Remote tomography and entanglement swapping via von Neumann–Arthurs–Kelly interaction

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We propose an interaction-based method for remote tomography and entanglement swapping. Alice arranges a von Neumann-Arthurs-Kelly interaction between a system particle P and two apparatus particles A_1, A_2, and then transports the latter to Bob. Bob can reconstruct the unknown initial state of particle P not received by him by quadrature measurements on A_1, A_2. Further, if another particle P' in Alice's hands is EPR entangled with P, it will be EPR entangled with the distant pair A_1, A_2. This method will be contrasted with the usual teleportation protocols.

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I. INTRODUCTION

The idea of "quantum tracking" of a single system observable by an apparatus observable first occurred in the measurement theory of Von Neumann [1], and generalized to two canonically conjugate observables by Arthurs and Kelly, Jr. [2]. Suppose the initial state of the system-apparatus combine is factorized. If after interaction, the apparatus observable X has the same expectation value in the final state as the system observable A in the initial state, for an arbitrary initial state of the system, then X is said to track A. This nomenclature was probably used first by Arthurs and Goodman [3] who, as well as, Gudder *et al.* [3] proved the joint measurement uncertainty relation. The Arthurs-Kelly interaction can also enable exact measurements of some quantum correlations between position and momentum [4].

We shall be concerned here not with joint measurements but with the completely different ideas of "remote quantum tomography" and "entanglement swapping" for continuous variables. These are akin to "quantum teleportation" or the replication of an unknown quantum state of a particle at a distant location without physically transporting that particle. Teleportation, as first proposed by Bennett *et al.* [5] and generalized to continuous variables by Vaidman [6], usually involves four different technologies. (i) An EPR-pair E_1, E_2 is shared by observers A (Alice) and B (Bob) at distant locations. (ii) The system particle P with an unknown state is received by A who makes a Bell-state measurement on the joint state of that particle and E_1 and (iii) communicates the result via a classical channel to B, and (iv) B then makes a unitary transformation depending on the classical information on E_2 to replicate the unknown system state. Teleportation has been experimentally realized, e.g., by Bouwmeester *et al.* [7], and the methods and uses extensively reviewed, e.g., by Braunstein *et al.* [8]. In particular the density matrix of the system particle can be constructed by quadrature measurements on E_2 (remote tomography).

II. INTERACTION-BASED REMOTE TOMOGRAPHY AND TELEPORTATION OF EPR ENTANGLEMENT

We report here a method which replaces the above four technologies by a two-step process: (i) interaction between the system particle and two apparatus particles followed by (ii) quantum communication of the two apparatus particles. At Alice's location A, a system particle P with unknown state interacts via an Arthurs-Kelly interaction with two apparatus particles A_1, A_2 in a known state. When the particles are photons, the interaction can easily be generated (see, e.g., Stenholm [2]). The particles A_1, A_2 are then sent to a distant observer Bob (B). B makes quantum tomographic measurements on them (quadrature measurements in the case of photons) and reconstructs the exact initial density matrix of the system particle without ever having received that particle. Further, if another particle P' in Alice's hands is EPR entangled with P, it will be EPR entangled with the distant pair A_1, A_2. (See Fig. 1.) Practical implementation will require a quantum channel to send the two apparatus particles from location A to the distant location of B followed by tomographic measurements by B: for photons, a generalization of single photon *optical homodyne tomography* (see, e.g., [9], [10], and [11]) to two photons, which seems feasible and worthwhile.

From the "application point of view" why is it practically useful to transport the apparatus particles with the system state imprinted on it? Why can't Alice directly send the system particle to Bob? There can be several reasons. E.g., the system particle might be unstable; or in the case of a photon, it might have a frequency unsuitable for optical fiber transmission. The apparatus photons can be chosen to have a frequency in the telecom windows around 1300 or 1550 nm where optical fibers have very low absorption facilitating long distance transmission. The scheme we propose exploits the entanglement between the system photon and the apparatus photons generated by the three-particle Arthurs-Kelly interaction. Multiparticle interactions to generate entanglement have previously been exploited for quantum enhanced metrology [12]. We proceed now to put our method on a rigorous footing.

III. A SYMMETRY PROPERTY

We shall use the Arthurs-Kelly system-apparatus interaction Hamiltonian, which is invariant under a class of simultaneous transformations on the system and apparatus specified below,

$$H = K(\hat{q}\hat{p}_1 + \hat{p}\hat{p}_2) = K(\hat{q}_\theta \hat{p}_{1,\theta} + \hat{p}_\theta \hat{p}_{2,\theta}), \tag{1}$$

where K is a coupling constant, $\hat{q}, \hat{p}$ are position and momentum operators of the system, respectively, and $\hat{x}_1, \hat{x}_2$ are

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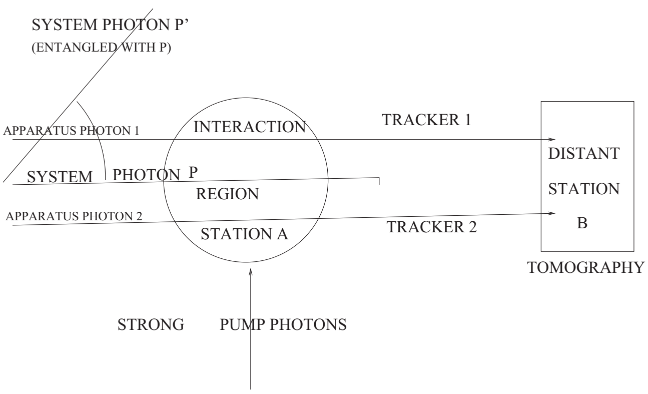

FIG. 1. Remote tomography and entanglement swapping via a Von Neumann-Arthurs-Kelly interaction between system photon P and tracker photons. If the photon P' is EPR entangled with P, the tracker photons become entangled with P'.

two commuting position operators of the apparatus (e.g., two photons), with conjugate momenta $\hat{p}_1,\hat{p}_2$ which are coupled to $\hat{q}$ and $\hat{p}$, respectively. The rotated quadrature operators with subscript θ are defined using the rotation matrix R,

$$\begin{pmatrix}\hat{q}_\theta\\ \hat{p}_\theta\end{pmatrix}=R\begin{pmatrix}\hat{q}\\ \hat{p}\end{pmatrix},\quad \begin{pmatrix}\hat{p}_{1,\theta}\\ \hat{p}_{2,\theta}\end{pmatrix}=R\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix},$$
$$R=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}. \tag{2}$$

The operators $\hat{p}_{j,\theta}$ are seen to be just the commuting momentum operators of the apparatus particles corresponding to rotated coordinates $x_{j,\theta}$, for $j=1,2$, $x_{1,\theta}+ix_{2,\theta}=\exp(-i\theta)(x_1+ix_2), \hat{p}_{j,\theta}=-i\partial/\partial x_{j,\theta}$. We also define, $\hat{x}_{1,\theta}+i\hat{x}_{2,\theta}=\exp(-i\theta)(\hat{x}_1+i\hat{x}_2)$. Then, in the case of the apparatus being two photons with annihilation operators $a_i, i=1,2$, $\hat{x}_{i,\theta}=a_i\exp(-i\theta)/\sqrt{2}+\text{H.c.}$, $\hat{p}_{i,\theta}=\hat{x}_{i,\theta+\pi/2}$.

IV. EXACT SOLUTION OF THE SCHRÖDINGER EQUATION WITH GENERALIZED INITIAL CONDITIONS

We assume the constant K to be so large that the free Hamiltonians of the system and the apparatus are negligible compared to H during interaction time T. We start from an initial factorized state,

$$\langle q|\langle x_1,x_2|\psi(t=0)\rangle\rangle=\langle q|\phi\rangle\chi(x_1,x_2), \tag{3}$$

where $\langle q|\phi\rangle$ is the unknown system wave function, and the apparatus wave function is chosen to be a product of two Gaussians, $\chi(x_1,x_2)=\chi_1(x_1)\chi_2(x_2)$,

$$\chi_1(x_1)=\pi^{-1/4}b_1^{-1/2}\exp\left[-x_1^2/\left(2b_1^2\right)\right],$$
$$\chi_2(x_2)=\pi^{-1/4}(2b_2)^{1/2}\exp\left[-2b_2^2x_2^2\right]. \tag{4}$$

Arthurs and Kelly chose $b_2=b_1=b$. We solve the Schrödinger equation with arbitrary b_1,b_2; we need $b_1\neq b_2$ to utilize the symmetry of the Hamiltonian.

The commutator of the two terms in H, in fact, commutes with each of the terms. Hence,

$$\exp(-iHt)=\exp(-iKt\hat{q}\hat{p}_1)\exp(-iKt\hat{p}\hat{p}_2)$$
$$\times\exp(iK^2t^2\hat{p}_1\hat{p}_2/2). \tag{5}$$

If we work in the q,x_1,p_2 representation, the three exponentials on the right-hand side successively translate x_1,q,x_1 acting on the initial wave function. Hence the exact solution of the Schrödinger equation is

$$\langle q,x_1,p_2|t\rangle=\chi_1[x_1-qKt+(1/2)p_2K^2t^2]$$
$$\times\tilde{\chi}_2(p_2)\phi(q-p_2Kt), \tag{6}$$

where $\tilde{\chi}_2$ denotes a Fourier transform of χ_2. The coordinate space wave function is given by a Fourier transform. Choosing $KT=1$ we obtain,

$$\psi(q,x_1,x_2)=\int\psi(q,x_1,x_2,\xi)d\xi, \tag{7}$$

where

$$\psi(q,x_1,x_2,\xi)=\phi(\xi)\exp\left[i(q-\xi)x_2\right]/(2\pi\sqrt{b_1b_2})$$
$$\times\exp\left(-\frac{(2x_1-q-\xi)^2}{8b_1^2}-\frac{(q-\xi)^2}{8b_2^2}\right). \tag{8}$$

Tracing the system-apparatus density matrix over the system coordinate we obtain the apparatus density matrix at time T,

$$\langle x_1,x_2|\rho_{APP}(T)|x_1'x_2'\rangle=\int\psi(q,x_1,x_2,\xi)$$
$$\times\psi^*(q,x_1',x_2',\xi')dqd\xi d\xi'\ . \tag{9}$$

The probability densities $P_1(x_1)$ and $P_2(x_2)$ for x_1 and x_2 are obtained by integrating the diagonal elements of this density operator over x_2 and x_1, respectively. In fact $P_1(x_1)$ and $P_2(x_2)$ can be obtained from the Arthurs-Kelly expressions by $b^2\to(b_1^2+b_2^2)/2$ and $b^{-2}\to(b_1^{-2}+b_2^{-2})/2$, respectively. The resulting expectation values of x_1,x_2 equal those of q,p, respectively, but the dispersions are higher, $(\Delta x_1)^2=(\Delta q)^2+(b_1^2+b_2^2)/2$, $(\Delta x_2)^2=(\Delta p)^2+(b_1^2+b_2^2)/(8b_1^2b_2^2)$.

Our key results require $b_1\neq b_2$. First, integrating the off-diagonal elements of the apparatus density matrix over x_2,x_2',

$$\int\langle x_1,x_2|\rho_{APP}(T)|x_1'x_2'\rangle dx_2dx_2'$$
$$=\frac{1}{b_1b_2}\int|\phi(q)|^2\exp\left(-\frac{(x_1-q)^2+(x_1'-q)^2}{2b_1^2}\right)dq\ . \tag{10}$$

This shows that we can extract the exact initial system position probability density from the final apparatus density matrix as the expectation value of an apparatus observable:

$$|\langle q=x_1|\phi\rangle|^2=\lim_{b_1\to0}\frac{b_2}{\sqrt{\pi}}\int dx_2dx_2'\langle x_1,x_2|\rho_{APP}(T)|x_1x_2'\rangle$$
$$=\lim_{b_1\to0}\text{Tr}\rho_{APP}(T)Y(x_1), \tag{11}$$

where $Y(x_1)$ is the apparatus observable,

$$Y(x_1)=\frac{b_2}{\sqrt{\pi}}|x_1\rangle\langle x_1|\int|x_2'\rangle\langle x_2''|dx_2'dx_2''$$
$$=2b_2\sqrt{\pi}(|x_1\rangle\langle x_1|)(|\hat{p}_2=0\rangle\langle\hat{p}_2=0|). \tag{12}$$

Similarly, the exact initial system momentum probability density is an expectation value of an apparatus observable in the final apparatus density matrix,

$$|\langle p = x_2|\phi\rangle|^2 = \lim_{b_2\to\infty} \frac{1}{2b_1\sqrt{\pi}} \int dx_1 dx_1' \times \langle x_1,x_2|\rho_{APP}(T)|x_1' x_2\rangle = \lim_{b_2\to\infty} \mathrm{Tr}\rho_{APP}(T)Z(x_2), \quad (13)$$

where $Z(x_2)$ is the apparatus observable, $Z(x_2) = (\sqrt{\pi}/b_1)(|x_2\rangle\langle x_2|)(|\hat{p}_1 = 0\rangle\langle\hat{p}_1 = 0|)$. In the limit, $b_1 \to 0$, $b_2 \to \infty$, we have faithful tracking of both system position and system momentum, since $Y(x_1)$ tracks the position projectors $|\hat{q} = x_1\rangle\langle\hat{q} = x_1|$ for all x_1 and $Z(x_2)$ tracks the system momentum projectors $|\hat{p} = x_2\rangle\langle\hat{p} = x_2|$ for all x_2.

Further, the Wigner function of the initial system state can be calculated exactly from the final apparatus density matrix,

$$W(x_1,x_2) = \lim_{b_1\to 0, b_2\to\infty} \frac{b_2}{2\pi b_1} \times \int dx_1' dx_2' \langle x_1,x_2|\rho_{APP}(T)|x_1' x_2'\rangle. \quad (14)$$

We now show that we can indeed measure a continuous infinity of apparatus observables on the final state to obtain the initial Wigner function of the system particle.

V. ROTATED QUADRATURES AND QUANTUM TOMOGRAPHY

In order to harness the symmetry property mentioned above, we need a corresponding symmetry property of the initial apparatus state, $\chi(x_1,x_2) = \chi(x_{1,\theta},x_{2,\theta})$. Therefore we are forced to use initial apparatus states very different from Arthurs and Kelly. We need

$$2b_1b_2 = 1;\ \chi(x_1,x_2) = \chi(x_{1,\theta},x_{2,\theta}) = \pi^{-1/2} b_1^{-1} \exp\left[-\left(x_1^2 + x_2^2\right)/\left(2b_1^2\right)\right]. \quad (15)$$

For this choice, the system-apparatus initial state can be rewritten for arbitrary θ as

$$\langle\hat{q}_\theta = q_\theta|\langle\hat{x}_{1,\theta} = x_{1,\theta},\hat{x}_{2,\theta} = x_{2,\theta}|\psi(t=0)\rangle = \langle\hat{q}_\theta = q_\theta|\,\phi\rangle\chi(x_{1,\theta},x_{2,\theta}), \quad (16)$$

with the obvious notation $(\hat{q}_\theta - q_\theta)|\hat{q}_\theta = q_\theta\rangle = 0$. Since the Hamiltonian H and the initial apparatus states have exactly the same form in terms of the rotated variables as in terms of the original variables, we can repeat the previous calculations with $\hat{q}_\theta,\hat{p}_\theta,q_\theta,p_\theta,x_{1,\theta},x_{2,\theta}$ replacing $\hat{q},\hat{p},q,p,x_1,x_2$, respectively. Hence the matrix elements of $\rho_{APP.}$ are obtained by replacing in the previously obtained expressions

$$q,p,x_1,x_2,x_1',x_2' \to q_\theta,p_\theta,x_{1,\theta},x_{2,\theta},x_{1,\theta}',x_{2,\theta}'.$$

Thus, we obtain for arbitrary θ,

$$|\langle\hat{q}_\theta = u|\phi\rangle|^2 = \lim_{b_1\to 0} \mathrm{Tr}\rho_{APP}(T)Y_\theta(u), \quad (17)$$

$$Y_\theta(u) \equiv \frac{\sqrt{\pi}}{b_1}|\hat{x}_{1,\theta} = u\rangle\langle\hat{x}_{1,\theta} = u||\hat{p}_{2,\theta} = 0\rangle\langle\hat{p}_{2,\theta} = 0|. \quad (18)$$

Since $\hat{p}_\theta = \hat{q}_{\theta+\pi/2}$, the initial system probability densities for it are obtained from above just by replacing $\theta \to \theta + \pi/2$.

Conceptually, $b_1 \to 0, b_2 = 1/(2b_1) \to \infty$ is the limit of the initial apparatus state having arbitrarily narrow spread in x_1 and x_2. We have proved that in this limit we can recover exactly the initial system probability densities of arbitrary Hermitian linear combinations $\hat{q}_\theta$,

$$\langle\hat{q}_\theta = u|\,\rho_S|\hat{q}_\theta = u\rangle = |\langle\hat{q}_\theta = u|\phi\rangle|^2 \quad (19)$$

and hence the initial Wigner function, by measuring expectation values of Hermitian operators in the same final state of the apparatus after interaction.

VI. RECONSTRUCTION OF THE INITIAL DENSITY MATRIX OF THE SYSTEM FROM THE FINAL APPARATUS DENSITY MATRIX

Quantum tomography is completed by calculating the Wigner function $W(q,p)$ as an inverse Radon transform,

$$W(q,p) = (2\pi)^{-2} \int_0^\infty \eta d\eta \int_0^{2\pi} d\theta \int_{-\infty}^\infty du \times \exp\{i\eta[u - (q\cos\theta + p\sin\theta)]\} \times \langle\hat{q}_\theta = u|\,\rho_S|\hat{q}_\theta = u\rangle, \quad (20)$$

and from that the density operator,

$$\langle q|\rho_S|q'\rangle = (2\pi)^{-1} \int_0^\pi |q - q'|d\theta(\sin\theta)^{-2} \times \exp\{[-i(q^2 - q'^2)\cot\theta]/2\} \int_{-\infty}^\infty du \times \exp[iu(q - q')/\sin\theta]\langle\hat{q}_\theta = u|\,\rho_S|\hat{q}_\theta = u\rangle. \quad (21)$$

VII. ACCOUNTING FOR TIME EVOLUTION OF THE APPARATUS PHOTONS DURING TRANSIT TIME τ TO DISTANT LOCATION B

Note that

$$\mathrm{Tr}\rho_{APP}(T)Y_\theta(u) = \mathrm{Tr}\rho_{APP}(T+\tau) \times \exp(-iH_0\tau)Y_\theta(u)\exp(iH_0\tau), \quad (22)$$

where the Hamiltonian $H_0 = \omega(a_1\dagger a_1 + a_2\dagger a_2 + 1)$, if the photons have the same frequency ω. Hence the $\langle\hat{q}_\theta = u|\,\rho_S|\hat{q}_\theta = u\rangle$ are equivalently given by replacing

$$\rho_{APP}(T),\hat{x}_{1,\theta},\hat{p}_{2,\theta} \ \to\ \rho_{APP}(T+\tau),$$
$$\cos(\omega\tau)\hat{x}_{1,\theta} - \sin(\omega\tau)\hat{p}_{1,\theta},\cos(\omega\tau)\hat{p}_{2,\theta} + \sin(\omega\tau)\hat{x}_{2,\theta},$$

respectively. We just have to measure different quadratures for the apparatus photons depending on the transit time τ.

VIII. QUANTITATIVE COMPARISONS FOR THE THIRD EXCITED STATE OF THE OSCILLATOR

We estimate here how small b_1 has to be for reasonably accurate reconstruction of the initial state which, in this example, is chosen to be the highly nonclassical third excited state of the oscillator. The Wigner function is

$$W(q,p) = [4(q^2+p^2)^3 - 18(q^2+p^2)^2 + 18(q^2+p^2) - 3] \times \frac{\exp -(q^2+p^2)}{3\pi}. \quad (23)$$

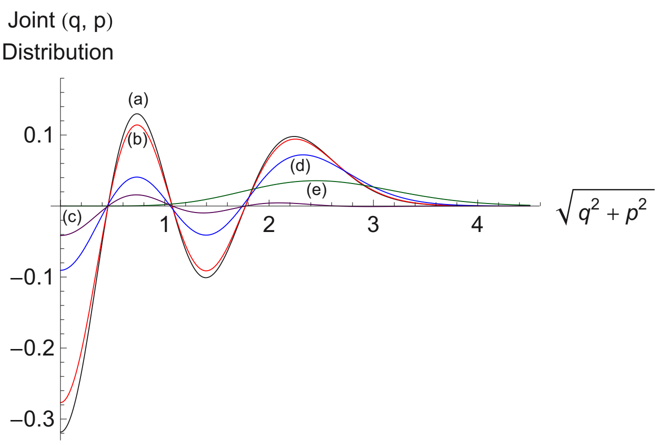

FIG. 2. (Color online) Joint distributions in (q,p) for the third excited state of the oscillator as a function of $\sqrt{q^2+p^2}$. (a) Wigner function. (b) Reconstructed Wigner function with $b_1 = 0.1$. (c) Difference between curves (a) and (b). (d) Reconstructed Wigner function with $b_1 = 0.3$. (e) Arthurs-Kelly probability distribution.

It is a function of $q^2 + p^2 \equiv d$ only, and hence we may write

$$W(d) = \exp(-d)[4d^3 - 18d^2 + 18d - 3]/(3\pi). \quad (24)$$

The reconstructed Wigner function in this case is

$$W_{b_1}(d) = \frac{(2b_1^2+1)^{-7}}{3\pi}\exp\left(-\frac{d}{(2b_1^2+1)}\right) \times \left[4d^3 + 18d^2(4b_1^4-1) + 18d(4b_1^4-1)^2 + 3(4b_1^4-1)^3\right], \quad (25)$$

and the Arthurs-Kelly probability distribution, with $b_1 = b_2 = \frac{1}{\sqrt{2}}$, is

$$P_{AK}(d) = \frac{d^3}{96\pi}\exp\left(-\frac{d}{2}\right). \quad (26)$$

In Fig. 2 we make quantitative comparisons between the Wigner function, our reconstructed Wigner function with $2b_1b_2 = 1$ (for $b_1 = \{0.1, 0.3\}$), and the Arthurs-Kelly probability distribution. In Fig. 3 we compare the position probabilities derived from the reconstructed Wigner function with the true quantum probability density and with that obtained from the Arthurs-Kelly probability distribution. A well-known measure of the distance between two probability distributions is given by the Kolmogorov-Smirnov distance, $D(K\text{-}S) = \max_x |F_1(x) - F_2(x)|$, where $F_i(x)$ is the cumulative probability for the variable $X \leqslant x$ for the ith probability distribution. This distance between the pseudoprobabilities given by the Wigner function and the reconstructed Wigner function, as well as for the corresponding position probabilities derived from them, are plotted in Fig. 4. The distance (especially for the position probability) is very small even up to $b_1 = 0.2$ though the theorem of exact equality is only in the limit $b_1 \to 0$.

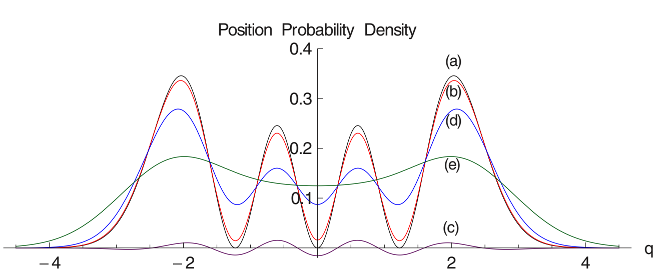

FIG. 3. (Color online) Position probability densities for the third excited state. (a) Quantum probability density of the state. (b) Obtained from reconstructed Wigner function with $b_1 = 0.1$. (c) Difference between curves (a) and (b). (d) Obtained from reconstructed Wigner function with $b_1 = 0.3$. (e) Obtained from Arthurs-Kelly probability distribution.

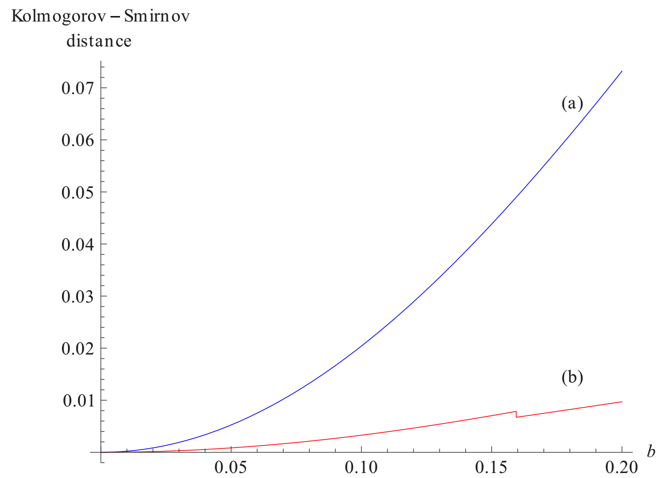

FIG. 4. (Color online) Plots for the Kolmogorov-Smirnov (K-S) distance between (a) the Wigner function and the reconstructed Wigner function and (b) the position probability density and the reconstructed density versus b_1. Even when b_1 is as large as 0.2, the K-S distance in case (a) reaches a value of only 0.072. The agreement is even better in case (b) (the small discontinuity in the K-S distance at $b_1 = 0.16$ is due to the shifting of the position where the maximum K-S distance is reached).

IX. TELEPORTATION OF ENTANGLEMENT

If the photon P with coordinate q is EPR entangled with another photon P' with coordinate q' with initial wave function $\phi(q,q')$, the density matrix for particles $1,2,P'$ after interaction can be shown to obey [13] analogues of Eqs. (11), (12) with $\langle q = x_1|\phi\rangle$ replaced by $\langle q = x_1, q'|\phi\rangle$, and $Y(x_1)$ replaced by $Y(x_1)|q'\rangle\langle q'|$; i.e.,

$$|\langle q = x_1, q'|\phi\rangle|^2 = \lim_{b_1\to 0}\frac{b_2}{\sqrt{\pi}}\int dx_2 dx_2' \times \langle x_1, x_2|\rho_{APP}(T)|x_1 x_2'\rangle = \lim_{b_1\to 0}\mathrm{Tr}\rho_{APP}(T)Y(x_1)|q'\rangle\langle q'|, \quad (27)$$

where

$$Y(x_1) = 2b_2\sqrt{\pi}(|x_1\rangle\langle x_1|)(|\hat{p}_2 = 0\rangle\langle\hat{p}_2 = 0|). \quad (28)$$

Thus the apparatus photons after interaction with P become entangled with P' achieving interaction-based teleportation of EPR entanglement. The exact initial probability densities for

q, q' (and similarly for p, p'), i.e., the exact EPR correlations, can be retrieved from this final entangled state.

X. CONCLUSIONS AND OUTLOOK

(i) We have shown that the Arthurs-Kelly interaction between an unknown state of a photon P and chosen initial state of two apparatus photons, followed by quantum communication of the two apparatus photons, enables a two-step remote tomographic reconstruction of the unknown initial state of P, as well as teleportation of its entanglement with another photon P', instead of the usual four step process. It is practically feasible because apparatus photon frequencies can be chosen in the telecom windows, and the technology of generating the Arthurs-Kelly interaction quantum optically is well established (see, e.g., Stenholm in [2]).

(ii) Remote tomography requires the measurement of the two photon observable $Y_\theta(u)$ which is just a product of two commuting quadrature operators for the apparatus photons, each of the kind usually measured for a single photon. This generalization of optical homodyning to the two teleported photons will by itself be a stimulating development.

(iii) The Arthurs-Goodman result on impossibility of simultaneous accurate tracking of position and momentum by commuting observables of the apparatus is not violated. The secret is that the apparatus observables tracking position and momentum do not commute, $[Y(x_1), Z(x_2)] \neq 0$. This is not a problem since we are interested in tomography, not in the simultaneous measurement of position and momentum.

(iv) The final density operator of the system can also be exactly calculated, and it can be seen that $\langle q \rangle_T = \langle q \rangle_0$, $\Delta q_T^2 = \Delta q_0^2 + 2b_2^2$; since the final system state is different from the initial state, and depends on the initial states of both the system and the apparatus, the no-cloning [14] and no-hiding theorems [15] are respected.

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