Effects of dipole-dipole interaction on the transmitted spectrum of two-level atoms trapped in an optical cavity

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The transmission spectrum of two dipole-coupled atoms interacting with a single-mode optical cavity in the strong coupling regime is investigated theoretically for the lower and higher excitation cases. The dressed states containing the dipole-dipole interaction (DDI) are obtained by transforming the two-atom system into an effective single-atom system. We find that the DDI can enhance the effects resulting from the positive atom-cavity detunings but weakens them for the negative detuning cases for the lower excitation, which causes the spectrum to exhibit two asymmetric peaks with shifted heights and positions. For the higher excitation cases, the DDI augments the atomic saturation and leads to the deforming of the spectrum. Furthermore, the large DDI can cause the atom and the cavity to decouple, producing a singlet of the normal-mode spectrum.

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I. INTRODUCTION

Realization of the strong coupling regime (SCR) in cavity quantum electrodynamics has created new fields of exploration in quantum mechanics [1,2]. SCR is very attractive, because photons emitted by atoms inside the cavity mode can be reabsorbed and reemitted and leads to Rabi oscillations [3], which gives rise to a normal-mode splitting in the eigenvalue spectrum of the atom-cavity system [4–6]. This normal-mode splitting has been observed with atomic beams passing through a cavity in both the microwave regime [7–9] and optical regime [10], as well as with trapped atoms in optical cavities [11–14]. During an experiment, the normal-mode splitting is detected by probing the transmitted spectrum at a low excitation. With increased excitation, the spectrum presents hysteresis, and subsequently forms a closed structure. When the atoms are saturated, they decouple from the cavity and only a single peak appears in the spectrum [15,16].

However, few studies have considered the DDI, even though several studies have explored atom-cavity systems with DDIs [17–23] because of their long-range and anisotropic nature. In fact, the DDI can profoundly affect the light absorption characteristics and lead to a shift of the atomic energy levels [24]. The renormalization of the atomic resonance frequency caused by the DDI can result in the optical bistability of the atomic system [25,26]. Thus, it is natural to expect that the transmitted spectrum may present several novel characteristics caused by the DDI. Fortunately, substantial progress has been made on solid materials [27,28] and ultracold atoms [29–32] in recent years, which have proven to be a good platform for the study of the DDI. In this study we go one step further and investigate the transmitted spectrum of two two-level atomic systems with one excited and the other in the ground state trapped and strongly coupled to an optical resonator. The two two-level atoms can interact with each other through photon exchange. A photon emitted by the excited atom could be absorbed by the other atom in the ground state. This kind of energy exchange results in an "interaction" between two two-level atoms, which is equivalent to the one derived from the dipole term of the classical electromagnetic interaction, therefore named "dipole-dipole" interaction. It is noteworthy that very similar models were widely used recently to discuss other physical effects, namely photon entanglement and other correlation phenomena, because this simplified situation is sufficient to illustrate the main aspects of the way in which a cavity environment can modify the nature of collisions. In our study, the behavior of the spectrum in equilibrium is studied for a wide range of DDI intensities and atom-cavity detunings. The relation and distinction of their effects on the spectrum both in the weak excitation limit and for higher excitations are both explored.

The paper is organized as follows: Sec. II presents the theoretical model under consideration and provides the steady-state solution by solving the main equation. Section III is devoted to the study of the transmitted spectrum in the weak excitation limit. Section IV describes the structural characteristics of the transmitted spectrum for a strong driving intensity. The effects of both the detuning and the DDI on the spectrum are discussed. Finally, we present our conclusions in Sec. V.

II. MODEL

We consider two identical dipole-coupled two-level atoms interacting with a single-mode high-finesse optical cavity (Fig. 1). The system is pumped along the cavity axis by a coherent laser field of frequency ω_p and an effective amplitude η. The Hamiltonian for the system in the rotating wave and electric dipole approximations is given by [23]

$$H = -\Delta_c a^\dagger a - \sum_{k=1}^{2}[\Delta_a \sigma_k^\dagger \sigma_k - g(a^\dagger \sigma_k + a\sigma_k^\dagger)] + J(\sigma_1\sigma_2^\dagger + \sigma_2^\dagger \sigma_1) + \eta(a + a^\dagger), \tag{2.1}$$

where $\Delta_c = \omega_p - \omega_c$ and $\Delta_a = \omega_p - \omega_a$. ω_c and ω_a are the resonance frequencies of the cavity field and the atoms, respectively. $a^\dagger$ and a are the field creation and annihilation operators, and $\sigma_k^\dagger$ and σ_k represent the raising and lowering

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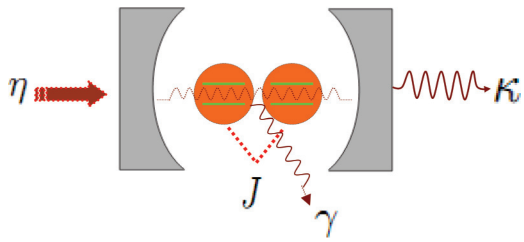

FIG. 1. (Color online) Schematic representation of two two-level atoms with dipole-dipole interaction intensity J interacting with a single-mode high-finesse cavity. The decay rates of atoms and cavity field are γ and κ, respectively. The cavity is pumped by a coherent laser field with strength η.

operators of the atom k ($k = 1,2$). The first term of the Hamiltonian (2.1) is the free Hamiltonian of the cavity. The atomic free Hamiltonian and the interaction Hamiltonian of the atoms and the cavity with coupling strength g are shown in the second term. The third term describes the DDI between atoms and the last term is the pump field Hamiltonian. The DDI is defined in the form [18]

$$J = \tfrac{3}{4}\left(\Gamma_0 c^3/\omega_a^3 r^3\right)(1 - 3\cos^2\varphi), \tag{2.2}$$

where r is the distance between the atoms and φ is the atomic dipole moments with respect to the interatomic axis. Γ_0 denotes the atomic spontaneous emission rate in free space. Here we assume the dipole moments of the two atoms are parallel to each other and are polarized in the direction perpendicular to the interatomic axis. Thus, $\cos\varphi = 0$, and the DDI intensity only depends on the positions of the two atoms in the cavity. Dissipation results from an excitation's spontaneous emission, and cavity photonic leakage can be taken into account within the master quantum equation of the density matrix ρ. It is expressed in the usual Lindblad form in the Born-Markov approximation ($\hbar = 1$) [33]:

$$\begin{aligned}
\dot{\rho} &= -i[H,\rho] + L_\kappa\rho + L_\gamma\rho + L_{\gamma'}\rho,\\
L_\kappa\rho &= \kappa[2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a],\\
L_\gamma\rho &= \sum_{k=1}^{2}\gamma(2\sigma_k\rho\sigma_k^\dagger - \sigma_k^\dagger\sigma_k\rho - \rho\sigma_k^\dagger\sigma_k),\\
L_{\gamma'}\rho &= \gamma'(2\sigma_1\rho\sigma_2^\dagger - \sigma_1^\dagger\sigma_2\rho - \rho\sigma_1^\dagger\sigma_2)\\
&\quad + \gamma'(2\sigma_2\rho\sigma_1^\dagger - \sigma_2^\dagger\sigma_1\rho - \rho\sigma_2^\dagger\sigma_1).
\end{aligned} \tag{2.3}$$

Here the nonunitary parts $L_\kappa\rho$, $L_\gamma\rho$, and $L_{\gamma'}\rho$ describe the coupling of the field mode and the atoms to the environment. The coefficients κ and γ are the decay rates of the cavity field and the atoms, respectively. The atom-atom cooperation induced by their coupling with a common reservoir is given by γ' [18,21,34] and is important only when the atomic distances are small relative to the radiation wavelength.

The time evolution of the operator's expectation values for the atom-cavity system can be obtained with the master equation

$$\begin{aligned}
\langle\dot{a}\rangle &= i(\tilde{\Delta}_c\langle a\rangle - g\langle\sigma_1\rangle - g\langle\sigma_2\rangle - \eta),\\
\langle\dot{\sigma}_1\rangle &= i(\tilde{\Delta}_a\langle\sigma_1\rangle + g\langle a\sigma_{1z}\rangle + \tilde{J}\langle\sigma_1\rangle\langle\sigma_{2z}\rangle),\\
\langle\dot{\sigma}_2\rangle &= i(\tilde{\Delta}_a\langle\sigma_2\rangle + g\langle a\sigma_{2z}\rangle + \tilde{J}\langle\sigma_2\rangle\langle\sigma_{1z}\rangle),\\
\langle\dot{\sigma}_{1z}\rangle &= 2ig(\langle a^\dagger\sigma_1\rangle - \langle a\sigma_1^\dagger\rangle) - 2\gamma(1 + \langle\sigma_{1z}\rangle),\\
\langle\dot{\sigma}_{2z}\rangle &= 2ig(\langle a^\dagger\sigma_2\rangle - \langle a\sigma_2^\dagger\rangle) - 2\gamma(1 + \langle\sigma_{2z}\rangle),
\end{aligned} \tag{2.4}$$

where $\tilde{\Delta}_a = \Delta_a + i\gamma$, $\tilde{\Delta}_c = \Delta_c + i\kappa$, and $\tilde{J} = J - i\gamma'$. Note that, in the equation of motion (2.4), a decorrelation between operators of the atoms can be used to explain the splitting classically, and only mean value correlations of concern, i.e., correlations of the operators, are lost in this picture [14,35]. In addition, the splitting here is different than with linear dispersion theory. The normal-mode resonances are a consequence of the phase shift and depend on both the cavity length and the refractive index caused by the presence of the atom, i.e., the interplay between the cavity field and the atom is nonlinear, which is in contrast to the nonlinear atom-atom interaction. The steady-state solutions of the nonlinear Eq. (2.4) can be obtained by setting $\langle\dot{a}\rangle = \langle\dot{\sigma}_1\rangle = \langle\dot{\sigma}_2\rangle = \langle\dot{\sigma}_{1z}\rangle = \langle\dot{\sigma}_{1z}\rangle = 0$.

When $g \gg (\gamma,\kappa)$, the atom-cavity system reaches a strong coupling regime. The new eigenstates of the system are described by the dressed states, which are linear combinations of pairs of bare atom states and cavity field states. However, it is difficult to determine the dressed states of the atom-cavity system with two dipole-dipole coupled atoms. In fact, when the excitation of the atoms is very low, we can adopt the methods in [18] and simplify the two-atom system to an effective single-atom system. The effective form of Hamiltonian H in Eq. (2.1) can then be written as

$$\begin{aligned}
H_{\text{eff}} &= -\Delta_c a_1^\dagger a - (\Delta_a - J)\sigma_1^\dagger\sigma_1\\
&\quad + \sqrt{2}g(a^\dagger\sigma_1 + a\sigma_1^\dagger) + \eta(a + a^\dagger).
\end{aligned} \tag{2.5}$$

In the transformed Hamiltonian, the dipole-coupled atoms are denoted by two fictitious atoms. Only one of them couples to the field mode with frequencies $\omega_a + J$ and an effective coupling strength $\sqrt{2}g$, but the other atom freely evolves decoupling from the field. As a result, the dressed states of the transformed system are similar to that of the single-atom system

$$\begin{aligned}
&|0\rangle = |g\rangle|0\rangle, \quad |n_-\rangle = \sin\frac{\theta_n}{2}|e,n-1\rangle - \cos\frac{\theta_n}{2}|g,n\rangle,\\
&|n_+\rangle = \cos\frac{\theta_n}{2}|e,n-1\rangle + \sin\frac{\theta_n}{2}|g,n\rangle,
\end{aligned} \tag{2.6}$$

where $\sqrt{n}$ is a photon number state, $\theta_n = \arctan 2\sqrt{2}g\sqrt{n}/(\Delta + J)$, and $\Delta = \omega_a - \omega_c$ is the detuning between atom and field. The corresponding eigenenergies are

$$\begin{aligned}
E_0 &= 0,\\
E_{n\pm} &= \omega_c + \frac{\Delta + J}{2} \pm \frac{1}{2}\sqrt{(\Delta + J)^2 + 8g^2 n}.
\end{aligned} \tag{2.7}$$

The spectrum of the first doublet of these states in a degenerate system (for $\omega_a = \omega_c$) splits into two new resonances and is called the normal-mode or vacuum-Rabi splitting. The observation of the normal-mode splitting, in fact, is also an indicator that a system has reached the SCR of cavity QED.

To investigate the steady-state normal-mode spectrum, we introduce the intracavity photon number [35]

$$\langle a^{\dagger}a\rangle_0 = |\langle a\rangle_0|^2, \tag{2.8}$$

which is given by the modulus square of $\langle a\rangle_0$ and is sufficient to calculate a spectrum of the coupled atoms-cavity system.

III. NORMAL-MODE SPECTRUM IN THE LOW EXCITATION LIMIT

By choosing an appropriate pump beam, we can maintain a weak pump intensity and, thus, a low atomic excitation can be achieved. In this situation, $\langle\sigma_{1z}\rangle,\langle\sigma_{2z}\rangle \to -1$, and, thus, we can set $\langle a\sigma_{1z}\rangle = \langle a\sigma_{2z}\rangle = -\langle a\rangle$. The steady-state solution of Eq. (2.4) and the steady-state intracavity photon number can then be given as

$$\langle a\rangle_0 = \frac{\eta}{\tilde{\Delta}_c}\frac{1}{1-v}, \tag{3.1}$$

$$\langle\sigma_1\rangle_0 = \frac{\eta v}{g}\frac{1}{1-v}, \tag{3.2}$$

$$\langle\sigma_2\rangle_0 = \frac{\eta v}{g}\frac{1}{1-v}, \tag{3.3}$$

$$\langle a^{\dagger}a\rangle_0 = \frac{\eta^2}{|\tilde{\Delta}_c|^2}\frac{1}{|1-v|^2}, \tag{3.4}$$

where

$$v = \frac{2g^2}{\tilde{\Delta}_c[\tilde{\Delta}_a - \tilde{J}]}. \tag{3.5}$$

These results are based on the classical approximation [35], which converts the fermionic commutation relation to a bosonic form, treating both atoms and fields as linear harmonic oscillators and omitting the effects of saturation.

The two normal-mode resonances are characterized by the eigenfrequencies $\omega_{\pm}$,

$$\omega_{\pm} = -\tfrac{1}{2}(\tilde{\Delta}_a - \tilde{J} + \tilde{\Delta}_c) \pm \tfrac{1}{2}\sqrt{8g^2 + (\tilde{\Delta}_a - \tilde{J} - \tilde{\Delta}_c)^2}, \tag{3.6}$$

which are based on Eq. (2.5). The frequencies $\omega_{\pm}$ have complex values. The real part $\mathrm{Re}(\omega_{\pm})$ determines the position of the resonances, while the imaginary part describes their widths. However, in the strong-coupling regime, $g \gg (\gamma,\gamma',\kappa)$, and, thus, the effects of the decay on the position of the resonances can be neglected. The resonance frequencies $\omega_{\pm}$, in this condition, match the first pair of dressed states in Eq. (2.7). For $\Delta = J = 0$, the distance between the two resonances has a minimum value $\omega_+ - \omega_- \approx 2\sqrt{2}g$. When Δ and J are nonzero, their position and distance can be obtained from Eq. (2.7).

Because the two-atom system, to some extent, is analogous to the one-atom system, we cite the parameter's value in experiments [36], in which a photon blockade for the light transmitted by an optical cavity containing one trapped atom is observed. The decay rate γ' caused by the DDI is usually weak; therefore, without a loss of generality, we take it as $0.05g$ in this section.

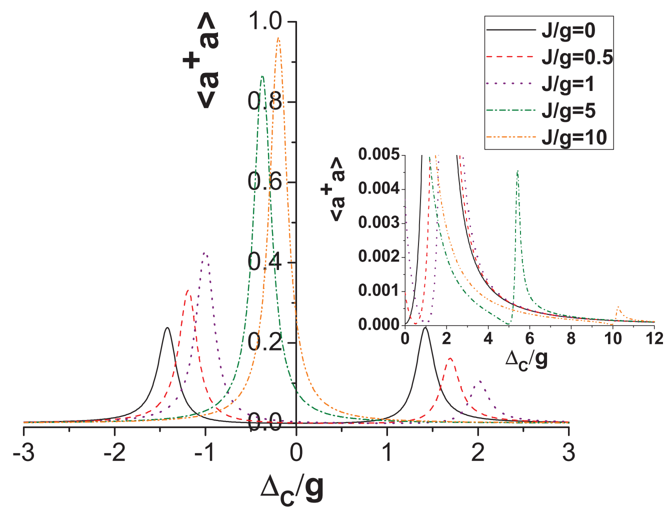

FIG. 2. (Color online) The normal-mode spectrums for different DDI intensities is shown. The parameters are $\Delta = 0$, $(\eta,\kappa,\gamma,\gamma') = (0.12,0.12,0.0767,0.05)g$.

In Fig. 2 the normal-mode spectrums for different DDI intensities is plotted. From Fig. 2 we see that when $J = 0$, the amplitudes of both resonances are equal and their position are symmetric about $\Delta_c = 0$ with a minimum distance of approximately $2\sqrt{2}g$. However, when $J \neq 0$, as the DDI intensity increases, the left peak becomes higher and approaches $\Delta_c = 0$, while the height of the right peak is greatly reduced and it departs from $\Delta_c = 0$. Moreover, the distance between the two peaks tends to increase. This is because for $\Delta = 0$, $J \neq 0$, and the distance between the two peaks is in accord with the expression $\sqrt{J^2 + 8g^2}$, which is a monotonically increasing function of J. In addition, their heights are determined by the value of $\sin\frac{\theta_n}{2}$ and $\cos\frac{\theta_n}{2}$ in Eq. (2.6). When $\Delta = J = 0$ the contributions from the atoms and the cavity states are equal in such a way that the normal modes have the same height. As the DDI intensity increases, the excitation probability of

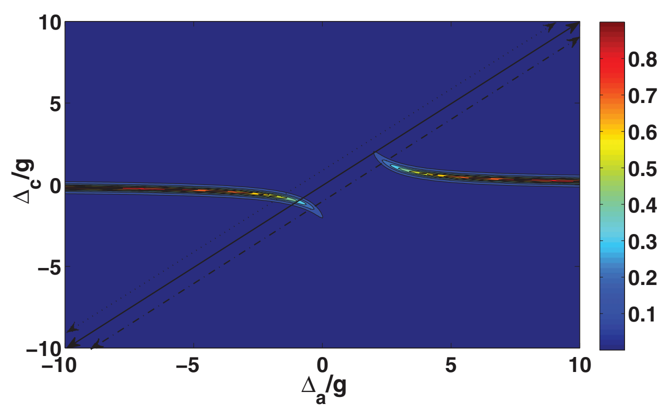

FIG. 3. (Color online) The normal modes form an avoided crossing between the resonances of the bare atoms and the bare cavity. Three different cases of $\Delta/g = 1$ (dashed-dotted), $\Delta = 0$ (solid), and $\Delta/g = -1$ (dotted) are shown. The dipole-dipole interaction strength is taken as $J/g = 1$. The parameters are $(\eta,\kappa,\gamma,\gamma') = (0.12,0.12,0.0767,0.05)g$.

the bare-cavity field state is enhanced for the lower dressed state $|1_-\rangle$, while for the bare atom states it is greatly reduced. The results are the opposite of what occurs for the higher state $|1_+\rangle$. It should be noted that the system is pumped by a coherent laser shining on one of the cavity mirrors, and, thus, the bare cavity states are more easily excited, leading to a better visibility of the "cavitylike" peak. However, when $J \gg g$, the probabilities $\sin^2(\frac{\theta_n}{2}) \approx 0$, $\cos^2(\frac{\theta_n}{2}) \approx 1$; therefore, the system is approximately in the state $|g,1\rangle$. The atoms are not being excited in this case, and the spectrum exhibits a single peak.

In Fig. 3 the normal-mode spectra are found to be functions of Δ_a and Δ_c. The atom-cavity detuning is defined as $\Delta = \omega_a - \omega_c$. An avoided crossing between the atomic transition and the resonant frequency of the cavity is shown. With increasing Δ_a and Δ_c, the atom-cavity system decouples, and the two resonances asymptotically approach the eigenfrequencies of the atoms and the cavity.

In Fig. 4, when $\Delta \neq 0$, both the position and height of the two resonances shift. In contrast, based on Eq. (2.7), when $J = 0$, $\Delta \neq 0$, and it is obvious that the position of the two peaks are dependent on the value of Δ, and the distance between them depends on $\sqrt{\Delta^2 + 8g^2}$. Moreover, the excitation of a dressed state is determined by the contribution of the cavity state to the dressed state. For positive detuning, based on Eq. (2.6), the excitation probability of the bare cavity field state of $|1_-\rangle$ increases with increasing Δ, but for $|1_+\rangle$, it reduces. In contrast, the results are opposite for negative detuning. Therefore, when $\Delta \neq 0$, the two resonances more likely to exhibit the cavitylike form with enlarged separation.

Interestingly, as seen in Figs. 2 and 4, this indicates that the DDI plays a similar role as does the positive detuning. The effect of the DDI is equivalent to increasing the positive detunings and decreasing the negative detunings. To confirm this conclusion, the cooperative action of Δ and J on the two resonances is shown in Fig. 5. This shows that the two resonances are symmetric with equal heights when $\Delta + J = 0$. For $\Delta > 0$, Δ and J have consistent effects on the two peaks. However, for $\Delta < 0$, their influences cancel each other, and the practical states rely on the larger absolute value of the two. In fact, these results are clear. Both for the dressed states in Eq. (2.6) and the corresponding eigenenergies in Eq. (2.7), Δ and J are present with the form "$\Delta + J$." Therefore, the position and height of the two peaks, as well as the distance between them, all depend on the cooperative action of $\Delta + J$.

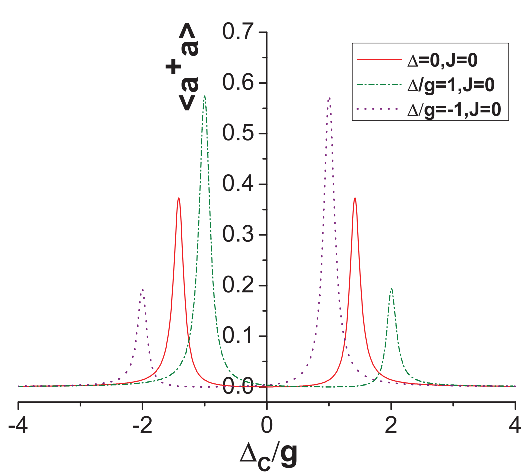

FIG. 4. (Color online) The normal-mode spectrums for different values of Δ and J are plotted. There is no dipole-dipole interaction between atoms, that is $J = 0$, $\gamma' = 0$.

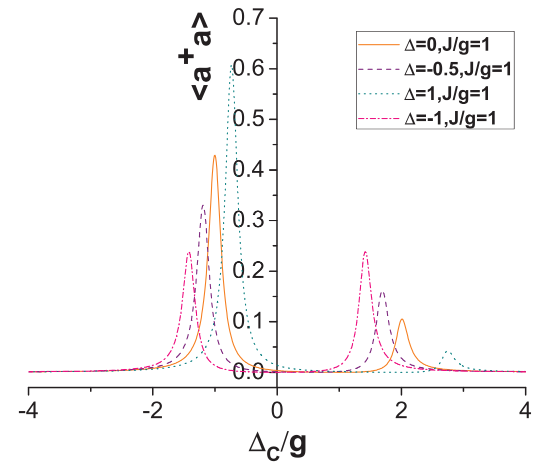

FIG. 5. (Color online) The normal-mode spectrums for different values of Δ and J are plotted. Both the detuning and the dipole-dipole interaction are considered. The dipole-dipole interaction intensity is taken as $J/g = 1$. Other parameters are $(\eta,\kappa,\gamma,\gamma') = (0.12,0.12,0.0767,0.05)g$.

IV. TRANSMISSION SPECTRUM FOR HIGHER PUMP INTENSITY

We have noted that the validity of the above results depends on the assumption of weak excitation. For higher pump intensities, the atomic saturation cannot be neglected. We can define $\langle\sigma_z\rangle_0 = -\frac{1}{1+s_0}$, and treat the cavity field classically by replacing a with $\langle a\rangle$. Therefore, $\langle a\sigma_z\rangle$ can be written as a product form $\langle a\rangle\langle\sigma_z\rangle$, and the steady state of Eq. (2.4) can be calculated:

$$\langle a\rangle_0 = \frac{\eta}{\tilde{\Delta}_c}\frac{1}{1-\mu}, \tag{4.1}$$

$$\langle\sigma\rangle_0 = \frac{\eta v}{g}\frac{1}{1-\mu}, \tag{4.2}$$

$$\mu = \frac{2g^2}{\tilde{\Delta}_c[\tilde{\Delta}_a(1+s_0) - \tilde{J}]}, \tag{4.3}$$

$$s_0 = \frac{2g^2(1+s_0)^2\langle a^\dagger a\rangle_0}{|\tilde{\Delta}_a(1+s_0) - \tilde{J}|^2} + \frac{2g^2(1+s_0)\langle a^\dagger a\rangle_0\gamma'}{|\tilde{\Delta}_a(1+s_0) - \tilde{J}|^2\gamma}, \tag{4.4}$$

where s_0 is the saturation parameter. Notice that $s_0 \to 0$ corresponds to the low saturation limit.

In Fig. 6, by using Eqs. (4.1)–(4.4), the spectrum for the DDI atoms with increased pump intensity is plotted. This shows similar behavior as the system without the DDI [15]. These results are not surprising, because the two dipole-coupled atoms can be treated as an effective atom with a renormalized atomic frequency and atom-cavity coupling intensity. As the pump intensity increases, atoms begin to saturate, and the peaks of the two resonances shift their position and deform,

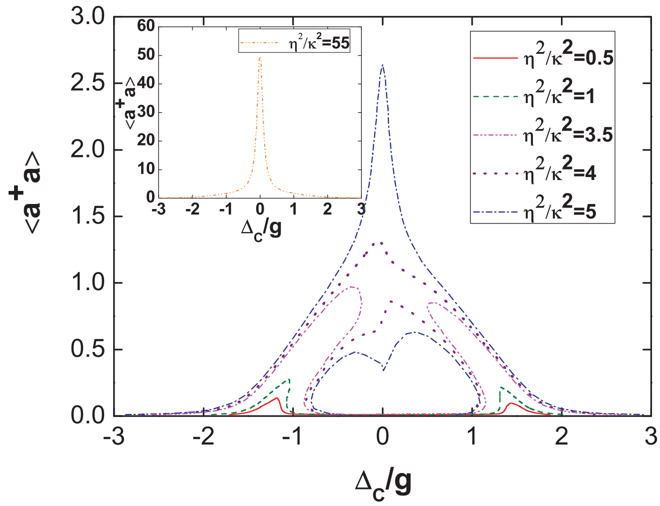

FIG. 6. (Color online) The normal-mode structure is revealed for different pump intensities. With increase of pump intensities, atoms tend to saturation, and the two peaks bend towards the center. In the limit of the high excitation, the atoms are saturated and the spectrum shows a single peak, as is shown in the inset. $J = 0.5g$, $\kappa = \gamma = 0.1g$, $\Delta = 0$, $\gamma' = 0.01g$.

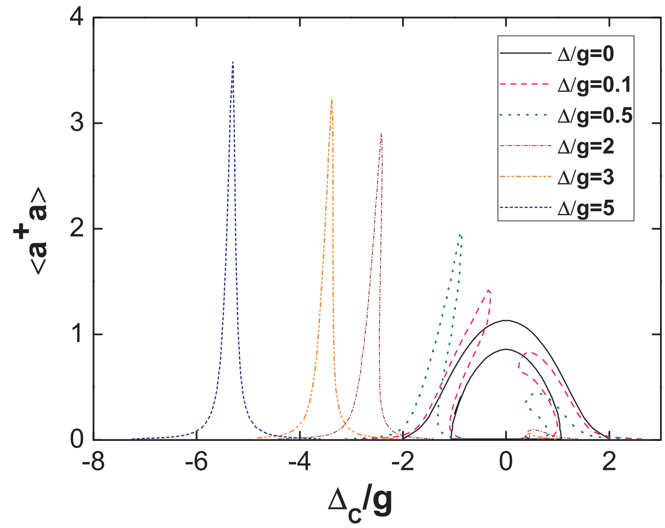

FIG. 7. (Color online) The effects of the atom-cavity detuning on the normal-mode structure of high excitation. The spectrum deformed and changes to a singlet in the limit of large detuning. $\eta^2/\kappa^2 = 4$, $J = 0$, $\kappa = \gamma = 0.1g$, $\gamma' = 0.1g$.

bending towards the center. Finally, they meet and form a closed structure. It is important to note that there are three possible expectation values for the operator $\langle a^\dagger a\rangle_0$ when $\eta^2/\kappa^2 \geqslant 1$. One of them is unstable, but the other two are stable. The amplitude of the intracavity field can switch between the two stable values, which is called bistability, and this predicts a nonlinear relation between the input and output intensity. The system, in this case, evolves from two coupled harmonic oscillators to highly deformed anharmonic oscillators. In the high excitation limit, as is shown in the inset, the atoms are saturated and do not contribute significantly to the dynamics of the system. The spectrum resembles that of an empty cavity, evolving from two peaks to a singlet. However, the spectrum no longer exhibits the same symmetry as the system without the DDI. Because we introduce the DDI into system, the shape of these curves becomes asymmetrical about $\Delta_c = 0$.

In the weak excitation limit, the DDI influences both the height and position of the two peaks and produces effects similar to the positive detunings. Therefore, for higher excitations, the effects of Δ and J on the spectrum are also worth exploring. In the following section we begin from the spectrum with a closed structure for higher excitations to study their effects.

In Fig. 7 an interesting phenomenon arises with increased atom-cavity detuning. The original closed spectrum begins to separate and splits into two peaks. This can be inferred from Eqs. (4.1) and (4.4). Based on these two eqnarrays, we can find

$$s_0 = \frac{2g^2\eta^2(1+s_0)^2\left(1+\frac{\gamma'}{\gamma}\right)}{|\tilde{\Delta}_c[(\Delta_c - \Delta + i\gamma)(1+s_0) - \tilde{J}] - 2g^2|^2}. \quad (4.5)$$

When $J = 0$, with increasing Δ, the atomic saturation parameter s_0 decreases. It is not difficult to understand that the detuning can cause the atom-cavity coupling to become weaker and thus reduces the atomic saturation. Therefore, with increasing Δ, the spectrum gradually returns to the cases of weaker excitation. When $\Delta/g \simeq 5$ the right peak nearly vanishes, while the right peak becomes more distinct. The system, in this case, is mainly dominated by Δ and shows a decoupled tendency.

However, as seen in Fig. 8, the spectrum as a function of J for the higher excitation exhibits a different behavior. The original closed spectrum only deforms, but it does not separate. This result maybe explained as follows. As mentioned above, the DDI can cause the atomic frequency to be renormalized and changes the atom-cavity coupling intensity from g to $\sqrt{2}g$. According to Eq. (4.5), s_0 increases with increasing J for

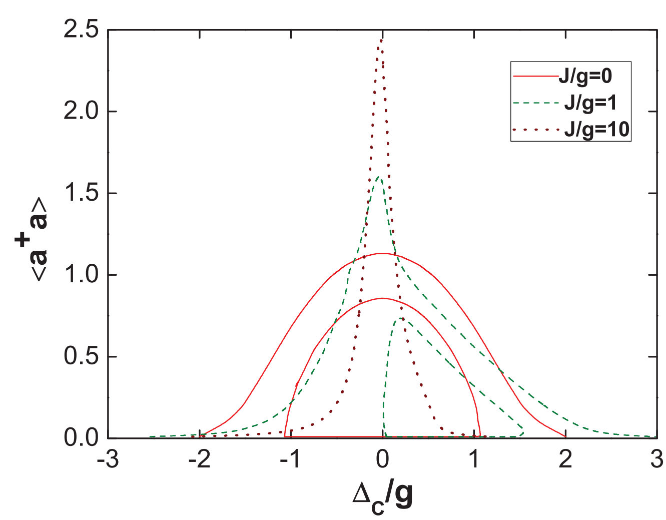

FIG. 8. (Color online) The effects of dipole-dipole interaction on the normal-mode structure of high excitation. The spectrum deformed and changes to a singlet in the limit of high dipole-dipole interaction intensity. $\eta^2/\kappa^2 = 4$, $\Delta = 0$, $\kappa = \gamma = 0.1g$, $\gamma' = 0.1g$.

$\Delta = 0$. For high excitations, the renormalization of the atomic frequency resulting from the small DDI does not generate pronounced influences on the excitation of the system and the spectrum only deforms in this case. However, for cases where $J \gg g$, a large atom-cavity detuning forms because of the renormalization of the atomic frequency and plays a dominant role in the atom-cavity system. Photons coming from a cavity mirror cannot populate the atomic states, making the spectrum exhibit a singlet.

V. CONCLUSION

We have characterized the transmission spectrum properties of two dipole-coupled two-level atoms strongly coupled to a single-mode optical cavity. In the low excitation limit, the DDI, acting as an atom-cavity detuning, can modify the positions and heights of the two peaks. However, the DDI exhibits effects similar to a positive detuning, and exhibits the opposite behavior of negative detuning. The dressed states have also been derived by transforming the two-atom system to an effective single-atom system. For higher excitation, the atom-cavity detuning can reduce the atomic saturation, making the original closed structure separate. The DDI can increase the atomic saturation, leading to the deforming of the original closed structure. Interestingly, except for in the high excitation and large detuning limit, the strong DDI can cause the decoupling of the atoms and cavity, which leads to the spectrum exhibiting a singlet. We expect that these results will be useful in understanding the quantum electrodynamics of the atom-cavity system with DDI.

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