Transient Cherenkov radiation from an inhomogeneous string excited by an ultrashort laser pulse at superluminal velocity

R. M. Arkhipov

Weierstrass Institute for Applied Analysis and Stochastics, Leibniz Institute in Forschungsverbund Berlin, Mohrenstrasse 39, 10117 Berlin, Germany and Faculty of Physics, St. Petersburg State University, Ulyanovskaya 1, Petrodvoretz, St. Petersburg 198504, Russia

I. Babushkin

Institute of Mathematics, Humboldt University of Berlin, Rudower Chaussee 25, 12489 Berlin, Germany

M. K. Lebedev, Yu. A. Tolmachev, and M. V. Arkhipov

Faculty of Physics, St. Petersburg State University, Ulyanovskaya 1, Petrodvoretz, St. Petersburg 198504, Russia

(Received 9 October 2013; published 9 April 2014)

The optical response of a one-dimensional string made of dipoles with a periodically varying density excited by a spot of light moving along the string at superluminal (subluminal) velocity is studied theoretically. The Cherenkov radiation in such a system is rather unusual, possessing both transient and resonant character. We show that, under certain conditions, in addition to the resonant Cherenkov peak, another Doppler-like frequency appears in the radiation spectrum. Both linear (small-signal) and nonlinear regimes as well as different string topologies are considered.

DOI: 10.1103/PhysRevA.89.043811 PACS number(s): 42.25.Fx, 41.60.Bq, 42.25.Hz, 42.65.Ky

I. INTRODUCTION

The problem of superluminal motion and its existence in nature has attracted the attention of various researchers for rather a long time (see [1–12] and references therein). At the turn of the 19th to the 20th century, Heaviside [1] and Sommerfeld [2] considered radiation from charged particles moving in vacuum at a velocity greater than the velocity of light in vacuum, c. However, their work was forgotten for many years because the special theory of relativity "bans" such motion. Further analysis has shown that only those motions that involve signal (information) transfer at the superluminal velocity are "prohibited"; this strong restriction is related to the violation of the causality principle [3,4,11,13].

If a charged particle moves faster than light in some medium, so-called Cherenkov radiation occurs. It is emitted typically into a cone with the angle depending on the ratio of the particle velocity and the speed of light in the medium. Similar conical emission can appear also in nonlinear optical parametric processes [14–18]. Not only particles but also spots of light can propagate faster than the phase velocity of light in a particular medium [10,19–23]. Those can be optical pulses and solitons in fibers or filaments [24–31] as well as in other optical systems [32–41]. Cherenkov radiation in various periodically modulated media, with modulations in both space [32,42,43] and time [44], has also been considered. The intersection point of two wave fronts can also move at a velocity exceeding that of light [10,45]. A similar situation occurs when a short plane-wave pulse crosses a flat screen (or a plane diffraction grating) [10,42]. In this case, the intersection of the pulse and the screen moves along the screen at the velocity $V = c/\sin\beta > c$ (here β is the angle of wave incidence) [46,47].

Despite the great number of different configurations studied in the context of Cherenkov radiation, in all those cases such radiation has the same nature. Namely, it is a result of interference of the secondary waves emitted by the moving "particle."

The temporal shape of the radiating wave and thus the spectrum of radiation can be significantly different depending on the particular situation. For instance, the spectrum of radiation induced by a charged particle moving faster than the phase velocity of light is rather unstructured [3,4]. In many other cases, resonances may occur [10,42]. One important example is the so-called Purcell-Smith radiation [42,48] appearing as a charged particle moves in the vicinity of a periodic structure. Also, a moving and at the same time oscillating dipole emits Cherenkov radiation characterized by a well-defined resonance [38], and a similar situation is realized for optical soliton propagation [49].

In the present article, we consider in detail the Cherenkov-type radiation in the case of a one-dimensional (1D) string formed of two-level atoms with a spatially periodic modulated number density. This system is excited at a superluminal (subluminal) velocity at the point of intersection of the string with a moving spot of light. This geometry is imposed by recent advances in optical technologies which allow for the reliable control of matter properties on the spatial level of the order of the light wavelength or even much smaller, which allows the creation of quasi-1D objects (see [50,51] and references therein). Although our consideration is rather general, we bear in mind the spatial properties of nanoantenna or quantum dot arrays [50–57] as well as thin microcapillaries. A similar geometry was recently realized experimentally in [22].

As we show, in the system considered here the Cherenkov emission has a somewhat unusual character. It possesses a narrowband spectrum, with the central frequency at the resonance of the dipoles of which the string is comprised. In the presence of inhomogeneities of the dipole density, a second Doppler-like frequency appears in the spectrum even in the linear regime, that is, when the pump remains weak. We show that this effect does not depend on the string geometry by considering both a straight and a circularly shaped string. When the pump power is increased, the nonlinear response of two-level atoms becomes important, and this additional frequency may even significantly exceed the resonant one.

The structure of the paper is as follows: In the Sec. II the system is described and the possible physical realizations are considered; Sec. III describes the linear response of the string whereas in Sec. IV nonlinear dynamics is considered. Concluding remarks are presented in Sec. V.

II. PHYSICAL CONSIDERATIONS

The geometry we would like to consider is illustrated in Fig. 1(a). A short spectrally broadband optical plane-wave pulse is emitted by a source 1, passing through lenses 2 and 3 which makes the pulse spatially extended. The source must produce a significantly broadband and flat spectrum [58–66], which includes also the resonance frequency ω_0 of the dipoles forming the string. This spatially extended pulse, short in time and in the axial direction, has the form of a thin "sheet of light" 4, which illuminates at the angle β the string medium parallel to the z axis.

We also assume that the string consists of oscillators (dipoles) with the resonance frequency ω_0 and decay constant γ, whose number density $N(z)$ varies periodically along the string with the spatial period Λ_z. Moreover, we assume that the string is thin; its thickness is less than the wavelength of light corresponding to the resonance frequency ω_0. Such quasi-1D geometry of the system suggests that, if the angles β and φ are not zero [cf. Fig. 1(a)], the secondary radiation emitted by any dipole will never hit another dipole on its way to the observer.

It should be noted that even for a string whose thickness is less than the light wavelength (at resonant frequency) not only linear but also nonlinear response is possible, leading to optical bistability as well as nontrivial dynamical regimes such as pulsations or chaos [67–76].

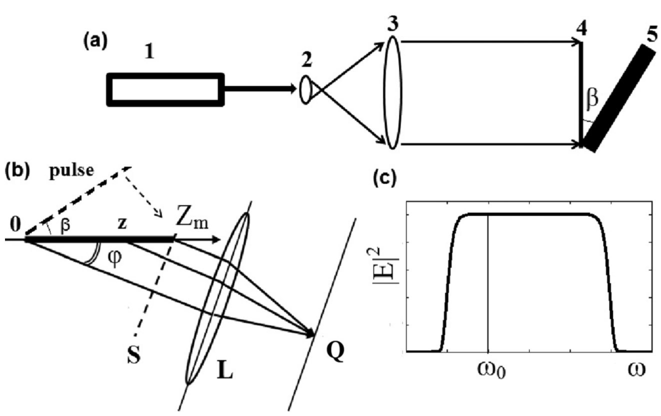

FIG. 1. (a) Excitation of a string at superluminal velocity. 1, a short spectrally broadband laser pulse source; 2,3, lenses; 4, the pump pulse. The intersection of the plane pulse and the medium moves along the string at the velocity $V = c/\sin\beta > c$. (b) The observation geometry of the string of length Z_m. The observer is placed far away from the string or in the focus of a lens L which collects the radiation of the string parallel to the z axis. (c) The source must produce a significantly broadband and flat spectrum [58–66], which includes also the resonance frequency ω_0 of the dipoles forming the string.

The electric field created by the string excitation observed at the remote position Q is determined by the solution of the wave equation $\Box\mathbf{E} = \mu_0\partial_{tt}\mathbf{P}$, where $\Box = \partial_{xx} + \partial_{yy} + \partial_{zz} - 1/c^2\partial_{tt}$ is the d'Alembert operator, and c is the velocity of light in vacuum. In particular, if the source is a single dipole, that is, $\mathbf{P} \propto \delta(\mathbf{r})$, the observed secondary emission at the point $\mathbf{r}'$ is

$$\mathbf{E}(\mathbf{r}',t) \propto \partial_{tt}\mathbf{P}(\mathbf{r},t - |\mathbf{r} - \mathbf{r}'|/c). \tag{1}$$

In the following, we normalize the last relation in such a way that the coefficient of proportionality between $\mathbf{E}$ and $\omega_0^2\mathbf{P}$ is 1.

Under those circumstances, the response of the dipoles to some excitation $\mathbf{E}_e(t)$ as seen from the point Q is described by the sum of the responses of the separate dipoles over the whole string, taking into account that the response comes to the point Q delayed in time. If the response of a single dipole to the excitation is described by the function $\mathbf{g}_e(t)$, the resulting expression will be:

$$\mathbf{E}(t,Q) = \int_0^{Z_m} N(z)\mathbf{g}_e(\tilde{t},z)dz, \tag{2}$$

where $\tilde{t}$ is the delay time which depends on the geometry of the system, and $N(z)$ describes the dipole density.

The physical nature of the oscillators forming the string can be very different. In particular, one can use a string of nanoantennas made of conducting material. Such nanoantennas indeed have resonance frequencies defined by their plasmonic resonances, which are highly flexible and are determined by the geometry and size of the structures [50–53]. Using such structures the resonance frequency ω_0 can be tuned in a wide range from terahertz up to the visible one. Semiconductor quantum dot arrays [54–57] can also be used. One may remark that in the case of semiconductors, strong nonlinearities accomplished by the possibility of electric pumping allow the use of such thinner-than-wavelength layers as active elements in quantum well and quantum dot lasers [77–87].

If the exciting pulse is weak and therefore the dipole response is linear, its response to the excitation pulse $\mathbf{E}_e(t)$ is described by the polarization $\mathbf{P}(t)$,

$$\ddot{\mathbf{P}} + \gamma\dot{\mathbf{P}} + \omega_0^2\mathbf{P} = g\mathbf{E}_e(t), \tag{3}$$

where g is the coupling strength to the field. Considering the small-signal (linear) regime, we assume also that the excitation pulse is shorter than the resonant period of the oscillators, so that its spectrum not only includes ω_0 but, at the same time, is significantly broad and flat [see Fig. 1(c)]. Such pulses can be obtained, for instance, in the terahertz range using a gaseous ionization-based source pumped by ultrashort optical pulses [58–66]. Under these conditions and also assuming that $\gamma \ll \omega_0$, the nonsingular part of the response of the oscillators can be to a good precision described by an excitation function

$$g_e(t) \approxeq e^{-\gamma t}\cos(\omega_0 t)\Theta(t), \tag{4}$$

where $\Theta(t)$ is the Heaviside step function.

In the case when the signal has large enough amplitude, the response of the dipoles becomes nonlinear. The nonlinear response in the case of a linear polarization of the incident field is described by the Bloch equations [88,89]

$$\frac{du(t,z)}{dt} = -\Delta\omega v(t,z) - \frac{1}{T_2}u(t,z), \tag{5}$$

$$\frac{dv(t,z)}{dt} = -\Delta\omega u(t,z) - \frac{1}{T_2}v(t,z) + \Omega_R(t,z)w(t,z), \tag{6}$$

$$\frac{dw(t,z)}{dt} = -\frac{1}{T_1}[w(t,z)+1] - \Omega_R(t,z)v(t,z). \tag{7}$$

Here the u and v components of the medium polarization are in phase and out of phase with the driving E field, respectively, w is the population difference, T_1 is the time relaxation of the population difference, T_2 is the time relaxation of the polarization, $\Delta\omega$ is the frequency detuning between the electric field and the resonance frequency of the medium, $\Omega_R = \frac{d_{12}E(t)}{\hbar}$ is the Rabi frequency of the driving field, d_{12} is the transition dipole moment, and E_0 is the amplitude of the driving field. This system of equations describes the interaction of optical pulses with a two-level medium [88,89]. We remark that Eqs. (5)–(7) are derived in the rotating-wave approximation, which is inapplicable for broad spectra and short pulses mentioned above. Thus, in studying the nonlinear response we limit ourselves to relatively long pump pulses. Namely, we assume in this case that the excitation pulse has a Gaussian shape given by $E(t) = E_0\exp(\frac{-t^2}{\tau_p^2})$. In the case of $\Delta\omega = 0$ and if the duration of the excitation pulse is much shorter than T_1,T_2 ($\tau_p \ll T_1,T_2$), one can obtain the following solution of the optical Bloch equations for the polarization $P(t,z)$ and population difference $n(t,z) = n_0 w(t,z)$ (n_0 is the concentration of the two-level atoms):

$$n(t,z) = n_0 w(t,z) = -n_0\cos\Phi(t,z), \tag{8}$$

$$P(t,z) = d_{12}n_0 u(t,z) = -d_{12}n_0\sin\Phi(t,z), \tag{9}$$

where

$$\Phi(t,z) = \frac{d_{12}}{\hbar}\int_{-\infty}^{t} E(t',z)dt' \tag{10}$$

is the pulse area [88,89]. The change of n and P in time can be represented as the rotation of a unit vector in the (x,y) plane in such a way that the x component of the vector corresponds to v, and the y component to w. Then the function Φ defined by Eq. (10) is the angle of rotation of this vector: $\Phi = \pi$ corresponds to a complete transition of the atom to the excited level (π pulse), and $\Phi = 2\pi$ corresponds to a transition to excited level and then back to the ground one (2π pulse).

III. LINEAR RESPONSE

A. Straight string: General considerations

In this section we consider the case when our string has the form of a straight line of length Z_m [Fig. 1(b)]. The observer is located at a very large distance from the string or at the focal point Q of the lens L. The string consists of identical dipole oscillators having the resonance frequency ω_0 (λ_0 is the corresponding wavelength) and the decay rate γ. The number density of oscillators along the Oz axis varies periodically with the period Λ_z:

$$N(z) = \frac{1}{2}\left(1 + a\cos\frac{2\pi}{\Lambda_z}z\right), \tag{11}$$

where $a \leqslant 1$ is the amplitude of density oscillations. This equation describes a sort of 1D diffraction grating. In the following, for simplicity, we take $a = 1$.

At the initial time $t = 0$ the excitation crosses the point $z = 0$ and starts to propagate at the velocity V along the string towards its other end. We suppose that the exciting pulse is linearly polarized and the polarization direction is perpendicular to the plane of Fig. 1(b). The oscillators start to emit electromagnetic radiation according to the response law Eqs. (1)–(4). We now consider this secondary radiation propagating at the angle φ to the string, which reaches the detector at the point Q [Fig. 1(b)]. Under those circumstances we can restrict ourself to a single linear polarization, thus obtaining a scalar problem.

The electric field emitted by the oscillator located at z (as observed near the same point) is proportional, according to Eq. (4), to

$$E(t,z) \approxeq \exp\left[-\frac{\gamma}{2}\left(t - \frac{z}{V}\right)\right]\cos\left[\omega_0\left(t - \frac{z}{V}\right)\right]\Theta\left[t - \frac{z}{V}\right], \tag{12}$$

where the delayed argument describes the fact that the excitation appears at a time moment delayed by z/V. Instead of point Q one may consider the electric field at the plane S orthogonal to the direction φ passing through the point Z_m. The propagation time from this reference plane to Q is constant and will be omitted in the following analysis.

The light propagation time from the point z to the reference plane is given by $\frac{Z_m - z}{c}\cos\varphi$. Thus, the electric field emitted at the point z will have at the reference plane the value

$$E_{\text{ref}}(t,z) \approxeq \exp\left[-\frac{\gamma}{2}f(t,z)\right]\cos[\omega_0 f(t,z)]\Theta[f(t,z)], \tag{13}$$

where $f(t,z) = t - \frac{z}{V} - \frac{Z_m - z}{c}\cos\varphi$.

The total field observed at Q is obtained by the integration of Eq. (13) over the whole string:

$$E(t,\varphi) = \int_0^{Z_m} N(z)\exp\left[-\frac{\gamma}{2}f(t,z)\right]\cos[\omega_0 f(t,z)] \times \Theta\left[f(t,z)\right]dz. \tag{14}$$

The analytical solution of Eq. (14) in the case of $\gamma = 0$ is given in the Appendix. As one can see from the analytical calculation in Appendix [see Eq. (A6)] the response contains the resonance frequency of the oscillators, ω_0, together with a new component given by the expression

$$\Omega_1 = 2\pi\frac{V/\Lambda_z}{\left|\frac{V}{c}\cos\varphi - 1\right|}. \tag{15}$$

The inverse numerator of Eq. (15) is the time interval which the excitation spot needs to cross a single oscillation period of $N(z)$. When this time is equal to the period of the oscillations ($V/c = \Lambda_z/\lambda_0$) formula (15) leads to

$$\Omega_{1_D} = \frac{\omega_0}{\left|\frac{V}{c}\cos\varphi - 1\right|}. \tag{16}$$

This relation formally coincides with that for the Doppler frequency shift [46,47], so we will call it the Doppler-like frequency.

More general Eq. (15) is valid for arbitrary V and has the same form as the one appearing in the case of Purcell-Smith radiation. The appearance of this frequency and other related questions will be studied in detail in the next subsection.

B. Straight string: The linear response dynamics

Now we explore the temporal and spectral shape of the linear string response defined by Eq. (14) and its dependence on the system parameters. We start from the numerical simulations of Eq. (14) for some "typical" parameter values. Namely, we choose the normalized parameters as $\frac{V}{c} = 2$, $\frac{Z_m}{\Lambda_z} = 9.55$, $\frac{\Lambda_z}{\lambda_0} = 5$, and $\frac{\omega_0}{\gamma} = 22.22$. The real-world values of the parameters corresponding to this set depend on the resonance frequency of the oscillators in the string. For instance, assuming $\omega_0 = 2\pi \times 10\ \text{ps}^{-1}$ [the frequency for which the δ-function assumption from Fig. 1(c) is especially easy fulfilled], we will have $\Lambda_z = 150\ \mu\text{m}$, $Z_m = 1.4$ mm, and $\gamma = 2.8\ \text{ps}^{-1}$. Another example is a pump at optical frequencies $\omega_0 = 2\pi \times 375\ \text{ps}^{-1}$, $\Lambda_z = 4\ \mu\text{m}$, $Z_m = 40\ \mu\text{m}$, and $\gamma = 106.04\ \text{ps}^{-1}$.

The numerical solution of the integral Eq. (14) and its spectrum are shown in Fig. 2 (in normalized units) for the above-mentioned parameters, and assuming $\varphi = 0$ (the observation point is on the same line as the string) in Figs. 2(a) and 2(b) and the Cherenkov angle $\varphi = 60^\circ$ in Figs. 2(c) and 2(d).

As one can see, the resonant response at $\omega = \omega_0$ dominates in both cases. Nevertheless, for $\varphi = 0$ an additional frequency arises. As is seen in Fig. 2(a), this frequency appears in the transient process for the time interval from approximately $t_1 \approx 20.0T_0$ to approximately $t_2 \approx 47.0T_0$, at the moment t_1 the excitation spot reaches the end of the string. During the period t_1 to t_2, the radiation from the points $z = Z_m$ to $z = 0$ arrives to the observer. As a result of the interference of the incoming waves, a transition process occurs. It lasts until the moment t_2. Only decaying emission with the frequency ω_0 remains at the later time.

For the superluminal velocity of the excitation the denominator of Eq. (15) approaches zero if

$$\cos\varphi_0 = c/V, \tag{17}$$

which coincides with the condition for Cherenkov radiation. Figure 2(c) corresponds to the Cherenkov emission angle. This angle corresponds also to the zero-order diffraction peak of the grating formed by $N(z)$. Under the parameters of Fig. 1, $\varphi_0 = 60^\circ$. When the condition Eq. (17) is fulfilled, we have $\Omega_1 = \infty$, and the radiation from all points of the grating (the resonance medium) comes to the reference plane simultaneously; thus no transient process occurs.

Analogously, the diffraction orders 1 and -1 are defined by the relation

$$\cos\varphi_{\pm 1} = \frac{\pm\lambda_0}{\Lambda_z} + \frac{c}{V}, \tag{18}$$

which for the parameters of Fig. 3(a) gives the angles $\varphi_{+1} = 45.57^\circ$ and $\varphi_{-1} = 72.54^\circ$, respectively. For those angles, we have $\omega_0 = \Omega_1$ as well. For all values of φ different from the one given by Eq. (18), the Doppler-like frequency Ω_1 is not equal to ω_0. It should be noted, however, that in this case the spectral intensity is smaller than the one at the Cherenkov angle.

The dependence of the spectrum of the solution of Eq. (14) on the system parameters is presented in Fig. 3. In particular, we are varying the observation angle φ in Fig. 3(a), the excitation speed V in Fig. 3(b), and the grating period Λ_z [cf. Eq. (11)] in Fig. 3(c). One can clearly see the frequency branch corresponding to the resonance $\omega = \omega_0$, as well as the another one corresponding to the frequency given by Eq. (15).

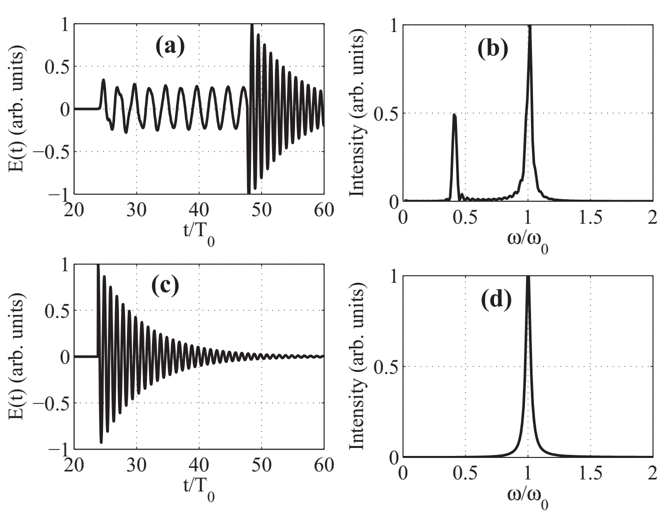

FIG. 2. Time dependence of the field $E(t)$ according to Eq. (14) (a),(c) and its spectral intensity $I(\omega)$ (b),(d) normalized to their maximal values vs normalized time t/T_0 and frequency ω/ω_0, for $V/c = 2$, $Z_m/\Lambda_z = 9.55$, $\Lambda_z/\lambda_0 = 5$, and $\omega_0/\gamma = 22.22$ for the observation angle $\varphi = 0$ (a),(b) and $\varphi = 60^\circ$ (c),(d); the latter corresponds to the Cherenkov emission angle.

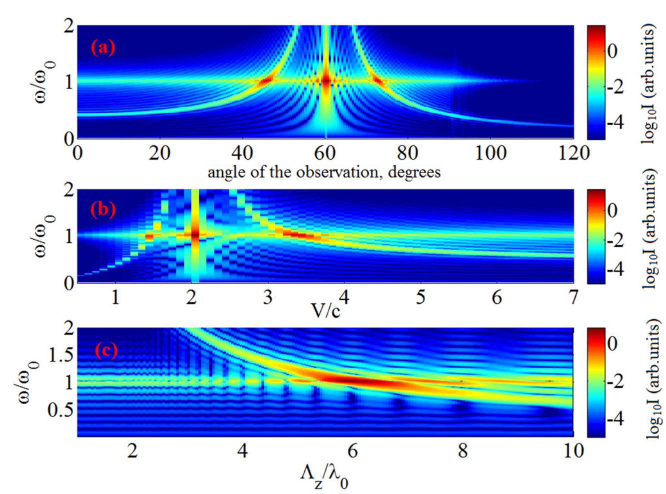

FIG. 3. (Color online) Dependence of the spectral intensity $I(\omega)$ of the string response according to Eq. (14) on the observation angle φ (a), the excitation velocity V (b), and the string density modulation period Λ_z (c). The other parameters coincide with those given in Figs. 2(c) and 2(d). The spectral intensity is presented on a logarithmic scale.

According to Eq. (15) Ω_1 decreases with increasing V/c for $V/c > 2$ and increases for $V/c < 2$. From Eq. (15) it also follows that $\Omega_1 \to \infty$ for $V \to 2c$ (when $\varphi = 60^\circ$). This also coincides with the typical behavior of the Doppler frequency shift. As can be seen from Fig. 3(c), Ω_1 decreases with increase of Λ_z/λ_0.

Up to now we have considered the case when the string is excited by a spot of light moving at superluminal velocity. Another interesting case appears when the velocity is subluminal.

An example of numerical solution of the integral Eq. (14) assuming $V/c = 0.7$ and $\varphi = 0$ is shown in Fig. 4. The other parameters are the same as in Fig. 2. One can see that the additional frequency component arises during the transient process from approximately $t_1 \approx 47.0T_0$ to approximately $t_2 \approx 70.0T_0$. At the time t_1 the radiation from the point $z = 0$ reaches the end of the medium. At the time t_2 the radiation from the point $z = Z_m$ appears at the observation point. Later on, only decaying oscillations at frequency ω_0 remain.

The dependence of the string response spectrum on the observation angle φ as well as on the grating period Λ_z is presented in Fig. 5. As one can see, the situation in the case $V < c$ is in many respects similar to the case of superluminal velocity. In particular, Ω_1 decreases with increase of φ as well as with increase of Λ_z. On the other hand, the Cherenkov angle (at which $\Omega_1 = \infty$) is never achieved.

C. Circular string: General considerations

In this section we consider a different topology of the string as depicted in Fig. 6. Namely, the string made of dipoles with the same resonance frequency ω_0 as before is arranged along a circle of radius R. The dipole density is modulated along the string in a periodic way with the angular period Λ_ϕ as

$$N(\phi) = \frac{1}{2}\left[1 + a\cos\left(\frac{2\pi}{\Lambda_\phi}\phi\right)\right]. \tag{19}$$

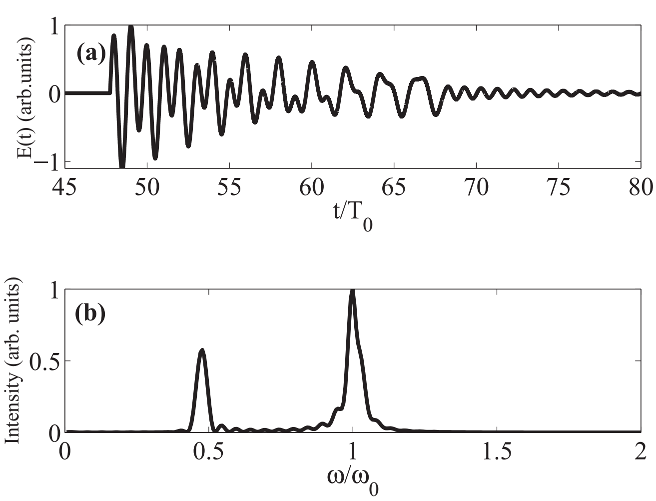

FIG. 4. Time dependence (a) of the string response field $E(t)$ according to Eq. (14), and (b) of its spectral intensity $I(\omega)$ normalized to the maximum value vs normalized time t/T_0 and frequency ω/ω_0, for $V/c = 0.7$, $Z_m/\Lambda_z = 9.55$, $\Lambda_z/\lambda_0 = 5$, and $\omega_0/\gamma = 22.22$ and observation angle $\varphi = 0$.

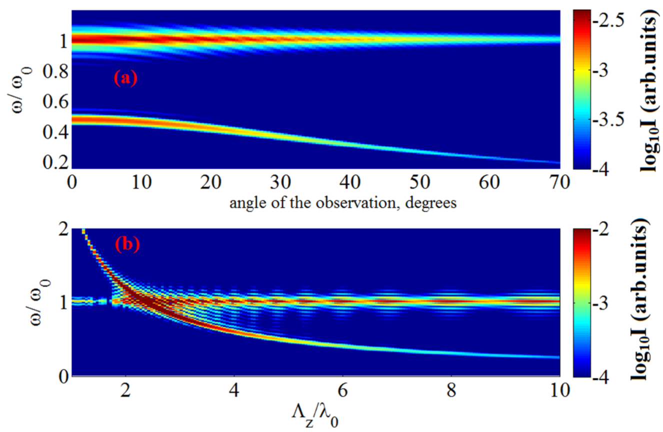

FIG. 5. (Color online) Dependence of the spectral intensity $I(\omega)$ of the string response according to Eq. (14) on the observation angle φ (a) and the string density modulation period Λ_z (b). The other parameters coincide with the ones in Fig. 4. Note the logarithmic scale in the plot.

As in Eq. (11), we assume the modulation amplitude $a = 1$. In the center of the circle a source of a short spectrally broad optical pulse [see Fig. 1(c)] is located, which quickly rotates, so that the crossing point (yellow point in Fig. 6) moves at velocity V along the circle.

The dipoles of the string respond to the excitation, emitting secondary waves. Here we will concentrate on the behavior of the string response field $E(t)$ observed in the center of the circle. The electric field formed in the center of the circle by the element dE_ϕ located at the point whose angular coordinate ϕ is given by

$$dE_\phi(t) = N(\phi)\exp\left[-\frac{\gamma}{2}f_\phi(t,\phi)\right]\cos[\omega_0 f_\phi(t,\phi)] \times \Theta[f_\phi(t,\phi)]d\phi, \tag{20}$$

where $f_\phi(t,\phi) = t - \frac{R\phi}{V} - \frac{R}{c}$. For one round-trip pass of the excitation, the total electric field is obtained by integration

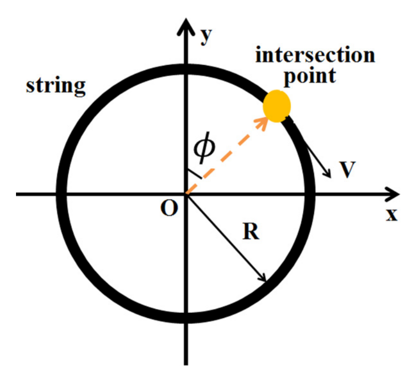

FIG. 6. (Color online) Circular geometry of the string. The source of a short pulse with a broad spectrum (c) is located in the center of the circle and quickly rotates. The crossing of the pulse and medium (yellow dot) moves at the velocity v along the string (black circle). As in the previous case, the string is made of dipoles characterized by the resonance frequency ω_0 and the dipole number density is modulated along the string periodically with the angular period Λ_ϕ.

of (20) over ϕ:

$$E(t,\phi) = \int_0^{2\pi} N(\phi)\exp\left[-\frac{\gamma}{2} f_\phi(t,\phi)\right]\cos[\omega_0 f_\phi(t,\phi)] \times \Theta[f_\phi(t,\phi)]d\phi. \quad (21)$$

The analytical solution of Eq. (21) in the case of $\gamma = 0$ is given in the Appendix. As one can see from Eq. (A8), the response contains the resonance frequency of the oscillators, ω_0, together with another component given by the expression

$$\Omega_2 = 2\pi \frac{V/\Lambda_\phi}{R}. \quad (22)$$

Equation (22) also has a simple physical meaning, namely, this is the frequency at which the intersection point crosses the inhomogeneity oscillations. Under the condition

$$\frac{V}{c} = \frac{\Lambda_\phi R}{\lambda_0}, \quad (23)$$

the new frequency is equal to the resonance one.

Note also that Eq. (22) is valid if the observer is located anywhere on the axis passing through the center of the circle perpendicularly to its plane.

D. Circular string: The linear response dynamics

We start from a typical situation when the frequency Ω_2 is clearly visible in the spectrum. Namely, we take the following parameters: $V/c = 3.75$, $\frac{\Lambda_\phi R}{\lambda_0} = 2$, $\omega_0/\gamma = 22.2$, and $\omega_0/\Omega_2 = 0.53$. The same $\omega_0 = 2\pi \times 10\ \mathrm{ps}^{-1}$ as in Sec. III B and $R = 3$ cm will in this case correspond to $\Lambda_\phi = 0.002\ \mathrm{rad}^{-1}$, $\gamma = 2.8\ \mathrm{ps}^{-1}$. The transient process for these parameters calculated using Eq. (21) is shown in Fig. 7. For these particular parameters, the frequency of the oscillators

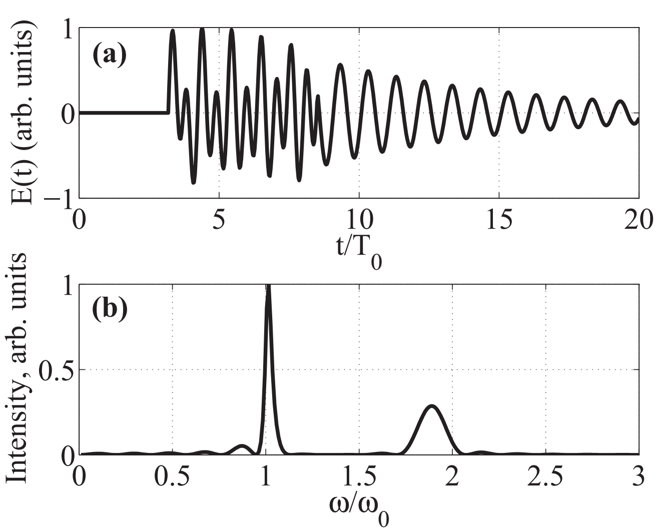

FIG. 7. (a) Time dependence of the electric field $E(t)$ excited by the string and (b) the corresponding intensity spectrum $I(\omega)$ in the center of the circle for the circular scheme depicted in Fig. 6 and the parameters $V/c = 3.75$, $\Lambda_\phi R/\lambda_0 = 2$, $\omega_0/\gamma = 22.2$, and $\omega_0/\Omega_2 = 0.53$.

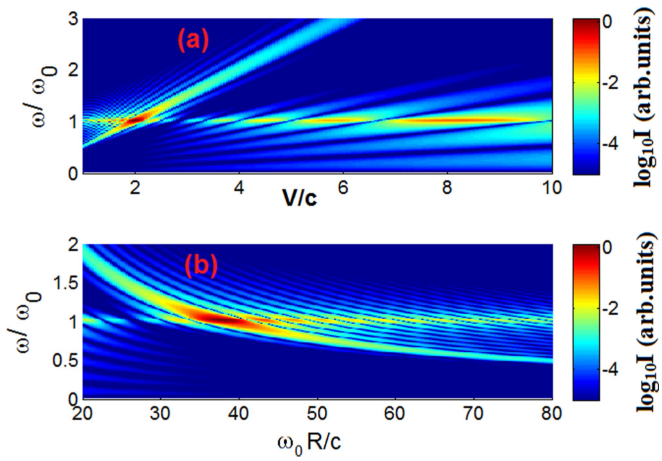

FIG. 8. (Color online) (a) Dependence of the radiation spectrum on the normalized propagation speed V of the excitation (a) and the radius of the circle R (b). Other parameters are as in Fig. 7.

practically doubles that of the transient emission, resulting in the high-amplitude beating clearly seen in Fig. 7. Once the transition process is finished, the observer at O sees only the ordinary decaying oscillations. This conclusion is also valid in the case when the excitation pulse moves at subluminal velocity or even precisely at the velocity of light, in contrast to the straight string. In all these cases, the radiation spectrum at the center of the circle will possess an additional frequency, different from ω_0, with only the exception given by Eq. (23).

In order to illustrate the dependence Ω_2 on the parameters of the system, we present the radiation spectrum in dependence on V [Fig. 8(a)] and R [Fig. 8(b)], whereas the other parameters are taken as in Fig. 7. As can be easily seen from Eq. (22), this frequency increases with V and decreases with R.

In the circular case considered here, the Cherenkov resonance corresponds to the R value defined by Eq. (23).

IV. STRONG PUMP AND NONLINEAR DYNAMICS

In the present section we investigate the response of a straight string in the regime when the pump is so strong that the response of the string becomes significantly nonlinear. Assuming the scalar approximation and also taking into account that the secondary radiation never comes back to the string and thus no nonlinear propagation takes place (unless the observation angle $\varphi = 0$), it is described by Eq. (2). Taking into account Eq. (9) we obtain

$$E(t,\varphi) = \int_0^{Z_m} N(z)P\,[f(t,z)]\sin[\omega_0 f(t,z)]\,dz. \quad (24)$$

In this section we consider pulses with a relatively narrow spectrum, to be consistent with the approximations for which Eq. (9) was derived. The pulses we use are nevertheless still short enough to clearly observe the frequency Ω_1. The result of numerical solution of the integral Eq. (24), assuming the parameters of Fig. 2 and the observation angle $\varphi = 71^\circ$, total pulse area $\Phi = \pi/2$, Rabi frequency $\Omega_R = 0.07\omega_0$, and Gaussian pulse durarion $\tau_p = 2T_0$, is presented in Fig. 9.

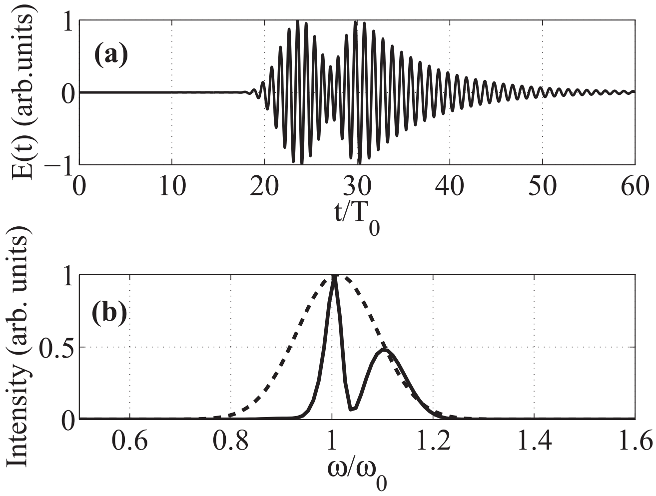

FIG. 9. (a) Time dependence of the string response field $E(t)$ according to Eq. (24). (b) S line: its spectral intensity $I(\omega)$ normalized to the maximum values vs normalized time t/T_0 and frequency ω/ω_0, for the parameters of Fig. 2 and total pulse area $\Phi = \pi/2$, $\Omega_R = 0.07\omega_0$, $\tau_p = 2T_0$; dashed line: spectrum of the excitation pulse.

As in the linear case, two frequencies are observed as shown in Fig. 9(b). Note that the peak corresponding to the frequency Ω_1 is more pronounced than in the linear case.

Figure 10 illustrates the dependence of the secondary emission spectrum on the total pulse area Φ [cf. Eq. (10)]. The pulse area was changed via modification of the Rabi frequency (pulse amplitude), keeping the pulse duration constant. In Fig. 10 one can observe two branches corresponding to the resonance frequency ω_0 and to the Doppler-like one Ω_1. Analysis of Fig. 10 shows that in the strongly nonlinear regime when the pulse area is large the radiation at the resonance frequency ω_0 has even smaller intensity than that at the frequency Ω_1. The periodic structure revealed in Fig. 10 in dependence on Φ is explained by the phase relations between the periodic term $\sin\Phi$ entering $P(t,z)$ [cf. Eqs. (9), (10), and (24)] and the period of spatial inhomogeneity of the dipole density $N(z)$.

V. CONCLUSION

In this paper, the secondary radiation excited by a moving intersection of a short spectrally broadband pulse and a resonant string made of identical dipoles is discussed for linear and circular string geometry. In such a situation, Cherenkov radiation naturally appears. In contrast to many other cases where the Cherenkov radiation is unstructured and has no clear frequency resonance, the present one demonstrates obvious resonant properties. That is, the response spectrum is centered at the resonant frequency of the dipoles comprising the string. In addition, as our analysis shows, an additional frequency appears in the presence of the string density oscillations, which has the meaning of a Doppler shift.

We point out also that this additional Doppler-like frequency (Ω_1 in the case of the straight string and Ω_2 in the circular case) appears in the transient regime, before all secondary waves excited by the pump pulse have reached the observation plane. The dynamics of the radiation after this moment is trivial and contains only decaying oscillations with the resonant frequency ω_0. In the strong-signal regime, when the nonlinearity in the string response becomes significant, the Doppler-like frequency may even be significantly larger in amplitude than the resonant one.

The behavior described here can find applications, for instance, in gaining control over the spectrum and pulse shape of broadband pulses using a rather compact setup.

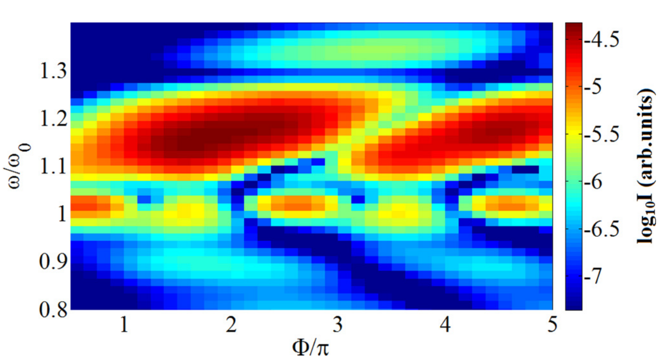

FIG. 10. (Color online) Dependence of the radiation spectrum on the pulse area Φ. Other parameters are as in Fig. 9.

ACKNOWLEDGMENTS

R.M.A. would like to acknowledge the support of EU FP7 ITN PROPHET, Grant No. 264687. I.B. is grateful to the German Research Foundation (DFG) for financial support in the framework of the Collaborative Research Center SFB 910 and Project No. BA 41561-1.

APPENDIX: ANALYTICAL SOLUTIONS OF EQS. (14) AND (21)

In this appendix we provide analytical solutions of Eqs. (14) and (21) when $\gamma = 0$. To obtain these expressions we first rearrange the argument of the Θ function in Eq. (14) as $f(t,z) = t - \frac{z}{V} - \frac{Z_m - z}{c}\cos\varphi = t - z/W - \frac{Z_m}{c}\cos\varphi$, where W is the effective velocity defined as

$$\frac{1}{W} = \frac{1}{V} - \frac{1}{c/\cos\varphi}. \quad \text{(A1)}$$

One can see that W can be interpreted as the velocity of the projection of the crossing point to the axis parallel to the observation plane in Fig. 1(b). Using this parameter we can rewrite the integral for the pulse response in the form

$$E(t) = \int N(z) h_0\left(t' - \frac{z}{W}\right) dz, \quad \text{(A2)}$$

where $t' = t - \frac{Z_m \cos\varphi}{c}$, and the function $h_0(t)$ denotes the response of a dipole located at $z = 0$ and excited with an excitation in the form of a delta function $\delta(t)$: $h_0(t) = \cos(\omega_0 t)\Theta(t)$.

The integral in Eq. (A2) has different forms depending on the sign of W:

$$E(t) = \int_0^{Wt'} N(z) h_0\left(t' - \frac{z}{W}\right) dz \quad \text{for } W > 0, \quad \text{(A3)}$$

$$E(t) = \int_{Z_m}^{Wt'} N(z) h_0\left(t' - \frac{z}{W}\right) dz \quad \text{for } W < 0. \quad \text{(A4)}$$

If $W > 0$ the emitting element of the string moves in the positive direction starting from zero as seen by the observer. In the opposite situation, when $W < 0$, it is seen as moving in the negative direction from Z_m to 0.

Equations (A3) and (A4) are valid for $0 < t < Z_m/|W|$ (transient regime) assuming $\nu_z = \frac{2\pi}{\Lambda_z}$. In particular, for $W > 0$ one can obtain

$$E(t) = \frac{W}{\omega_0}\sin(\omega_0 t') + \frac{W}{W^2\nu_z^2 - \omega_0^2}[\nu_z W \sin(\nu_z W t') - \omega_0 \sin(\omega_0 t')]. \tag{A5}$$

On the other hand, for $W < 0$ we have

$$E(t) = \frac{W}{\omega_0}\sin\left(\omega_0\left(t' - \frac{Z_m}{W}\right)\right) + \frac{W^2\nu_z \sin(\nu_z W t') - \nu_z W^2 \sin(\nu_z Z_m)\cos\left[\omega_0\left(t' - \frac{Z_m}{W}\right)\right]}{\nu_z^2 W^2 - \omega_0^2} - \frac{\omega_0 W \cos(\nu_z Z_m)\sin\left[\omega_0\left(t' - \frac{Z_m}{W}\right)\right]}{\nu_z^2 W^2 - \omega_0^2}. \tag{A6}$$

The last equation contains oscillating terms with the frequencies ω_0 and $\Omega_1 = \nu_z W$, which coincides with Eq. (15).

For $t \geqslant Z_m/W$, that is, when the excitation pulse comes out of the string, we have

$$\begin{aligned} E(t) &= \int_0^{Z_m} N(z)\cos\left[\omega_0\left(t' - \frac{z}{W}\right)\right]dz \\ &= \frac{W}{\omega_0}\left[\sin(\omega_0 t') - \sin\left(\omega_0\left(t' - \frac{Z_m}{W}\right)\right)\right] + \frac{\nu_z W^2 \sin(\nu_z Z_m)\cos\left[\omega_0\left(t' - \frac{Z_m}{W}\right)\right]}{\nu_z^2 W^2 - \omega_0^2} \\ &\quad + \frac{\omega_0 W \cos(\nu_z Z_m)\sin\left[\omega_0\left(t' - \frac{Z_m}{W}\right)\right] - W\omega_0 \sin(\omega_0 t')}{\nu_z^2 W^2 - \omega_0^2}. \end{aligned} \tag{A7}$$

This term describes oscillations with the frequency ω_0 after the transition process stops.

In the case of circular geometry, for the transient process ($\frac{R}{c} < t < \frac{2\pi R}{V} + \frac{R}{c}$ if $V > c$) one can obtain

$$\begin{aligned} E(t) &= \int_{Vt''/R}^{2\pi} N(\phi)\cos\left[\omega_0\left(t'' - \frac{R\phi}{V}\right)\right]d\phi = -\frac{V}{R\omega_0}\sin\left(\omega_0\left(t'' - \frac{2\pi R}{V}\right)\right) \\ &\quad + \frac{V^2\nu_\phi \sin(2\pi\nu_\phi)\cos\left[\omega_0\left(t'' - \frac{2\pi R}{V}\right)\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2} + \frac{R\omega_0 V \cos(2\pi\nu_\phi)\sin\left[\omega_0\left(t'' - \frac{2\pi R}{V}\right)\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2} - \frac{V^2\nu_\phi \sin(\Omega_2 t'')}{\nu_\phi^2 V^2 - R^2\omega_0^2}. \end{aligned} \tag{A8}$$

Here $t'' = t - \frac{R}{c}$ and $\nu_\phi = \frac{2\pi}{\Lambda_\phi}$. The last expression contains terms oscillating at the frequencies Ω_2 and ω_0. After the transition process ends ($V > c, t > \frac{2\pi R}{V} + \frac{R}{c}$), we have

$$\begin{aligned} E(t) &= \int_0^{2\pi} N(\phi)\cos\left[\omega_0\left(t'' - \frac{R\phi}{V}\right)\right]d\phi = \frac{V}{R\omega_0}\left\{\sin(\omega_0 t'') - \sin\left[\omega_0\left(t'' - \frac{2\pi R}{V}\right)\right]\right\} \\ &\quad + \frac{V^2\nu_\phi \sin(2\pi\nu_\phi)\cos\left[\omega_0\left(t'' - \frac{2\pi R}{V}\right)\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2} + \frac{R\omega_0 V\left[\cos(2\pi\nu_\phi)\sin\left[\omega_0\left(t'' - \frac{2\pi R}{V}\right)\right] - \sin(\omega_0 t'')\right]}{\nu_\phi^2 V^2 - R^2\omega_0^2}, \end{aligned} \tag{A9}$$

which contains only terms oscillating with the frequency ω_0.

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