Measuring the parity of N distant atoms with linear optics

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It is known that parity measurement, together with single-qubit rotation, is sufficient for implementing scalable quantum computation. In this Brief Report, we propose a scheme for a projective measurement of the parity operator $P_z = \otimes_{i=1}^{N}\sigma_{i,z}$ of N distant atoms trapped in spatially separated cavities. Instead of direct interaction between the atoms, quantum interference of polarized photons decaying from the optical cavities is used to realize expected measurement without resorting to a sequence of single- and two-qubit operations. It is shown that parity measurement can be implemented repeatedly until success without destroying the qubits at any stage of the operation.

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Introduction. Quantum computing has attracted much interest since it is, in principle, able to solve hard computational problems more efficiently than present classical computers [1]. Most of the research in constructing a universal quantum computer is based on the realization of two-qubit controlled gates and one-qubit gates with the coherently controlled qubit-qubit interactions [2]. However, such an approach induces some potentially experimental problems: (1) The addition of an extra qubit to a system may disrupt the physical settings that have been put in place for quantum computation. (2) The qubits must be closed enough so that a two-qubit logic can be implemented, but in this case, individual addressing cannot be achieved. Another promising way was proposed to realize quantum logic gates without controllable interaction, in which entangling operation between qubits was obtained by using ancillary entangled states and performing appropriate measurement on qubits [3–6]. For example, it has been shown that together with single-qubit rotation, parity measurement is sufficient for implementing scalable quantum computation [6]. Indeed, parity measurement is a basic operation in quantum information processing and can be applied to many problems, ranging from the generation of multiqubit entangled states [7] to the realization of Bell measurement and quantum error-correcting codes [8]. This also motivated experimental implementations for parity measurements in various systems [9].

Among a variety of systems being explored for hardware implementations of quantum computers, cavity quantum electrodynamics (QED) is favored since cold and localized atoms are well suited for storing quantum information in their long-lived internal states, and the photons are a natural source for fast and reliable transport of quantum information over long distances [10]. Several schemes have been proposed for creating two-qubit Einstein-Podolsky-Rosen state [11] and multiqubit Greenberger-Horne-Zeilinger, W, Dicke [12], and cluster states [13,14] between distant atoms. Furthermore, it has also been shown that a quantum logic gate between two distant atoms can be conditionally implemented [15,16]. In particular, in Ref. [16], a repeat-until-success scheme is proposed by Lim *et al.* for realizing the conditional quantum phase gate without destroying the qubits at any stage of the computation. In Ref. [13], an odd-parity measurement was introduced for generating a cluster state. But this scheme is intrinsically nondeterministic. It is not clear whether these entangled operations [13,15,16] can be directly extended to realize a multiqubit quantum operation without resorting to a sequence of single- and two-qubit operations, which is of importance for reducing the complexity of physical realization of some practical quantum computation and quantum algorithms.

In this Brief Report, we propose an alternative scheme to realize a parity measurement of N atoms trapped in distant optical cavities. The present protocol has the following favorable features: (1) We show that the parity measurement can be implemented repeatedly until success without destroying the qubits at any stage of the operation, i.e., the success probability of our protocol approaches unity in the ideal case. (2) The scheme is insensitive to some practical quantum noise, such as photon loss, inefficiency of the photon detectors, and the phase accumulated by the photons on their way from the cavities to the place where they are detected, which only decrease the success probability but exert no influence on the fidelity of expected operation. (3) The scheme can be directly used to realize the N-qubit parity measurement without resorting to a sequence of single- and two-qubit operations, which is of importance for reducing the complexity of some practical operations.

Basic model. A single-sided optical cavity with a single trapped atom plays the role of a basic building block in our protocol. The level structure of the trapped atom is shown in Fig. 1, which is a double-Λ configuration, and has been proposed to implement quantum computation [17] and engineer entanglement of one-photon wave packets [18]. For concreteness, we consider a possible implementation using $^{40}Ca^{+}$, whose usefulness in the quantum information context has been demonstrated in recent experiments [19,20]. We encode the ground states $|g_H\rangle$ and $|g_V\rangle$ as logic zero and one states, i.e., $|g_H\rangle = |0\rangle$ and $|g_V\rangle = |1\rangle$. The atomic transitions $|s_H\rangle \longleftrightarrow |e_H\rangle$ and $|s_V\rangle \longleftrightarrow |e_V\rangle$ are coupled to horizontal and vertical polarization modes, respectively, while the transitions $|e_H\rangle \longleftrightarrow |g_H\rangle$ and $|e_V\rangle \longleftrightarrow |g_V\rangle$ are driven by two classical fields with vertical and horizontal polarizations, respectively. The classical fields and cavity modes are detuned from their respective transitions by a same amount Δ.

In the case of large detuning the excited states can be eliminated adiabatically to obtain an effective interaction (in

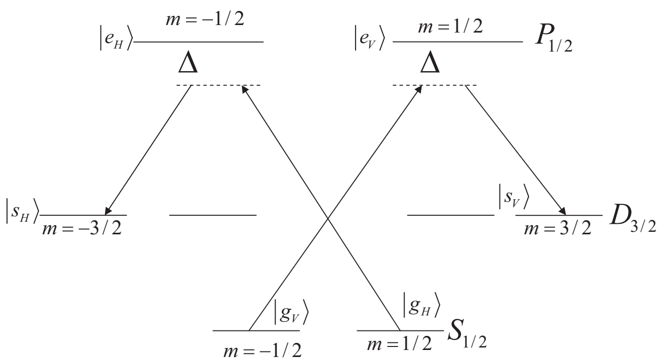

FIG. 1. Relevant level structure and atomic transitions of the trapped atom.

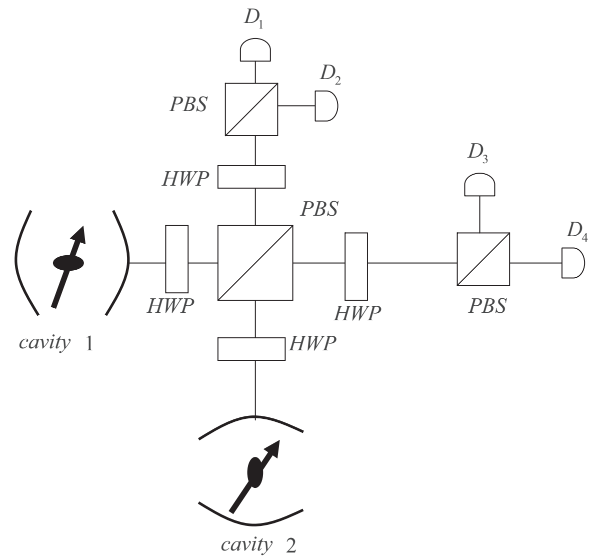

FIG. 2. Schematic setup for a parity measurement of two distant atoms. It includes three polarization beam splitters (PBS), which transmit the horizontal polarization and reflect vertical polarization, four half-wave plates (HWP), which implement transformation $|H\rangle \rightarrow (|H\rangle + |V\rangle)/\sqrt{2}$, $|V\rangle \rightarrow (|H\rangle - |V\rangle)/\sqrt{2}$, and four photon detectors D.

the interaction picture)

$$H = \Omega(a_H^\dagger |s_H\rangle\langle g_H| + a_V^\dagger |s_V\rangle\langle g_V| + \text{H.c.}), \tag{1}$$

where a_H and a_V ($a_H^\dagger$ and $a_V^\dagger$) are the annihilation (creation) operators of the horizontal and vertical polarization modes. $\Omega = g_c\Omega_c/\Delta$ denotes the effective coupling constant. Here g_c and Ω_c are the interaction strengths of the atom coupled to their cavity fields (classical fields), which can be assumed to be the same. In order to investigate the quantum dynamics of the system, it is convenient to follow a quantum trajectory description [21]. The evolution of the system's wave function is governed by a non-Hermitian Hamiltonian

$$H' = H - i\kappa(a_H^\dagger a_H + a_V^\dagger a_V) \tag{2}$$

as long as no photon decays from the cavity. The single trapped atom is initially prepared in the state

$$\alpha|g_H\rangle + \beta|g_V\rangle, \tag{3}$$

and both polarization modes are initially in the vacuum states $|0,0\rangle$, where $|m,n\rangle$ denotes m and n photons in the horizontal and vertical polarization mode, respectively. Now we switch on the Hamiltonian (1) in the atom-cavity system for a time τ. If no photon is emitted from the cavity, at the time $\tan(\Omega_\kappa\tau) = 2\Omega_\kappa/\kappa$ with $\Omega_\kappa = \sqrt{\Omega^2 - \kappa^2/4}$, the atom-cavity state evolves to

$$|\Psi\rangle_m = \alpha|s_H\rangle|1,0\rangle + \beta|s_V\rangle|0,1\rangle, \tag{4}$$

which can be transformed into

$$|\Psi\rangle = \alpha|g_H\rangle|1,0\rangle + \beta|g_V\rangle|0,1\rangle \tag{5}$$

by applying two fast Raman transitions to drive the atom. The success probability of this evolution procedure is given by

$$P_{\text{single}} = \frac{e^{-\kappa\tau}\sin^2(\Omega_\kappa\tau)\Omega^2}{\Omega_\kappa^2}. \tag{6}$$

In order to make the success probability as large as possible, the coupling parameters g_c,Ω_c and detuning Δ have to be adjusted to satisfy a condition $\Omega = g_c\Omega_c/\Delta \gg \kappa$, i.e., $\Omega_\kappa \gg \kappa$, which makes $P_{\text{single}} \approx 1$. This corresponds to the requirement of strong-coupling cavity QED. In the cavity QED experiment using $^{40}Ca^+$ [19], the parameters $g_c = 0.92$ MHz and $\kappa = 1.2$ MHz have been reported, which cannot satisfy the condition of scheme. Thus to satisfy the requirement of the scheme, the experimental setup needs to be further improved. In the cavity QED, parameters g_c and κ are fixed by the hardware of the system, which can be modified through the length L and finesse F of the cavity [19]: $g_c \sim L^{-3/4}$ and $\kappa \sim (FL)^{-1}$. If we can increase the cavity finesse of Ref. [19] by two orders of magnitude and decreasing the length of the cavity to about 2 mm, we have the parameters $g_c \approx 2.6$ MHz and $\kappa \approx 0.048$MHz, which is enough for the requirement of the scheme.

Parity measurement of two atoms. We first explain the scheme by considering the parity measurement of two atoms. The schematic setup is shown in Fig. 2. The photon leakage from cavities 1 and 2 is first sent through two half-wave plates, and then interfered at a polarization beam splitter, with the outputs detected by four photon detectors after two half-wave plates and two polarization beam splitters. If a photon detector D_j ($j = 1,2,3,4$) detects a photon, the coherent evolution of the system is interrupted by a quantum jump, which can be formulated with the operators b_j on the joint state vectors of two atom-cavity systems

$$\begin{aligned} b_1 &= \tfrac{1}{2}(a_{1H} - a_{1V} + a_{2H} + a_{2V}), \\ b_2 &= \tfrac{1}{2}(a_{1V} - a_{1H} + a_{2H} + a_{2V}), \\ b_3 &= \tfrac{1}{2}(a_{1H} + a_{1V} - a_{2H} + a_{2V}), \\ b_4 &= \tfrac{1}{2}(a_{1H} + a_{1V} + a_{2H} - a_{2V}). \end{aligned} \tag{7}$$

We now analyze the scheme in detail. Initially, two atoms are prepared in the state $|\Psi\rangle_{\text{in}} = |\Psi(0)\rangle_1 \otimes |\Psi(0)\rangle_2$ with $|\Psi(0)\rangle_j = \alpha_j|g_H\rangle_j + \beta_j|g_V\rangle_j$, and all cavity modes are in the vacuum states. We switch on the Hamiltonian (1) in each atom-cavity system for a time τ, and prepare the joint state of two atom-cavity systems as follows: $|\Psi\rangle_{\text{prep}} = |\Psi\rangle_1 \otimes |\Psi\rangle_2$

where the state $|\Psi\rangle_j$ is given by Eq. (5). The probability success in this stage is given by $P_{\rm prep} = P_{\rm single}^2$.

Next we consider the detection stage of the scheme, in which we make a photon number measurement with four photon detectors D_j ($j = 1,2,3,4$) on the output modes of the setup. We assume that photons are detected at time t. This assumption is posed to calculate the system's time evolution during this time interval in a consistent way with the *no-photon-emission* Hamiltonian (2). The joint state of the total system evolves into

$$|\Psi(t)\rangle = |\Psi(t)\rangle_1 \otimes |\Psi(t)\rangle_2, \quad (8)$$

with

$$|\Psi(t)\rangle_j = \alpha_j e^{-\kappa t}|g_H\rangle_j|1,0\rangle_j + \beta_j e^{-\kappa t}|g_V\rangle_j|0,1\rangle_j. \quad (9)$$

The detection of one photon with the detector D_j can be formulated with the operator b_j on the joint state $|\Psi(t)\rangle$. If D_1 and D_4 detect one photon and D_2 and D_3 do not detect any photon, or vice versa, the state of the total system is projected into

$$\alpha_1\alpha_2|g_H\rangle_1|g_H\rangle_2 + \beta_1\beta_2|g_V\rangle_1|g_V\rangle_2 = (1 + \sigma_{1,z}\sigma_{2,z})|\Psi\rangle_{\rm in}, \quad (10)$$

where $\sigma_{j,z} = |g_H\rangle_j\langle g_H| - |g_V\rangle_j\langle g_H|$. It is easy to prove that Eq. (10) is the eigenstate of parity operator $\sigma_{1,z} \otimes \sigma_{2,z}$ with the eigenvalue $+1$. If D_1 and D_3 detect one photon and D_2 and D_4 do not detect any photon during that time interval, or vice versa, the state of the total system becomes

$$\alpha_1\beta_2|g_H\rangle_1|g_V\rangle_2 + \beta_1\alpha_2|g_V\rangle_1|g_H\rangle_2 = (1 - \sigma_{1,z}\sigma_{2,z})|\Psi\rangle_{\rm in}, \quad (11)$$

which is an eigenstate of the parity operator $\sigma_{1,z} \otimes \sigma_{2,z}$ with the eigenvalue -1. Equations (10) and (11) demonstrate the conditional implementation of the parity measurement of two atoms with a success probability $P_{\rm succ} = (1 - e^{-2\kappa t})^2/2$.

Other detecting events should be also considered. Due to quantum interference, it is impossible for detectors D_1 and D_2 (D_3 and D_4) to detect one photon respectively, so that we only consider the case that two photons are detected in the same detectors D_1, D_2, D_3, or D_4. If D_1 or D_2 detects two photons, the state of the total system is projected into

$$(\alpha_1|g_H\rangle_1 - \beta_1|g_V\rangle_1)(\alpha_2|g_H\rangle_2 + \beta_2|g_V\rangle_2) = \sigma_{1,z}|\psi\rangle_{\rm in}. \quad (12)$$

If D_3 or D_4 detects two photons, we have

$$(\alpha_1|g_H\rangle_1 + \beta_1|g_V\rangle_1)(\alpha_2|g_H\rangle_2 - \beta_2|g_V\rangle_2) = \sigma_{2,z}|\psi\rangle_{\rm in}. \quad (13)$$

The probability of conditionally obtaining Eqs. (12) and (13) is given by $P_{\rm non} = (1 - e^{-2\kappa t})^2/2$. It is easily seen that Eqs. (12) and (13) can be transformed into the initial state $|\Psi\rangle_{\rm in}$ by using a local operation. Thus, based on the measurement result, we can switch on the atom-cavity interaction and repeat the above procedure until success in realizing the parity measurement of the initial state $|\Psi\rangle_{\rm in}$. The total success probability of the protocol is given by

$$\begin{aligned} P &= P_{\rm prep}P_{\rm succ} + P_{\rm prep}P_{\rm succ}P_{\rm prep}P_{\rm non} \\ &\quad + P_{\rm prep}P_{\rm succ}(P_{\rm prep}P_{\rm non})^2 + \cdots \\ &= \frac{P_{\rm prep}P_{\rm succ}}{1 - P_{\rm prep}P_{\rm non}}. \end{aligned} \quad (14)$$

If we have the parameters $g_c \approx 2.6$ MHz, $\kappa \approx 0.048$ MHz, $\Omega_c = 2.8$ MHz, and $\Delta = 6.76$ MHz and choose the interaction time $t = 80\ \mu$s, we find the total success probability of the scheme reaches about 0.97, which is a little lower than the ideal unit success probability because the preparation stage has an extremely small chance of failure.

We now give a brief discussion on the influence of some practical noise on the scheme. First, the scheme is inherently robust to photon loss, which includes the contribution from channel attenuation, and the inefficiency of the photon detectors. All these kinds of noise can be considered by an overall photon loss probability η [11]. If one photon is lost, a click from each of the detectors is never recorded. In this case, the scheme fails to realize the expected quantum operation. Therefore the photon loss only decreases the success probability $P_{\rm succ}$ and $P_{\rm non}$ by a factor of $(1-\eta)^2$, but has no influence on the fidelity of the expected operation. Second, the scheme is insensitive to the phase accumulated by the photons on their way from the ions to the place where they are detected. The phases $\varphi_1 = \int_0^{L_1} k(l)dl$ and $\varphi_2 = \int_0^{L_2} k(l)dl$, where k is the wave number and L_j are the optical lengths which photons travel from the jth ions towards the photon detectors, lead only to a multiplicative factor $e^{i(\varphi_1+\varphi_2)}$ in Eqs. (10)–(13). This result demonstrates that phases accumulated by the photons have no effect on the conditional implementation of the quantum operation. Third, the influence of atomic recoil on the implementation of a quantum phase gate could be suppressed. When an atom absorbs or emits photons, it is always accompanied by a recoil. In our scheme, both atoms absorb and emit photons with the same energy simultaneously; if one detects two photons at the same time, the influence of the atomic recoil on the scheme can thus be suppressed. Finally, it is pointed out that we should choose a sufficiently large detuning, so that excited states can be decoupled from the evolution and the scheme is immune to the effect of the atomic spontaneous emission.

Extension to the case of N atoms. Finally we demonstrate that the above-proposed procedure can be directly used to realize the parity measurement of N distant atoms, which is a fundamental operation in quantum information processing [8,22]. Based on the basic model, we first encode quantum state $|\Psi_{\rm in}\rangle_j = \alpha_j|g_H\rangle_j + \beta_j|g_V\rangle_j$ of the jth atom into an entangled state $|\Psi\rangle_j = \alpha_j|g_H\rangle_j|1,0\rangle_j + \beta_j|g_V\rangle_j|0,1\rangle_j$ ($j = 1,\ldots,N$). To realize expected operation, a photon, which originates from the state $|\Psi\rangle_j$, is injected into the jth input port of the setup shown in Fig. 3. We first consider the events that m detectors of $D_1, D_3, \ldots, D_{2N-1}$ and (N-m) detectors of $D_2, D_4, \ldots, D_{2N}$ detect one photon, respectively. If m is even, the state of the system is projected into

$$(1 + \otimes_{j=1}^N \sigma_{j,z}) \otimes_{j=1}^N |\Psi_{\rm in}\rangle_j, \quad (15)$$

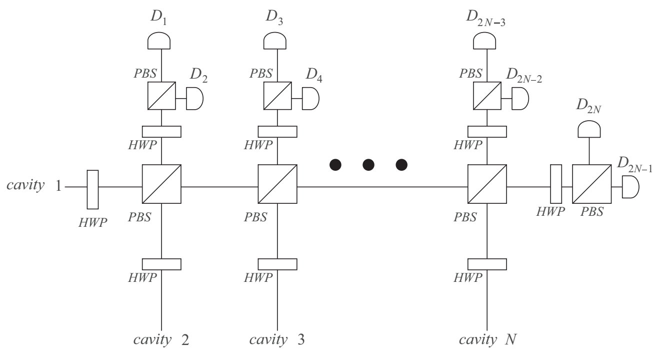

FIG. 3. Schematic setup for a parity measurement of N atoms trapped in distant cavities.

otherwise

$$(1 - \otimes_{j=1}^{N}\sigma_{j,z}) \otimes_{j=1}^{N} |\Psi_{\rm in}\rangle_j. \quad (16)$$

Equations (15) and (16) are eigenstates of parity operator $\otimes_{j=1}^{N}\sigma_{j,z}$ with the eigenvalues $+1$ and -1. For other detecting events, the state of the system is projected into the initial state $\otimes_{j=1}^{N}|\Psi_{\rm in}\rangle_j$ up to a local operation. For example, if D_1 detects two photons, and $D_3, D_5, \ldots, D_{2N-1}$ detect one photon, the state of the system is projected into $\sigma_{1,z} \otimes_{j=1}^{N} |\Psi_{\rm in}\rangle_j$, which can be transformed into the initial state by local operation. Thus, based on the measurement result, we can switch on the atom-cavity interaction and repeat the above procedure until success in realizing the parity measurement of N atoms.

Conclusion. In summary, a repeat-until-success scheme was proposed to implement the parity measurement of N distant atoms by combining cavity QED and linear optical elements. Instead of direct interaction between atoms, quantum interference of polarized photons is used to realize expected operation. The scheme is insensitive to some practical quantum noise, and can be directly used to realize the parity measurement of N distant atoms, which is important for reducing the complexity of physical realization of some practical quantum computation and quantum algorithms.

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[1] P. Shor, in *Proceedings of the 35th Annual Symposium on the Foundation of Computer Science* (IEEE Comp. Soc. Press, Los Alomitos, CA, 1994), p. 124; L. K. Grover, Phys. Rev. Lett. **80**, 4329 (1998).
[2] M. L. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[3] D. Gottesman and I. Chuang, Nature (London) **402**, 390 (1999).
[4] E. Knill, R. Laflamme, and G. Milburn, Nature (London) **409**, 46 (2001).
[5] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001); R. Raussendorf and H. J. Briegel, *ibid.* **86**, 5188 (2001).
[6] C. W. J. Beenakker, D. P. DiVincenzo, C. Emary, and M. Kindermann, Phys. Rev. Lett. **93**, 020501 (2004); K. Nemoto and W. J. Munro, *ibid.* **93**, 250502 (2004); A. Kolli, B. W. Lovett, S. C. Benjamin, and T. M. Stace, *ibid.* **97**, 250504 (2006).
[7] T. P. Bodiya and L. M. Duan, Phys. Rev. Lett. **97**, 143601 (2006); R. Ruskov and A. N. Korotkov, Phys. Rev. B **67**, 241305 (2003); B. Trauzettel, A. N. Jordan, C. W. J. Beenakker, and M. Büttiker, *ibid.* **73**, 235331 (2006); R. Ionicioiu, Phys. Rev. A **75**, 032339 (2007); N. S. Williams and A. N. Jordan, *ibid.* **78**, 062322 (2008); G. Haack, H. Förster, and M. Büttiker, Phys. Rev. B **82**, 155303 (2010).
[8] C. R. Myers, M. Silva, K. Nemoto, and W. J. Munro, Phys. Rev. A **76**, 012303 (2007); R. Ionicioiu, A. E. Popescu, W. J. Munro, and T. P. Spiller, *ibid.* **78**, 052326 (2008).
[9] W. Pfaff *et al.*, Nat. Phys. **9**, 29 (2013); D. Riste *et al.*, Nature (London) **502**, 350 (2013).
[10] H. Mabuchi *et al.*, Science **298**, 1372 (2002); H. Walther *et al.*, Rep. Prog. Phys. **69**, 1325 (2006).
[11] C. Cabrillo, J. I. Cirac, P. García-Fernández, and P. Zoller, Phys. Rev. A **59**, 1025 (1999); S. Bose, P. L. Knight, M. B. Plenio, and V. Vedral, Phys. Rev. Lett. **83**, 5158 (1999); X.-L. Feng, Z.-M. Zhang, X.-D. Li, S.-Q. Gong, and Z.-Z. Xu, *ibid.* **90**, 217902 (2003); L.-M. Duan and H. J. Kimble, *ibid.* **90**, 253601 (2003); D. E. Browne, M. B. Plenio, and S. F. Huelga, *ibid.* **91**, 067901 (2003); C. Simon and W. T. M. Irvine, *ibid.* **91**, 110405 (2003).
[12] C. D. Fidio *et al.*, J. Opt. B **5**, 105 (2003); X. B. Zou, K. Pahlke, and W. Mathis, Phys. Rev. A **68**, 024302 (2003); **69**, 052314 (2004); Y.-F. Xiao, X.-B. Zou, and G.-C. Guo, *ibid.* **75**, 012310 (2007).
[13] S. D. Barrett and P. Kok, Phys. Rev. A **71**, 060310 (2005).
[14] S. C. Benjamin *et al.*, New J. Phys. **7**, 194 (2005).
[15] I. E. Protsenko, G. Reymond, N. Schlosser, and P. Grangier, Phys. Rev. A **66**, 062306 (2002); X. B. Zou and W. Mathis, *ibid.* **71**, 042334 (2005).
[16] Y. L. Lim, A. Beige, and L. C. Kwek, Phys. Rev. Lett. **95**, 030505 (2005).
[17] T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **75**, 3788 (1995).
[18] K. M. Gheri, C. Saavedra, P. Törmä, J. I. Cirac, and P. Zoller, Phys. Rev. A **58**, R2627 (1998); C. Saavedra, K. M. Gheri, P. Torma, J. I. Cirac, and P. Zoller, *ibid.* **61**, 062311 (2000).
[19] F. Schmidt-Kaler *et al.*, Nature (London) **422**, 408 (2003); Ch. Roos *et al.*, Phys. Rev. Lett. **83**, 4713 (1999); A. B. Mundt *et al.*, *ibid.* **89**, 103001 (2002); M. Keller *et al.*, Nature (London) **431**, 1075 (2004); New J. Phys. **6**, 95 (2004).
[20] In the past decade, there has been great experimental progress in increasing the cavity quality. See the following papers: C. Sames, H. Chibani, C. Hamsen, P. A. Altin, T. Wilk, and G. Rempe, Phys. Rev. Lett. **112**, 043601 (2014); M. Khudaverdyan *et al.*, New J. Phys. **10**, 073023 (2008); A. Roy and M. D. Barrett Appl. Phys. Lett. **99**, 171112 (2011). In particular, cavity finesse in the order of a million has been reported in experiments.
[21] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998).
[22] B. Zeng, D. L. Zhou, and L. You, Phys. Rev. Lett. **95**, 110502 (2005).