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Quantum friction and fluctuation theorems

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We use general concepts of statistical mechanics to compute the quantum frictional force on an atom moving at constant velocity above a planar surface. We derive the zero-temperature frictional force using a nonequilibrium fluctuation-dissipation relation, and we show that in the large-time, steady-state regime, quantum friction scales as the cubic power of the atom's velocity. We also discuss how approaches based on Wigner-Weisskopf and quantum regression approximations fail to predict the correct steady-state zero-temperature frictional force, mainly due to the low-frequency nature of quantum friction.

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A remarkable example of fluctuation-induced interactions is quantum friction, the drag force experienced between two bodies in relative motion in vacuum, associated with the energy and momentum transfer from one body to the other mediated by the quantum electromagnetic field. Radiation-mediated friction is deeply rooted in the foundations of quantum mechanics, and it was already discussed by Einstein in his seminal 1917 paper on the blackbody spectrum [1]. Quantum friction has recently attracted attention in the context of macroscopic bodies and atoms in linear [2] or rotational [3] motion above a surface, Coulomb drag in electron transport phenomena [4], and as the dissipative counterpart of the dynamical Casimir effect [5]. Several authors [6–14] have obtained quite diverse results for the atom-surface drag at zero temperature, making different predictions as to its dependence on the velocity of the atom and the atom-surface separation. Here we revisit the problem of quantum friction using general concepts of quantum statistical mechanics. We derive a quantum nonequilibrium fluctuation-dissipation theorem (FDT) for an atom in steady-state motion above a surface and compare its predictions with the quantum regression theorem (QRT).

We first consider the prototype problem of a static atom above a planar material surface at zero temperature. The atom is described by an electric dipole operator $\hat{\mathbf{d}}$ located at position $\mathbf{r}_a$. In a simple two-state system model (ground state $|g\rangle$ and excited state $|e\rangle$), the atomic electric dipole operator is given by $\hat{\mathbf{d}} = \mathbf{d}\hat{\sigma}_1$, where $\mathbf{d}$ is the (real) dipole vector and $\hat{\sigma}_1 = |e\rangle\langle g| + |g\rangle\langle e|$ describes the internal degrees of freedom [15] (the generalization to multilevel atoms is straightforward [12,16]). Alternatively, in a model of the atom as a harmonic oscillator, $\hat{\mathbf{d}} = \mathbf{d}\hat{q}$, where $\hat{q}$ is a dimensionless position operator [17]. At any given time t, the force on the atom normal to the surface is given by $F_z(t) = \langle \hat{\mathbf{d}}(t)\cdot\partial_{z_a}\hat{\mathbf{E}}(\mathbf{r}_a,t)\rangle$. From the Maxwell equations, the electric field operator can be written as $\hat{\mathbf{E}}(\mathbf{r},t) = \hat{\mathbf{E}}_0^{(+)}(\mathbf{r},t) + (i/\pi)\int_0^\infty d\omega \int_0^t d\tau\, e^{-i\omega\tau}\underline{G}_I(\mathbf{r},\mathbf{r}_a,\omega)\cdot\hat{\mathbf{d}}(t-\tau) + \text{H.c.}$, where $\underline{G}$ is the electric Green tensor of the surface (the subscripts R and I will denote real and imaginary part), and $\hat{\mathbf{E}}_0^{(+)}$ denotes the positive-frequency solution for the electric field in the absence of the atom. We will assume that the initial atom + field-matter state is factorizable, $\hat{\rho}(0) = \hat{\rho}_{\rm a}(0)\bigotimes\hat{\rho}_{\rm fm}(0)$, with the joint field-matter subsystem in its vacuum state. Using normal ordering, the force can be written as

$$F_z(t) = \text{Re}\left\{\frac{2i}{\pi}\int_0^\infty d\omega \int_0^t d\tau\, e^{-i\omega\tau} \times \text{Tr}\left[\langle\hat{\mathbf{d}}(t)\hat{\mathbf{d}}(t-\tau)\rangle\cdot\partial_{z_a}\underline{G}_I(\mathbf{r}_a,\mathbf{r},\omega)|_{\mathbf{r}=\mathbf{r}_a}\right]\right\}, \quad (1)$$

where the trace is over the vector coordinates, and $\langle\cdots\rangle$ denotes the expectation value over the initial state. Note that in this equation, $\hat{\mathbf{d}}(t)$ represents the exact dynamics of the dipole operator, including backaction from the field and matter. The two-time correlation tensor $\underline{C}_{ij}(t,t-\tau) \equiv \langle\hat{\mathbf{d}}_i(t)\hat{\mathbf{d}}_j(t-\tau)\rangle$ will be a key quantity in what follows. For the equilibrium problem being considered, the stationary ($t\to\infty$) density matrix of the coupled atom-field-matter system has the Kubo-Martin-Schwinger (KMS) form $\hat{\rho}(\infty) = \hat{\rho}_{\rm KMS} \propto e^{-\beta\hat{H}}$ (β is the inverse temperature and $\hat{H}$ is the system's Hamiltonian); at zero temperature, $\hat{\rho}(\infty)$ corresponds to the ground state of the whole system. Hence, in the stationary state the two-time correlation tensor tends to $\underline{C}_{ij}(\tau) \equiv \text{tr}\{\hat{\mathbf{d}}_i(\tau)\hat{\mathbf{d}}_j(0)\hat{\rho}_{\rm KMS}\}$, and the zero-temperature FDT [18] relates the corresponding power spectrum $\underline{S}(\omega) = (2\pi)^{-1}\int_{-\infty}^\infty d\tau\, e^{i\omega\tau}\underline{C}(\tau)$ with the atom's polarizability tensor $\underline{\alpha}_{ij}(\tau) = (i/\hbar)\theta(\tau)\text{tr}\{[\hat{\mathbf{d}}_i(\tau),\hat{\mathbf{d}}_j(0)]\hat{\rho}_{\rm KMS}\}$,

$$\underline{S}(\omega) = \frac{\hbar}{\pi}\theta(\omega)\underline{\alpha}_I(\omega), \quad (2)$$

where $\theta(\omega)$ is the step function and $\underline{\alpha}(\omega)$ is the Fourier transform of $\underline{\alpha}(\tau)$. Equation (2) is valid for the two previous models for the atom, since the equilibrium FDT holds not only for linear but also for nonlinear systems [19,20], including an atom treated using a (nonlinear) two- or multilevel model. This can be seen in the following derivation of the FDT, showing its validity for an arbitrary (time-independent) system Hamiltonian $\hat{H}$ [21,22]. Let $\hat{A}$ and $\hat{B}$ be two observables, and define $M_{AB}(\tau) = \langle\hat{A}(\tau)\hat{B}(0)\rangle - \langle\hat{A}(0)\rangle\langle\hat{B}(0)\rangle$ and $\chi_{AB}(\tau) = (i/\hbar)\langle[\hat{A}(\tau),\hat{B}(0)]\rangle$. Then $\chi_{AB}(\tau) = (i/\hbar)[M_{AB}(\tau) - M_{BA}(-\tau)]$. For $\alpha_{AB}(\tau) = \theta(\tau)\chi_{AB}(\tau)$, it follows that $\alpha_{AB}(\omega) - \alpha^*_{BA}(\omega) = (i/\hbar)\int_{-\infty}^\infty d\tau\, e^{i\omega\tau}[M_{AB}(\tau) - M_{BA}(-\tau)]$. Using the equilibrium KMS condition $M_{BA}[-(\tau + i\hbar\beta)] = M_{AB}(\tau)$ [23,24],

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we have

$$S_{AB}(\omega) = \frac{\hbar}{2\pi i(1-e^{-\beta\hbar\omega})}[\alpha_{AB}(\omega) - \alpha^*_{BA}(\omega)], \qquad (3)$$

which reduces to (2) in our case. Both for the oscillator and the two-level atom, $\underline{C}$ and $\underline{\alpha}$ are symmetric tensors, and therefore the power spectrum $\underline{S}(\omega)$ is real. Note that $\underline{\alpha}$ is the nonperturbative polarizability that depends on the optical properties of the surrounding field, the atom, and the surface, and is a function of the atom's position $\mathbf{r}_a$ (omitted in the following for simplicity). Taking the large-time limit of (1) and using the FDT, one obtains the (nonperturbative and non-Markovian) Casimir-Polder force [25]

$$F_{\rm CP} = \frac{\hbar}{\pi}\int_0^\infty d\xi\ {\rm Tr}\{\underline{\alpha}(i\xi)\cdot\partial_{z_a}\underline{G}(\mathbf{r}_a,\mathbf{r},i\xi)|_{\mathbf{r}=\mathbf{r}_a}\}. \qquad (4)$$

Another commonly used fluctuation relation is the regression theorem [26] and its generalization to the quantum case, known as the quantum regression hypothesis (sometimes called "theorem") given by the Lax formula [27]. The quantum regression theorem (QRT) is approximate, valid only in the weak system-bath coupling limit and near a resonance (see, for example, [22,28]). Although successfully used in quantum optics within its range of validity, the QRT is known to fail whenever non-Markovian and off-resonance effects play an important role [29]: the broadband nature of fluctuation-induced interactions suggests that its use in this context is therefore questionable. Within the QRT, the two-time dipole correlation tensor for a two-state atom or a harmonic oscillator for $t\to\infty$ is given by $\underline{C}_{ij}(t,t-\tau) = \mathbf{d}_i\mathbf{d}_j e^{-i(\omega_a - i\gamma_a/2)\tau}$, where ω_a and γ_a are the atomic transition frequency and dissipation rate, respectively. Using this expression in (1) and taking the large-time limit, one obtains a Casimir-Polder force of the same form as (4), but with $\underline{\alpha}(i\xi)$ replaced by $[\underline{\tilde{\alpha}}(i\xi)+\underline{\tilde{\alpha}}(-i\xi)]/2$, where $\underline{\tilde{\alpha}}_{ij}(i\xi) = (\mathbf{d}_i\mathbf{d}_j/\hbar)[(\omega_a - i\xi - i\gamma_a/2)^{-1} + (\omega_a + i\xi + i\gamma_a/2)^{-1}]$ is the generalized ground-state atomic polarizability [16]. The QRT fails to give the expression (4) predicted by the FDT and the exact solution for the harmonic-oscillator model [30], which coincides with (4) and reduces to the well-known Lifshitz formula.

The mathematical reason for this discrepancy lies in the distinct large-time behavior of the correlation tensor $\underline{C}(\tau)$. While the QRT predicts an exponential decay, the exact FDT results in a power-law decay for large times $\tau\gamma_a \gg 1$ (and agrees with the QRT only for $\gamma_a\tau \lesssim 1$). For example, in the large-time limit, $\underline{C}(\tau)\propto\tau^{-2}$ for $\alpha_I(\omega)\propto\omega$ (Ohmic dissipation). Only in the weak-coupling limit ($\gamma_a\to 0$), corresponding to a second-order perturbative calculation in powers of the coupling strengths $\mathbf{d}$, does the QRT coincide with the FDT. A related phenomenon takes place in the spontaneous decay of an excited atom in vacuum, which in the Wigner-Weisskopf approximation is predicted to be exponential, but has large-time power-law corrections [31].

The previous analysis shows that, beyond the weak-coupling regime, the correct large-time behavior of the two-time correlation tensor strongly affects the steady-state Casimir-Polder force in (1). We show now that similar considerations also apply to the nonequilibrium situation of an atom moving parallel (along the x direction) to a flat

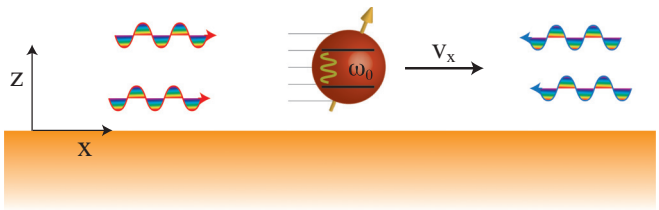

FIG. 1. (Color online) Quantum friction on an atom moving at constant velocity above a surface.

semi-infinite ($z\leqslant 0$) bulk (Fig. 1). As before, we model the atom by an electric dipole operator and treat its center-of-mass coordinate $\mathbf{r}_a(t)$ semiclassically. The quantum frictional force is given by $F_{\rm fric}(t) = \langle\hat{\mathbf{d}}(t)\cdot\partial_{x_a}\hat{\mathbf{E}}(\mathbf{r}_a(t),t)\rangle$, where the expectation value is taken with respect to an initial uncorrelated atom+field-matter state in which the subsystem field-matter is in its vacuum state [32]. The x dynamics is governed by $m_a\ddot{x}_a(t) = F_{\rm ext}(t) + F_{\rm fric}(t)$, where $F_{\rm ext}(t)$ is an external classical force on the atom that drives it from the initial rest state at $\mathbf{r}_a(t=0) = (x_a,y_a,z_a)$ to a steady state at time t_s, after which the atom moves at constant velocity v_x above the surface, $\mathbf{r}_a(t) = (x_a + v_x t, y_a, z_a)$. In the large-time limit, the stationary frictional force is given by

$$F_{\rm fric} = -{\rm Re}\left\{\frac{2}{\pi}\int\frac{d^2\mathbf{k}}{(2\pi)^2}k_x\int_0^\infty d\omega\int_0^\infty d\tau\ e^{-i(\omega-k_x v_x)\tau}\right.$$
$$\left.\times\,{\rm Tr}[\underline{C}(\tau;v_x)\cdot\underline{G}_I(\mathbf{k},z_a,z_a,\omega)]\right\}. \qquad (5)$$

Here $\underline{C}_{ij}(\tau;v_x) = {\rm tr}\{\hat{\mathbf{d}}_i(\tau)\hat{\mathbf{d}}_j(0)\hat{\rho}(\infty)\}$ is the two-time correlation tensor in the nonequilibrium stationary state $\hat{\rho}(\infty)$ of the coupled moving atom plus field and matter. Note that it depends on the velocity of the atom, which is denoted by the v_x dependency after the semicolon in the expression above. Once more, we emphasize that $\hat{\mathbf{d}}(\tau)$ contains the exact dynamics of the moving atomic dipole, i.e., including the backaction from the field and matter.

There is an extensive literature on nonequilibrium fluctuation theorems, trying to generalize fundamental equilibrium results such as the fluctuation-dissipation theorem to nonequilibrium steady-state configurations (see, for example, [33,34]). One of the challenges is to identify the form of the nonequilibrium stationary density matrix, which is no longer described by a KMS state but is model-dependent. Despite this limitation, we will show that it is still possible to draw general conclusions about the frictional force in the low-velocity limit. In analogy to the static case, we define a power spectrum $\underline{S}(\omega;v_x) = (2\pi)^{-1}\int_{-\infty}^\infty d\tau\, e^{i\omega\tau}\underline{C}(\tau;v_x)$, which is again a real and symmetric tensor since in our description $\underline{C}$ is symmetric. Using the symmetry properties of the Green tensor $\underline{G}$ for the homogeneous planar surface (see [35], for example), (5) can be rewritten as

$$F_{\rm fric} = -2\int\frac{d^2\mathbf{k}}{(2\pi)^2}k_x\int_0^\infty d\omega$$
$$\times\,{\rm Tr}[\underline{S}(k_x v_x - \omega;v_x)\cdot\underline{G}_I(\mathbf{k},z_a,z_a,\omega)]. \qquad (6)$$

Note that in this expression, the power spectrum $\underline{S}$ depends on the wave vector only through the Doppler-shifted frequency

$\omega - v_x k_x$. The friction is the momentum transfer $\hbar k_x$ to the atom weighted by its Doppler-shifted power spectrum and by the electromagnetic density of states, all integrated over frequency and momentum. As expected, the force vanishes for $v_x = 0$: since $\underline{S}$ is symmetric, only the symmetric part of $\underline{G}_I$ (even in k_x [35]) is relevant. The integral then vanishes for parity reasons.

Generally, one is interested in computing $F_{\rm fric}$ to leading order in v_x. For this, however, one needs to know the expression for $\underline{S}(\omega; v_x)$, which in general is not available (see, however, the harmonic-oscillator model below). Nevertheless, even without this knowledge, it is possible to prove that at zero temperature and in the stationary limit ($t \to \infty$) there are no linear in v_x terms in the friction force, independently of the model for the atom's polarizability. Indeed, terms proportional to v_x could only arise either from $\underline{S}(-\omega; v_x)$ or from $\underline{S}(k_x v_x - \omega; 0)$. The contribution of the former term cancels again for parity reasons upon integration over k_x. The latter term, corresponding to a stationary state $\hat{\rho}(\infty)$ in which the atom is static, can be evaluated using the equilibrium FDT (2), i.e., $\underline{S}(k_x v_x - \omega; 0) = (\hbar/\pi)\theta(k_x v_x - \omega)\underline{\alpha}_I(k_x v_x - \omega)$. Because of the motion-induced Doppler shift, only frequency modes $0 \leqslant \omega \leqslant k_x v_x$ contribute, implying that very low frequencies are relevant at small velocities. Since the atomic polarizability and the Green tensor are susceptibilities, they satisfy the crossing relation, and their imaginary parts, being odd in ω, vanish at $\omega = 0$ in our case [36]. An expansion for small v_x then leads to

$$F_{\rm fric} \approx -\frac{2\hbar v_x^3}{3(2\pi)^3}\int_{-\infty}^{\infty} dk_y \int_0^{\infty} dk_x k_x^4 {\rm Tr}[\underline{\alpha}_I'(0)\cdot\underline{G}_I'(\mathbf{k},0)]$$
$$\approx -\frac{45\hbar v_x^3}{256\pi^2\epsilon_0 z_a^7}\alpha_I'(z_a,0)\Delta_I'(0), \qquad (7)$$

where in the first line we omitted writing the z_a dependency of the Green tensor at coincidence. In the second line, we have considered the low-frequency (near-field) form of the Green tensor for a dielectric semispace described by a complex permittivity $\epsilon(\omega)$ (ϵ_0 in the vacuum permittivity), with $\Delta(\omega) \equiv [\epsilon(\omega) - 1]/[\epsilon(\omega) + 1]$, and we have used $\underline{\alpha}(z_a,\omega) = \delta_{ij}\alpha(z_a,\omega)$ (we have reintroduced z_a to underscore the dependency of the dressed polarizability on the position of the atom). The above argument proves that, within our description for the atom, the lowest-order expansion in velocity of the zero-temperature, stationary frictional force on an atom moving above a planar surface is at least cubic in v_x. In principle, however, in (7) there could be other v_x^3 contributions to the frictional force arising from v_x derivatives of $\underline{S}(k_x v_x - \omega; v_x)$. Also, when either of the ω derivatives of the two tensors in (7) vanishes at $\omega = 0$, higher-order terms in v_x must be considered.

Regarding the dependency of the stationary frictional force (7) on the atom-surface separation, it must be emphasized that the z_a^{-7} scaling arises solely from the z_a dependency of the Green tensor. In addition, as explained above, the power spectrum $\underline{S}$ and the polarizability $\underline{\alpha}$ depend implicitly on z_a via the exact dynamics of the coupled atom–field-matter system. In particular, these quantities are related to the atomic decay, which at short distances and to lowest order in perturbation theory scales as z_a^{-3}, leading in (7) to a total z_a^{-10} dependency of the frictional force. For systems with intrinsic dissipation (e.g., gold nanoparticles), the radiation-induced damping is generally negligible and the frictional force has therefore a milder dependency on separation [2].

In contrast to the FDT, the QRT predicts that for slow velocities, the quantum frictional force is linear in v_x. As shown above, such a dependency results in principle from contributions of $\underline{S}(k_x v_x - \omega; 0)$ in (6). Using the QRT expression for the two-time correlation tensor in the static case, $\underline{C}_{ij}(t, t-\tau; 0) = \mathbf{d}_i\mathbf{d}_j e^{-i(\omega_a - i\gamma_a/2)\tau}$, and taking the $t \to \infty$ limit, one obtains indeed at the leading-order expansion

$$F_{\rm fric}^{\rm QRT} \approx v_x \frac{2|\mathbf{d}|^2\gamma_a}{3\pi}\int \frac{d^2\mathbf{k}}{(2\pi)^2}k_x^2\int_0^{\infty} d\omega$$
$$\times \frac{\omega + \omega_a}{\left[(\omega+\omega_a)^2 + \gamma_a^2/4\right]^2}{\rm Tr}[\underline{G}_I(\mathbf{k}, z_a, z_a, \omega)], \qquad (8)$$

where, for simplicity, we assumed that the atom is isotropic [12]. As for the static Casimir-Polder force, the quantum regression hypothesis fails to give the correct quantum frictional force. Note, however, that once again both the FDT and the QRT give the same quantum frictional force in the limit $\gamma_a \to 0$, consistent with the observation before that the quantum regression hypothesis coincides with the exact fluctuation-dissipation theorem for systems near equilibrium in the weak-coupling limit. In this limit, the resulting force is exponentially suppressed in v_x^{-1} [10,13]. Linear-response relations in fluctuational electrodynamics, based on equilibrium fluctuations, can also be employed to study quantum friction for small perturbations around the equilibrium state [9–11,14,37,38]. In agreement with our analysis, at zero temperature the linear-in-velocity frictional force vanishes. However, far-from-equilibrium situations require fully nonequilibrium fluctuation relations.

The previous derivation uses general principles based on the fluctuation-dissipation theorem in nonequilibrium settings. In the following, we present an alternative derivation that does not resort to the FDT, and we compute quantum friction for the harmonic-oscillator model by directly solving the equations of motion for the atomic dipole in the stationary limit (see the supplemental material [43] for a similar derivation for the two-state atom). The dynamics of the dipole operator for the moving harmonic-oscillator atom can be solved for exactly. Its equation of motion, including the backreaction of the electromagnetic field, is given by $\ddot{\hat{q}}(t) + \omega_a^2\hat{q}(t) = (2\omega_a/\hbar)\mathbf{d}\cdot\hat{\mathbf{E}}(\mathbf{r}_a(t), t)$. Splitting the solution to Maxwell's equations for the total field $\hat{\mathbf{E}}$ as a sum of free ($\hat{\mathbf{E}}_0$, homogeneous solution) and source ($\hat{\mathbf{E}}_S$, particular solution) parts and taking the Fourier transform, the equation of motion can be rewritten as $[-\omega^2 + \omega_a^2 - \frac{2\omega_a}{\hbar}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\mathbf{d}\cdot\underline{G}(\mathbf{k}, z_a, z_a, \omega + k_x v_x)\cdot\mathbf{d}]\hat{q}(\omega) = \frac{2\omega_a}{\hbar}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\mathbf{d}\cdot\hat{\mathbf{E}}_0(\mathbf{k}, z_a, \omega + k_x v_x)e^{i(k_x x_a + k_y y_a)}$. The polarizability of the moving oscillator is then given by $\underline{\alpha}_{ij}(\omega; v_x) = \frac{2\omega_a}{\hbar}\mathbf{d}_i\mathbf{d}_j[-\omega^2 + \omega_a^2 - \frac{2\omega_a}{\hbar}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\mathbf{d}\cdot\underline{G}(\mathbf{k}, \omega + k_x v_x)\cdot\mathbf{d}]^{-1}$, where we have omitted the z_a dependency of the Green tensor. The dynamic power spectrum $\underline{S}(\omega; v_x)$ is computed starting from $\langle\hat{\mathbf{d}}_i(\omega)\hat{\mathbf{d}}_j(\omega')\rangle = \mathbf{d}_i\mathbf{d}_j\langle\hat{q}(\omega)\hat{q}(\omega')\rangle$ and using that $\underline{S}_{ij}(\omega; v_x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\langle\hat{\mathbf{d}}_i(\omega)\hat{\mathbf{d}}_j(\omega')\rangle$. The resulting exact

expression for the zero-temperature case is

$$\underline{S}(\omega; v_x) = \frac{\hbar}{\pi}\theta(\omega)\underline{\alpha}_I(\omega; v_x) - \frac{\hbar}{\pi}\underline{J}(\omega; v_x), \tag{9}$$

where the "current" $\underline{J}$ is given by

$$\underline{J}(\omega; v_x) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}[\theta(\omega) - \theta(\omega + k_x v_x)] \times \underline{\alpha}(\omega; v_x) \cdot \underline{G}_I(\mathbf{k}, \omega + k_x v_x) \cdot \underline{\alpha}^*(\omega; v_x). \tag{10}$$

Generalized FDT relations for nonequilibrium, stationary classical systems [33] have the same structure as (9). Since only the symmetric part of the Green tensor contributes to $\mathbf{d} \cdot \underline{G}(\mathbf{k}, \omega + k_x v_x) \cdot \mathbf{d}$, from the previous expressions for the polarizability and the current $\underline{J}$ we can deduce that the power spectrum is even in v_x. Using the identity $\underline{\alpha}_I(\omega; v_x) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\underline{\alpha}(\omega; v_x) \cdot \underline{G}_I(\mathbf{k}, \omega + k_x v_x) \cdot \underline{\alpha}^*(\omega; v_x)$, we rewrite the power spectrum (9) as $\underline{S}(\omega; v_x) = \frac{\hbar}{\pi}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\theta(\omega + k_x v_x)\underline{\alpha}(\omega, v_x) \cdot \underline{G}_I(\mathbf{k}, \omega + k_x v_x) \cdot \underline{\alpha}^*(\omega, v_x)$. An expansion at low velocity takes the form

$$\underline{S}(\omega; v_x) \approx \frac{\hbar}{\pi}\theta(\omega)\left[\underline{\alpha}_I(\omega; 0) + \underline{\eta}(\omega; 0)\frac{v_x^2}{2}\right] + O\left(v_x^4\right). \tag{11}$$

Here we have defined $\underline{\eta}(\omega; 0) = \underline{\alpha}''(\omega; 0) \cdot \underline{G}_I(\omega) \cdot \underline{\alpha}^*(\omega; 0) + \underline{\alpha}(\omega; 0) \cdot \underline{g}(\omega) \cdot \underline{\alpha}^*(\omega; 0) + \underline{\alpha}(\omega; 0) \cdot \underline{G}_I(\omega) \cdot [\underline{\alpha}''(\omega; 0)]^*$ (the double prime denotes second derivative with respect to velocity) and $\underline{g}(\omega) = \int d^2\mathbf{k}(2\pi)^{-2}k_x^2\partial_\omega^2\underline{G}_I(\mathbf{k}, \omega)$. The tensor $\underline{\eta}(\omega; 0)$ vanishes at $\omega = 0$ because it is a sum of terms proportional either to the imaginary part of the Green tensor or to its second derivative. Using (11) in (6), one can verify that to leading order in v_x the quantum frictional force for the harmonic-oscillator model is exactly given by (7), and the next order is proportional to v_x^5 (see the supplemental material [43]).

Our result for the v_x^3 dependence of the quantum friction force on a moving atom contrasts with some previous works in the literature that predicted a zero-temperature frictional force linear in v_x. In [12] the atom was modeled as a multilevel system and the dipole correlation function in (5) was computed using QRT, which led to a stationary friction force linear in velocity (8). Calculations of quantum friction based on QRT, Wigner-Weisskopf, or Markovian approximations encompass an exponential-only decay of the dipole correlation tensor, which is valid for times $t \lesssim \gamma_a^{-1}$. Importantly, they miss the power-law decay at larger times $t \gg \gamma_a^{-1}$, which strongly affects the low-frequency behavior of the spectrum. The discussion after (6) shows that, in the stationary case, quantum friction is a low-frequency phenomenon [see also the paragraph after (4)]. Therefore, it is not surprising that the above-mentioned approximations fail to predict the correct stationary behavior and lead to a different dependence of the force on the atom's velocity. On the other hand, in [13] the atom was modeled as a harmonic oscillator and, by calculating the power dissipated by the atom into pairs of surface plasmons using an approach based on standard perturbation theory, a linear-in-velocity frictional force similar to [12] was obtained (within the same approximations, an identical result is obtained for a two-level atom). This time-dependent perturbative approach assumed that the atom remains in its bare ground state and is valid for times not too long, for which decays are still exponential. In contrast, our previous calculation shows that in the large-time, nonequilibrium steady state, the quantum frictional force becomes cubic in velocity.

Due to the weak nature of quantum friction, its experimental detection is challenging. Indeed, in the near field our result (7) takes the form

$$F_{\text{fric}} \approx -\frac{90}{\pi^3}\frac{\hbar\rho^2\alpha_0^2}{(2z_a)^{10}}v_x^3, \tag{12}$$

where ρ is the surface's electrical resistivity and α_0 is the static atomic polarizability. As an example, for a ground state ^{87}Rb atom [$\alpha_0 = 5.26 \times 10^{-39}$ Hz/(V/m)^2 [39]] flying at $v_x = 340$ m/s at a distance $z_a = 10$ nm above a silicon semispace ($\rho = 6.4 \times 10^2\ \Omega\,\text{m}$), the zero-temperature drag force is $F_{\text{fric}} \approx -1.3 \times 10^{-20}$ N. Nevertheless, new experimental setups (e.g., new materials [4] and/or new geometries [40]) and techniques (e.g., atom interferometry) could make it accessible in the near future.

In summary, we have studied quantum friction using general concepts of quantum statistical mechanics. We have derived a generalized nonequilibrium fluctuation-dissipation relation for an atom in steady motion above a surface, and we showed that at low speeds the quantum frictional force is cubic in velocity. The analysis can be extended to include thermal fluctuations. In the high-temperature (classical) limit ($\hbar\beta\gamma \ll 1$ [22]), however, quantum regression agrees with the FDT [22,28,41], and the resulting frictional force scales linearly with velocity. A study similar to the one present here can be performed for the case of macroscopic bodies in relative motion [2,42]. Finally, we would like to stress that our discussion of the implications and limitations of the use of fluctuation relations in calculations of equilibrium and nonequilibrium atom-surface interactions can potentially impact a broad range of fields, such as atom interferometry and atom chips.

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[1] A. Einstein, Physik. Zeitschr. **18**, 121 (1917).
[2] A. I. Volokitin and B. N. J. Persson, Rev. Mod. Phys. **79**, 1291 (2007).
[3] R. Zhao, A. Manjavacas, F. J. García de Abajo, and J. B. Pendry, Phys. Rev. Lett. **109**, 123604 (2012).
[4] A. I. Volokitin and B. N. J. Persson, Phys. Rev. Lett. **106**, 094502 (2011).
[5] D. A. R. Dalvit and P. A. M. Neto, Phys. Rev. Lett. **84**, 798 (2000).
[6] J. Mahanty, J. Phys. B **13**, 4391 (1980).

[7] W. L. Schaich and J. Harris, J. Phys. F **11**, 65 (1981).
[8] M. S. Tomassone and A. Widom, Phys. Rev. B **56**, 4938 (1997).
[9] A. I. Volokitin and B. N. J. Persson, Phys. Rev. B **65**, 115419 (2002).
[10] G. Dedkov and A. Kyasov, Tech. Phys. Lett. **28**, 346 (2002).
[11] J. R. Zurita-Sánchez, J.-J. Greffet, and L. Novotny, Phys. Rev. A **69**, 022902 (2004).
[12] S. Scheel and S. Y. Buhmann, Phys. Rev. A **80**, 042902 (2009).
[13] G. Barton, New J. Phys. **12**, 113045 (2010).
[14] G. Pieplow and C. Henkel, New J. Phys. **15**, 023027 (2013).
[15] L. Allen and J. H. Eberly, *Optical Resonance and Two-level Atoms* (Dover, New York, 1975).
[16] S. Y. Buhmann, L. Knöll, D.-G. Welsch, and H. T. Dung, Phys. Rev. A **70**, 052117 (2004).
[17] The two-state atom free Hamiltonian is $\hat{H}_a = (\hbar\omega_a/2)\hat{\sigma}_3$, where $[\hat{\sigma}_i,\hat{\sigma}_j] = 2i\varepsilon_{ijk}\hat{\sigma}_k$. In the harmonic oscillator case, $\hat{H}_a = (\hbar\omega_a/4)(\hat{p}^2+\hat{q}^2)$ with $[\hat{q},\hat{p}] = 2i$.
[18] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).
[19] V. G. Polevoi and S. M. Rytov, Theor. Math. Phys. **25**, 1096 (1975).
[20] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 2008).
[21] R. Kubo, Rep. Prog. Phys. **29**, 255 (1966).
[22] P. Talkner, Ann. Phys. (NY) **167**, 390 (1986).
[23] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
[24] P. C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).
[25] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[26] L. Onsager, Phys. Rev. **38**, 2265 (1931).
[27] M. Lax, Phys. Rev. **129**, 2342 (1963).
[28] G. W. Ford and R. F. O'Connell, Phys. Rev. Lett. **77**, 798 (1996).
[29] G. Ford and R. O'Connell, Opt. Commun. **179**, 451 (2000).
[30] F. Intravaia, C. Henkel, and M. Antezza, in *Casimir Physics*, edited by D. A. R. Dalvit, P. W. Milonni, D. C. Roberts, and F. S. S. Rosa, Lecture Notes in Physics Vol. 834 (Springer, Berlin, 2011), pp. 345–391.
[31] P. R. Berman and G. W. Ford, in *Advances in Atomic, Molecular, and Optical Physics*, edited by E. Arimondo, P. R. Berman, and C. C. Lin (Academic, Amsterdam, 2010), Vol. 59, p. 175.
[32] We are not including the Röentgen interaction $-\hat{\mathbf{d}}\cdot\mathbf{v}\times\hat{\mathbf{B}}$, which, for the purpose of calculating the friction force, is negligible as compared to the nonretarded interaction $-\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}$, as shown in [12].
[33] R. Chetrite, G. Falkovich, and K. Gawedzki, J. Stat. Mech. (2008) P08005.
[34] M. Baiesi, C. Maes, and B. Wynants, Phys. Rev. Lett. **103**, 010602 (2009).
[35] J. M. Wylie and J. E. Sipe, Phys. Rev. A **30**, 1185 (1984).
[36] L. Landau and E. Lifshitz, *Course of Theoretical Physics: Statistical Physics, Pt. 1* (Pergamon, Oxford, 1980).
[37] M. F. Maghrebi, R. Golestanian, and M. Kardar, Phys. Rev. D **87**, 025016 (2013).
[38] V. A. Golyk, M. Krüger, and M. Kardar, Phys. Rev. B **88**, 155117 (2013).
[39] D. A. Steck, Rubidium 87 D Line Data, http://steck.us/alkalidata.
[40] F. Intravaia *et al.*, Nature Commun. **4**, 2515 (2013).
[41] G. Ford and R. O'Connell, Ann. Phys. (NY) **276**, 144 (1999).
[42] J. B. Pendry, J. Phys. Condens. Matter **9**, 10301 (1997).
[43] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.89.050101 for a similar derivation for the two-state system.