rev. Lett. **101**, 230801 (2008).
[34] C. J. Bordé, Phys. Lett. A **140**, 10 (1989).
[35] P. Cladé, private communication, LKB Paris.
[36] W. Vassen, private communication, VU University, Amsterdam.
[37] G. Ferrari, N. Poli, F. Sorrentino, and G. M. Tino, Phys. Rev. Lett. **97**, 060402 (2006).
[38] N. Poli, F.-Y. Wang, M. G. Tarallo, A. Alberti, M. Prevedelli, and G. M. Tino, Phys. Rev. Lett. **106**, 038501 (2011).
[39] G. M. Tino, in *Atom Interferometry, Proceedings of the International School of Physics "Enrico Fermi," Course CLXXXVIII, Varenna 2013*, edited by G. M. Tino and M. A. Kasevich (Societá Italiana di Fisica and IOS Press, Amsterdam, 2014).
[40] R. Rana, M. Höcker, and E. G. Myers, Phys. Rev. A **86**, 050502 (2012).
[41] G. M. Tino, F. Sorrentino *et al.*, Nucl. Phys. B (Proc. Suppl.) **243**, 203 (2013).
[42] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. **109**, 111807 (2012), and references therein.
[43] D. Nomura and T. Teubner, Nucl. Phys. B **867**, 236 (2013).