Stationary and quasistationary light pulses in three-level cold atomic systems

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We have studied stationary and quasistationary signal light pulses in cold Λ-type atomic media driven by counterpropagating control laser fields at the condition of electromagnetically induced transparency. By deriving a dispersion relation we present spectral and temporal properties of the signal light pulse and a significant influence of atomic decoherence on the coupled stationary light pulses for spatial splitting. Finally we discuss quasistationary light pulse evolution characterized by frozen spatial spreading for a robust coherent control of slow light pulses.

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I. INTRODUCTION

Quantum coherence control and deterministic manipulation of light pulses have become a topical issue in optics and optical quantum information science. Herein, using electromagnetically induced transparency (EIT) effect [1] is one of the promising tools for the coherent control of weak light fields even in a single photon level [2,3]. This technique is intensively elaborated for various applications of EIT-based quantum memories [4,5] and generation of nonclassical light fields [6]. EIT-based quantum coherence control of weak light is a strong candidate for ultralow power nonlinear optics [7] applicable to quantum information science such as implementation of two-qubit gates in the optical quantum computing [8–14]. Using the stationary light pulse (SLP) concept proposed recently under the EIT condition [15–20] seems to be especially promising for this purpose due to prolonged lifetime of the slow light pulses in the atomic media. The SLP coherent control with two counterpropagating light fields has been initially demonstrated in a hot atomic gas [15] and in solids [21]. Later this basic scheme was studied for cold atomic systems in a series of works [22–30]. Herein, the two counterpropagating control light fields lead to the excitation of many spatial gratings and long-lived atomic coherences [22,23,25]. The coherence gratings can result in the signal light dynamics of splitting the input light pulse into two counterpropagating light pulses [22,24–26,29]. Recent numerical studies [27–29] of nonadiabatic effects in SLP dynamics have revealed strong negative influence of the multiply excited coherence gratings on the SLP [19,30]. However, the spectral and temporal properties of SLP remain insufficiently elaborated for SLP outside of the adiabatic coherent control [24,26]. The problem of light-atom dynamics is due to complicated nonlinear Bragg scattering of the light fields on the three-level atomic system driven by intensive counterpropagating laser fields. Various atomic relaxation processes have brought in so many important issues of SLP but are still open and require further investigations for spectral properties and time-space dynamics as well as influence of arbitrary intensities of the control laser fields.

In this work, developing coupled Maxwell-Bloch equations for slowly varying amplitudes [22–24,26] with nonadiabatic evolution, we have derived the dispersion relation of stationary light field modes in cold atomic media. Using a simple dispersion relation we have revealed basic spectral properties of SLP and quasistationary light pulses. Herein, we have found a narrowed spectral domain providing realization of SLP, and analyzed the temporal properties of the stationary and quasistationary signal light pulses. Then, the drastic influence of the atomic decoherence and control laser fields on the basic properties of SLP is demonstrated. In particular, we have observed a regime of the quasistationary light pulse evolution characterized by frozen spatial spreading in the presence of the control laser fields with appropriate intensities. Finally, we describe a simple picture for generation of stationary light pulse in a cold Λ-type atomic medium.

II. PHYSICAL MODEL AND LIGHT-ATOM EQUATIONS

We study stationary light pulse interactions in a three-level atomic system composed of two lower long-lived levels, $|1\rangle$ and $|2\rangle$, and one optically excited level $|3\rangle$ (see Fig. 1). The transition $|1\rangle \leftrightarrow |3\rangle$ is resonant to the input weak signal light pulse while the transition $|2\rangle \leftrightarrow |3\rangle$ is driven by two counterpropagating laser fields. The laser fields are characterized by the Rabi frequencies Ω_+, Ω_-, and wave vectors K_+, K_- with frequency detuning $\Delta = \omega_c - \omega_{32}$; ω_{mn} is the frequency atomic transition $|m\rangle \leftrightarrow |n\rangle$; ω_p and ω_c are the frequency of probe and control fields.

For generation of SLP one can propose various scenarios of external coherent laser control. We briefly describe the temporal scheme analyzed recently in [28]. Here, the weak probe signal field $\hat{E}_+$ is launched into the atomic medium in the presence of one copropagating control laser field Ω_+. We characterize the probe field $\hat{E}_+$ by its wave vector $\bar{k}_+$ with spectral detuning from the optical transition $\Delta = \omega_p - \omega_{31}$. It is also assumed that initially all three-level atoms are prepared on the ground level $|1\rangle$. After complete arrival of the slow light pulse into the medium we switch off the control laser field following the well-known procedure [3] of EIT-based quantum

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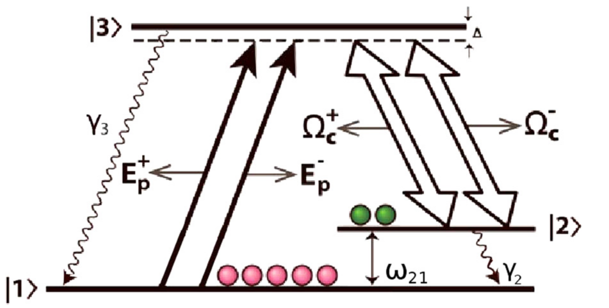

FIG. 1. (Color online) A schematic of atomic levels for resonant interactions with weak probe fields $E_p^{\pm}$ and two control fields $\Omega_c^{\pm}$; γ_2 and γ_3 are the decay constants of the atomic coherences $\hat{P}_{12}^j(t,z_j)$ and $\hat{P}_{13}^j(t,z_j)$; Δ is a spectral detuning from the optical transition.

storage on the long-lived atomic (spin) coherence of $|1\rangle \leftrightarrow |2\rangle$. Later we *adiabatically* switch on two counterpropagating control laser fields. Such switching will produce some band gap for the light fields in the frequency domain around ω_{31} [31]; at the same time the stored long-lived atomic coherence will be transformed into the optical coherence generating resonant light fields near the band-gap spectral range. Below we describe the light-atom dynamics of the excited light field after transient stage of its generation [30], i.e., SLP dynamics.

In order to clarify the basic properties of stationary and quasistationary light fields $\hat{E}_{\pm} = \sqrt{\hbar\omega_p/(2\varepsilon_0 V)}\hat{\mathcal{E}}_{\pm}(t,z)$ $e^{-i\omega_p(t\mp\frac{z}{c})} + \text{H.c.}$ (where $\hbar$ is Planck's constant, ε_0 is the electric permittivity, V is a quantization volume) we use coupled Maxwell-Bloch equations for slowly varied atomic coherences $\hat{P}_{12}^j$ and $\hat{P}_{13}^j$ and light field amplitudes $\hat{\mathcal{E}}_{\pm}(t,z)$:

$$\frac{\partial}{\partial t}\hat{P}_{13}^j(t,z_j) = -\gamma_3\hat{P}_{31}^j(t,z_j) + i\mathcal{G}\{\hat{\mathcal{E}}_+(t,z_j)e^{-i(\Delta t-kz_j)} + \hat{\mathcal{E}}_-(t,z_j)e^{-i(\Delta t+kz_j)}\} + i\{\Omega_+e^{-i(\Delta t-Kz_j)} + \Omega_-e^{-i(\Delta t+Kz_j)}\}\hat{P}_{12}^j(t,z_j), \tag{1}$$

$$\frac{\partial}{\partial t}\hat{P}_{12}^j(t,z_j) = -\gamma_2\hat{P}_{12}^j(t,z_j) + i\{\Omega_+^*e^{i(\Delta t-Kz_j)} + \Omega_-^*e^{i(\Delta t+Kz_j)}\}\hat{P}_{13}^j(t,z_j), \tag{2}$$

$$\left(\frac{\partial}{c\partial t} - ik_o + \frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_+(t,z) = \frac{iN\mathcal{G}}{c}\hat{P}_{13}(t,z)e^{i(\Delta t-kz)}, \tag{3}$$

$$\left(\frac{\partial}{c\partial t} - ik_0 - \frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_-(t,z) = \frac{iN\mathcal{G}}{c}\hat{P}_{13}(t,z)e^{i(\Delta t+kz)}, \tag{4}$$

where $\mathcal{G} = \wp_{13}\sqrt{\frac{\omega_+}{2\varepsilon_0\hbar V}}$ is the constant of atom-photon interaction [2]; N is the atomic density; $\wp_{13}$ is the dipole moment on the atomic transition $|1\rangle$-$|3\rangle$ $k_0 = \frac{\omega_{21}}{c}$, $k = \frac{\omega_p}{c}$, $K = \frac{\omega_c}{c}$, where γ_2 and γ_3 are relaxation constants of atomic coherences on the atomic transitions $|1\rangle$-$|2\rangle$ and $|1\rangle$-$|3\rangle$.

By using Fourier transformation, $\widetilde{P}_{nm}(\omega,z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{P}_{nm}(t,z)e^{-i\omega t}dt$, $\widetilde{\mathcal{E}}_{\pm}(\omega+\Delta,z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{\mathcal{E}}_{\pm}(t,z)$ $e^{-i\Delta t}e^{-i\omega t}dt$, from Eq. (2) we get

$$\widetilde{P}_{13}(\omega,z) = \frac{i\mathcal{G}(i\omega+\gamma_2)\{\widetilde{\mathcal{E}}_+(\omega+\Delta,z)e^{i(kz_j)} + \widetilde{\mathcal{E}}_-(\omega+\Delta,z)e^{-i(kz_j)}\}}{(i\omega+\gamma_3)(i\omega+\gamma_2) + \{\Omega_+^2+\Omega_-^2+2\Omega_+\Omega_-\cos(2Kz)\}}, \tag{5}$$

and after Fourier decomposition of the denominator in (6) we get

$$\widetilde{P}_{13}(\omega,z) = \{\widetilde{\mathcal{E}}_+(\omega+\Delta,z)e^{i(k_+z_j)} + \widetilde{\mathcal{E}}_-(\omega+\Delta,z)e^{-i(k_-z_j)}\}\frac{\mathcal{G}\,(i\omega+\gamma_2)}{\sqrt{\Gamma(\omega)^2+(2\Omega_+\Omega_-)^2}} \times\left\{1+\sum_{n=1}^{\infty}i^n(e^{in(2Kz)}+e^{-in(2Kz)})\frac{(2\Omega_+\Omega_-)^n}{[\Gamma(\omega)+\sqrt{\Gamma(\omega)^2+(2\Omega_+\Omega_-)^2}]^n}\right\}, \tag{6}$$

where $\Gamma(\omega) = \omega(\gamma_3+\gamma_2) - i(\gamma_3\gamma_2 - \omega^2 + \Omega_+^2 + \Omega_-^2)$.

Similar expressions in the adiabatic limit ($|\omega| \to 0$) have been used in [22,24] for analysis of SLP. Below we use Eq. (6) for analysis of stationary and quasistationary light pulses without the adiabatic approximation. By performing spatial Fourier transformation $\widetilde{\widetilde{A}}_{\pm}(\omega,\tilde{k}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\widetilde{A}_{\pm}(\omega,z)e^{i\tilde{k}z}dz$, we get from Eqs. (3) and (4) in the two mode approximation,

$$\left(\frac{\omega}{c} - k_0 - \tilde{k}\right)\widetilde{\widetilde{\mathcal{E}}}_+(\omega,\tilde{k}) = \alpha\widetilde{\widetilde{\mathcal{E}}}_+(\omega,\tilde{k}) + \beta\widetilde{\widetilde{\mathcal{E}}}_-(\omega,\tilde{k}), \tag{7}$$

$$\left(\frac{\omega}{c} - k_0 + \tilde{k}\right)\widetilde{\widetilde{\mathcal{E}}}_-(\omega,\tilde{k}) = \alpha\widetilde{\widetilde{\mathcal{E}}}_-(\omega,\tilde{k}) + \beta\widetilde{\widetilde{\mathcal{E}}}_+(\omega,\tilde{k}), \tag{8}$$

where $\alpha = \frac{N\mathcal{G}^2}{c}\frac{[i(\omega-\Delta)+\gamma_2]}{\sqrt{\Gamma(\omega-\Delta)^2+(2\Omega_+\Omega_-)^2}}$, $\beta = i\rho\alpha$, $\rho = \frac{(2\Omega_+\Omega_-)}{[\Gamma(\omega-\Delta)+\sqrt{\Gamma(\omega-\Delta)^2+(2\Omega_+\Omega_-)^2}]}$. It is worth noting a good acceptability of the two-wave approximation has been examined in [31] for analysis of photonic band gap.

Equations (7) and (8) are characterized by the following dispersion relation:

$$\tilde{k}_{\pm}(\omega) = \pm\sqrt{[\alpha(\omega)+k_0-\omega/c]^2 - \beta^2(\omega)} = \pm K(\omega), \tag{9}$$

with appropriate eigenstates of two light field modes $A_{\pm}(\omega,\tilde{k})$ of Eqs. (7) and (8):

$$A_+(\omega,\tilde{k};z) = \frac{\beta(\omega)}{\frac{\omega}{c}-\alpha(\omega)-k_0-K(\omega)}A\,(\omega,q)\,e^{ik_+(\omega)z} + A(\omega,q)e^{-ik_-(\omega)z}, \tag{10}$$

$$A_-(\omega,\tilde{k};z) = \mathcal{B}(\omega,q)e^{-ik_+(\omega)z} + \frac{\beta(\omega)}{\frac{\omega}{c}-\alpha(\omega)-k_0+K(\omega)} \times\mathcal{B}\,(\omega,q)\,e^{ik_-(\omega)z}, \tag{11}$$

where $k_+(\omega) = k + K(\omega)$ and $k_-(\omega) = k - K(\omega)$. As seen in Eqs. (7) and (8), the parameters $\rho(\omega)$ and $\beta(\omega)$, respectively, determine the coupling strength of two counterpropagating weak light fields related to its eigenstates. The whole SLP field will be in a superposition: $E(\omega,z)=\alpha_+A_+(\omega,\tilde{k};z)+\alpha_-A_-(\omega,\tilde{k};z)$ with some amplitudes $\alpha_+(\omega)$ and $\alpha_-(\omega)$ determined by the transient stage of SLP generation and initial condition.

We note a quite simple analytical formula in Eq. (9) that makes an easy general spectral analysis for both SLP and usual slow light (when $\Omega_-=0$ or $\Omega_+=0$), where Eq. (9) transforms into well-known EIT dispersion [2]. By using Eq. (9) we can also find analytically the group velocity of the coupled light fields $\upsilon_{gr}(\omega)=\frac{\partial\omega}{\partial K}$ for arbitrary strength of the control laser fields Ω_- and Ω_+. By neglecting small mis-phase-matching ($k_0\to 0$ [32]) and weak atomic decoherence $\gamma_2\to 0$ we find a very strong coupling regime for $\rho(\omega)>1$ (see details later). In the opposite case of weak coupling $|\beta^2(\omega)|\ll[\alpha(\omega)+\alpha k_0-\omega/c]^2$ (i.e., for $|\Omega_-|\ll|\Omega_+|$ or $|\Omega_+|\ll|\Omega_-|$) we get almost free uncoupled two light modes: $A_+(\omega,\tilde{k};z)\approx\frac{(\omega/c)-\alpha(\omega)-k_0}{2\beta(\omega)}A(\omega,q)e^{ik_+(\omega)z}$ and $A_-(\omega,\tilde{k};z)\approx\mathcal{B}(\omega,q)e^{-ik_+(\omega)z}$, which are counterpropagating each other along the z axis. More detailed analysis of the spectral and temporal properties of SLP and the quasistationary light pulse is discussed in the next sections.

III. PROPERTIES OF STATIONARY LIGHT PULSES

For a symmetric case of standing control laser field $\Omega_+=\Omega_-=\Omega$ we get a quite simple formula for the wave number $K(\omega)$ from Eq. (9):

$$K(\omega)=\left[\frac{\omega}{c}+\frac{N\mathcal{G}^2}{c}\frac{(\omega-i\gamma_2)}{\sqrt{\tilde{\Gamma}_0(\omega)^2-4\Omega^4}}\right]\times\sqrt{1-\left[\frac{2\Omega^2}{\tilde{\Gamma}_0(\omega)+\sqrt{\tilde{\Gamma}_0(\omega)^2-4\Omega^4}}\right]^2}, \quad (12)$$

where $\tilde{\Gamma}_0(\omega)=[(\gamma_2+i\omega)(\gamma_3+i\omega)+2\Omega^2]$.

Real (dispersion) and imaginary (absorption) parts of Eq. (12) are depicted in Fig. 2.

It is worth noting that the spectral behavior of Eq. (12) presented in Fig. 2 is in perfect agreement with the numerical results of [29]. For comparison we present the dispersion and absorption, Figs. 3 and 4, respectively, for stationary with the usual traveling control light fields. As shown in Fig. 3, dispersion for standing control field is characterized by more steep spectral behavior at the center line. Absorption for the standing control field (see Fig. 4) demonstrates remarkably linear character with a frequency detuning $|\omega|$ while there is quadratic dependence on the usual EIT absorption. These spectral behaviors of dispersion and absorption explain faster spreading and decay of SLP. From the basic physical point of view it becomes clear if taken into account in the usual EIT that the effect of control light field characterized by different amplitudes results in spreading of dispersion, group velocity, and spectral domain of weak absorption. Such spectral properties remain the EIT scheme for surface plasmons [33], where the intensive control plasmon field is also tightly varied in space.

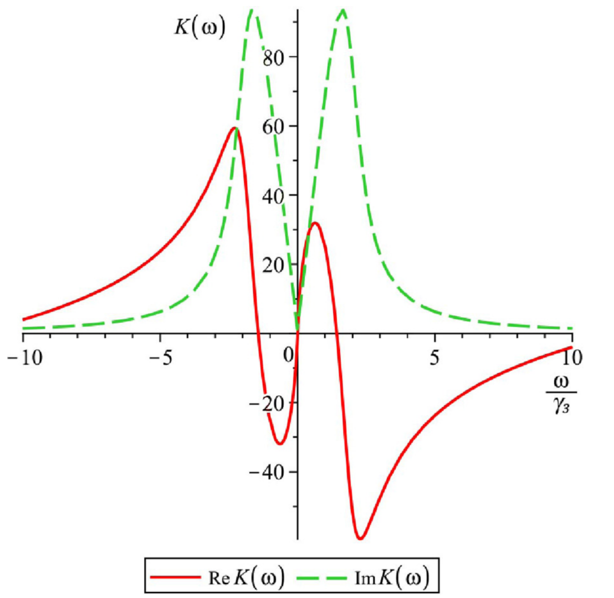

FIG. 2. (Color online) Dispersion: $(c/\gamma_3)\mathrm{Re}[K(\omega)]$ (red solid line) and absorption $(c/\gamma_3)\mathrm{Im}[K(\omega)]$ (green dashed line). The curves are presented in γ_3: $\Omega_+=\Omega_-=\gamma_3$, $\gamma_2\to 0$, $\Delta\to 0$.

Using Eq. (12) for negligibly weak relaxation $\gamma_2\to 0$ we get $K(\omega)\approx\frac{\omega^{3/4}}{2v_0}\sqrt[4]{\Omega^2/\gamma_3}e^{-i(\pi/8)}$ that determines the following spectral group velocity:

$$v_{gr}(\omega)=\frac{\partial\omega}{\partial k}=\frac{8}{3}\sqrt[4]{\frac{\gamma_3}{\Omega}}v_0\sqrt[4]{\frac{\omega}{\Omega}}\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right), \quad (13)$$

where $v_0=c\frac{\Omega^2}{N\mathcal{G}^2}$ is a group velocity in the usual EIT.

In accordance with Eq. (13), we get that SLP can be generated only in a narrow spectral range ($\omega/\Omega\ll 1$) where

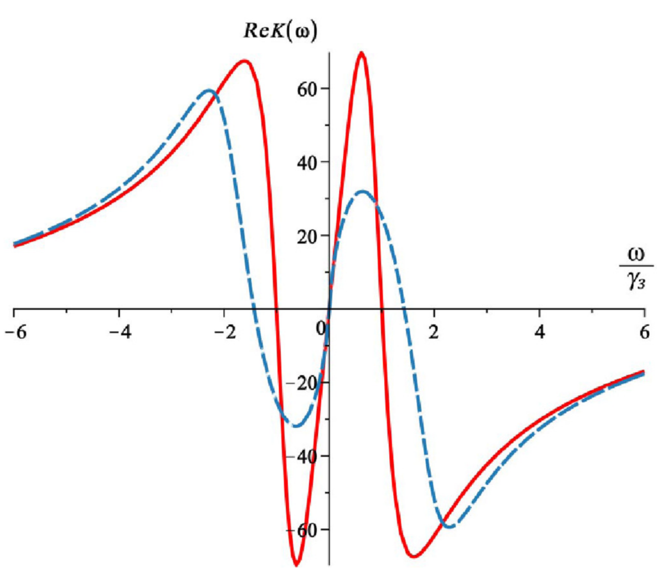

FIG. 3. (Color online) EIT dispersion $(c/\gamma_3)\mathrm{Re}[K(\omega)]$ for traveling (red solid line, $\Omega_+\neq 0,\Omega_-=0$) and standing (blue dashed line, $\Omega_+=\Omega_-$) control light fields. Other parameters are the same as in Fig. 2: $\Omega_+=\Omega_-=\gamma_3$, $\gamma_2\to 0$, $\Delta\to 0$, $\gamma_3=1$.

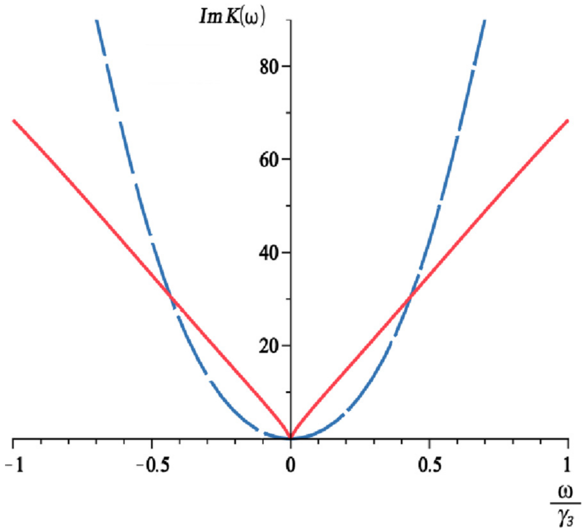

FIG. 4. (Color online) EIT absorption $(c/\gamma_3)\mathrm{Im}[K(\omega)]$ for standing (red solid line, $\Omega_+ = \Omega_-$) and traveling (blue dashed line, $\Omega_+ \neq 0, \Omega_- = 0$) control light fields; other parameters are the same as in Fig. 2: $\Omega_+ = \Omega_- = \gamma_3$, $\gamma_2 \to 0$, $\Delta \to 0$, $\gamma_3 = 1$.

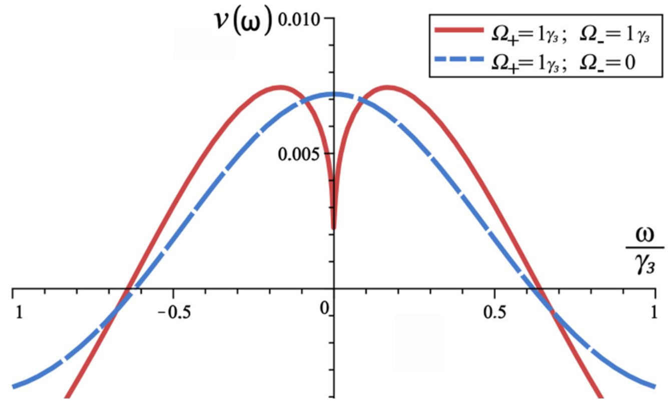

FIG. 6. (Color online) Group velocities for the standing ($\Omega_- = \Omega_+$, red solid line) and traveling ($\Omega_- = 0, \Omega_+ \neq 0$, blue dashed line) control laser fields in cold atomic systems. Other parameters are the same as in Figs. 2–4: $\gamma_2 \to 0$, $\Delta \to 0$, $\gamma_3 = 1$.

$v_{gr}(\omega) \ll v_o \to 0$ (see Fig. 5). The imaginary part in Eq. (13) is related to the light field absorption as it is depicted in Fig. 4.

For comparison we present a spectral behavior of the group velocities for the standing and traveling control laser fields in Fig. 6. It is clearly seen from Figs. 5 and 6 that the standing control field can provide SLP in much narrower spectral range of frequencies in comparison with the standard EIT scheme. These spectral properties clearly explain a previous result [29] that effective generation of SLP is possible only for large enough spatial length providing sufficiently narrow spectral width.

Spectral properties of group velocity and absorption for different amplitudes of standing control laser field are presented in Figs. 7 and 8, where $v_{gr}{\sim}\Omega^{3/2}|\omega|^{1/4}$ at the center line. In Fig. 8 linear spectral dependence of absorption is preserved for different amplitudes of standing control laser field. Here, the absorption decreases as standing control laser field increases. This is consistent with average Autler-Townes splitting of the excited optical level due to the driving standing light field. As a result we conclude that, with intense controlling standing light fields, stationary light is generated in more favorable spectral conditions. In this case we provide lower absorption with a small group velocity where $v_{gr}(\omega) \approx 0$. However, due to a very sharp spectral growth of the group velocity of the stationary light nonzero average group velocity results as shown in Fig. 9: one SLP mode $A_+(t,z) = \int d\omega\, f(\omega) A_+(\omega,\tilde{k};z) e^{-i\omega t}$ with Gaussian spectral shape $f(\omega) \sim \exp[-\omega^2/2\sigma^2]$. Figure 10 demonstrates dispersion and absorption effects on the SLP mode for the spectral width σ. All three $A_+(t,z)$-SLP modes in Fig. 10 move in $+z$ direction and wider initial spectral width of SLP leads to higher absorption and larger average group velocity. A similar result occurs for $A_-(t,z)$-SLP modes moving in "$-z$" direction (not shown). Therefore, taking into account a symmetry decomposition of the initial SLP on two

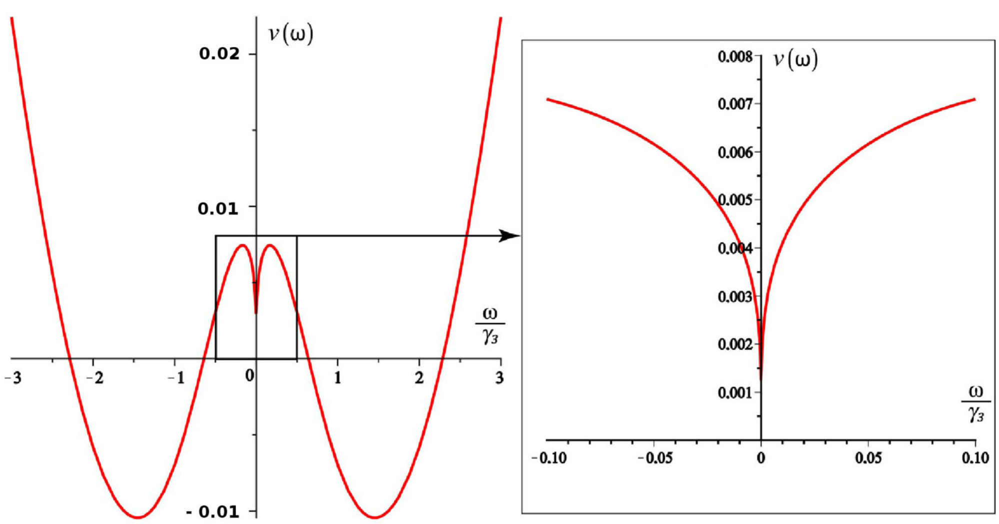

FIG. 5. (Color online) Spectral properties of group velocity for standing control laser field $\Omega_+ = \Omega_-$, other parameters are the same as in Figs. 2–4: $\Omega_\pm = \gamma_3$, $\gamma_2 \to 0$, $\Delta \to 0$, $\gamma_3 = 1$.

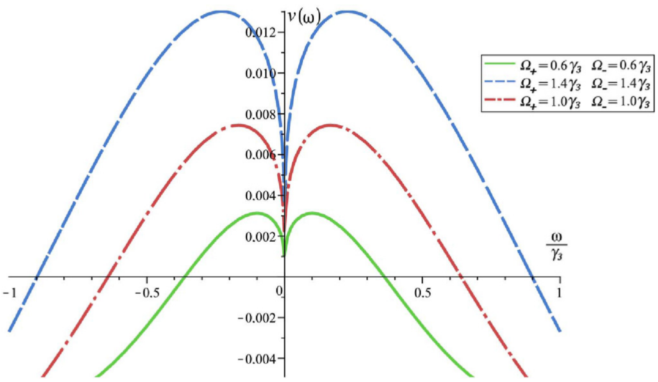

FIG. 7. (Color online) Group velocities for different amplitudes of standing control laser field ($\Omega_\pm = 0.6\gamma_3$, green solid line), ($\Omega_\pm = \gamma_3$, red dashed dotted line), ($\Omega_\pm = 1.4\gamma_3$, blue dashed line), $\gamma_2 \to 0$, $\Delta \to 0$, $\gamma_3 = 1$. Other parameters are the same as in Figs. 2–4.

field modes, $E(t=0,z) = \frac{1}{\sqrt{2}}\{A_+(t=0;z) + A_-(t=0;z)\}$, we find that the initial light pulse envelope splits into two pulses propagating in opposite directions that is in general agreement with the adiabatic limit [22,24], and the SLP splitting is accelerated for wider spectral width of the initial light pulse.

Let us also analyze the influence of atomic decoherence γ_2 between levels $|1\rangle$ and $|2\rangle$ on the group velocity of $A_+(t,z)$ SLP mode. Putting the small spectral detuning $|\omega| \ll \gamma_3$ in Eq. (12) we get

$$
\begin{aligned}
K(\omega) &= \frac{N\mathcal{G}^2}{\Omega^{3/2}} \frac{\omega - i\gamma_2}{2c} \frac{1}{\sqrt[4]{(\gamma_2 + i\omega)\gamma_3}} \\
&\approx \frac{N\mathcal{G}^2}{2c\sqrt[4]{\gamma_2\gamma_3}\,\Omega^{3/2}} \left\{ \frac{3}{4}\omega - i\gamma_2 \left[1 + \frac{3}{32}\left(\frac{\omega}{\gamma_2}\right)^2 \right] \right\},
\end{aligned} \tag{14}
$$

where Eq. (14) leads to nonzero group velocity (see also Fig. 11) even for central frequency domain $|\omega| < \gamma_2$:

$$
v_{gr}(|\omega| < \gamma_2) = \frac{1}{K'(0)} = \frac{8c\sqrt[4]{\gamma_2\gamma_3}\,\Omega^{3/2}}{3N\mathcal{G}^2} \sim \sqrt[4]{\gamma_2} > 0. \tag{15}
$$

Thus, the atomic decoherence ($\gamma_2 \neq 0$) makes it completely impossible to form a SLP mode for spectral region $\delta\omega_f < \gamma_2$,

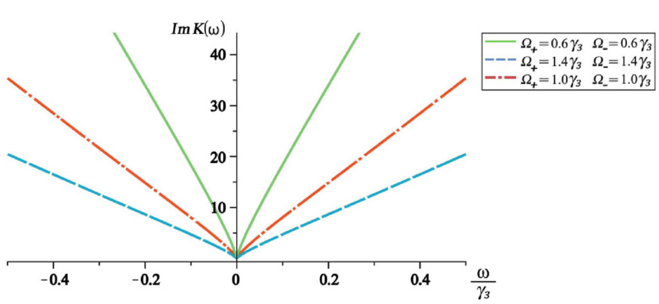

FIG. 8. (Color online) Spectral properties of the absorption for different amplitudes of standing control laser field. ($\Omega_\pm = 0.6\gamma_3$, green solid line), ($\Omega_\pm = \gamma_3$, red dashed dotted line), ($\Omega_\pm = 1.4\gamma_3$, blue dashed line), $\gamma_2 \to 0$, $\Delta \to 0$, $\gamma_3 = 1$. Other parameters are the same as in Figs. 2–4.

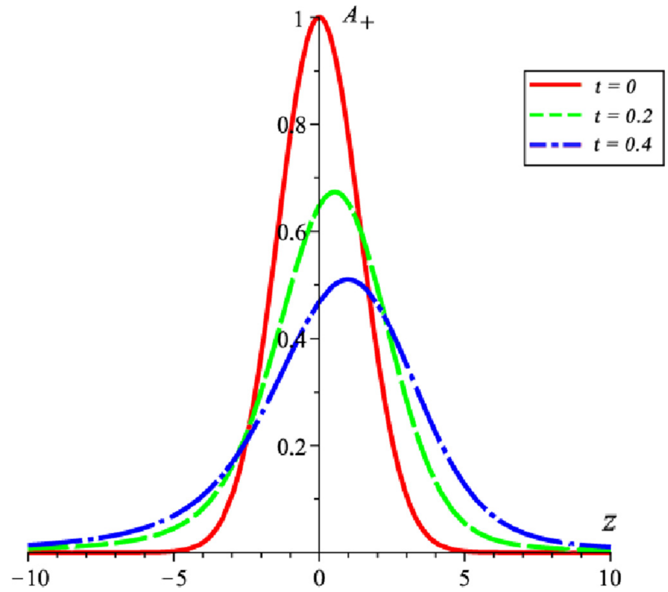

FIG. 9. (Color online) Spatial profile of SLP mode $A_+(t,z)$ for different moments of time $t = 0$ (red solid line), $t = 0.2$ (green dashed line), $t = 0.4$ (blue dashed dotted line). Time is denoted by $1/\gamma_3$; spectral width $\sigma = \gamma_3$.

where the atomic relaxation decouples the counterpropagating modes $\widetilde{\widetilde{\hat{\mathcal{E}}}}_+(\omega,\tilde{k})$ and $\widetilde{\widetilde{\hat{\mathcal{E}}}}_-(\omega,\tilde{k})$, so that the forward and backward parts $A_+(t,z)$ and $A_-(t,z)$ of the light pulse can freely move in opposite directions.

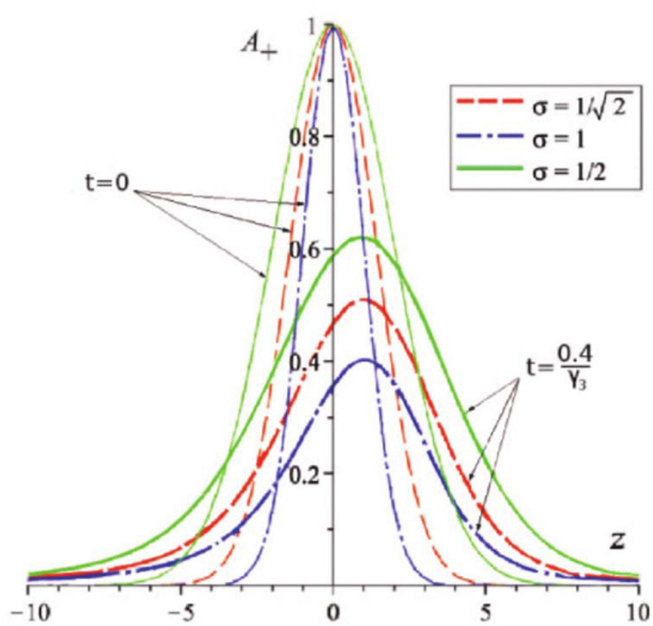

FIG. 10. (Color online) Spatial profile of SLP mode $A_+(t,z)$ for $t = 0$ and $t = 0.4$ for three spectral widths ($\sigma = 1/\sqrt{2}$, red dashed line), ($\sigma = 1$, blue dashed dotted line), ($\sigma = 1/2$, green solid line) in units of γ_3.

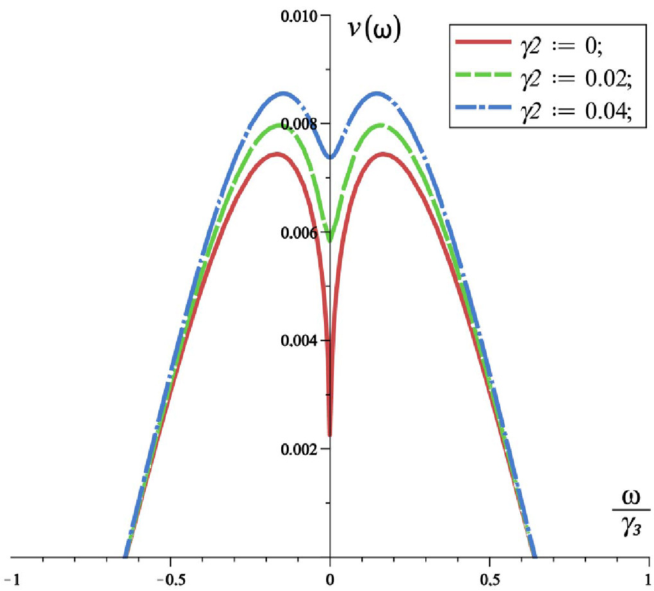

FIG. 11. (Color online) Spectral properties of group velocity $v_{gr}(\omega)$ for SLP mode $A_+(t,z)$ at different relaxation rates ($\gamma_2 = 0$, red solid line), ($\gamma_2 = 0.02$, green dashed line), ($\gamma_2 = 0.04$, blue dashed dotted line) in units of γ_3; large atomic relaxation excludes any formation of SLP even in narrow spectral range around $\omega = 0$; other parameters are the same as in Figs. 2–4: $\Omega_\pm = \gamma_3$, $\Delta \to 0$.

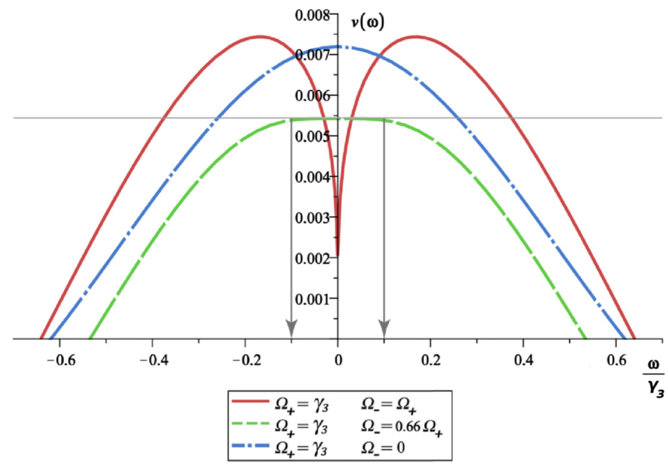

FIG. 12. (Color online) Spectral properties of group velocity in particular cases: SLP, $\Omega_- = \Omega_+$ (red solid line); usual slow light $\Omega_- = 0$, $\Omega_+ \neq 0$ (blue dashed dotted line); quasistationary light $\Omega_- = 0.66\Omega_+$ (green dashed line).

IV. LONG-LIVED QUASISTATIONARY LIGHT PULSES

As shown in Figs. 5–7 and 11, the standing control light field drastically changes the spectral properties of dispersion and absorption due to spatial dependence on the control laser field amplitude. Taking into account such influence it seems promising to find some regimes for such spatially inhomogeneous coherent control of slowdown light fields. Below we propose such a scheme for a quasistationary light field.

For control counterpropagating laser fields with different amplitudes ($\Omega_- < \Omega_+$) and weak atomic decoherence $\gamma_2 \ll |\Omega_+ - \Omega_-|$ we get from Eq. (9) in the limit of negligibly small spectral detuning $\omega \to 0$: $v_{gr}(\omega)|_{\omega\to 0} = \frac{1}{K'(0)} = \frac{c}{N\mathcal{G}^2}\Omega_+(\Omega_+^2 - \Omega_-^2)^{1/2} = v_+\sqrt{1-\Omega_-^2/\Omega_+^2} = v_+\sqrt{1-v_-^2/v_+^2}$ (where $v_\pm = \frac{c\Omega_\pm^2}{N\mathcal{G}^2}$ is a usual group velocity in EIT with one traveling control light field, Ω_+ or Ω_-) which coincides with previous results obtained in the adiabatic limit [22] (see also Refs. [24] and [26]). However, it is important to control the group velocity of SLP in a wider spectral range comparable with total resonant linewidth γ_3. Using Eq. (13), we have numerical analyses for the spectral behavior of group velocity as the function of ratio Ω_-/Ω_+ in Fig. 12, where an interesting example of the quasistationary light at $\Omega_-/\Omega_+ = 0.66$ is presented. From Fig. 12, we see that the quasistationary light can be characterized by flat spectral properties of the group velocity providing thereby weaker spatial spreading of slow light field. The flat behavior is a result of two opposite effects—of the usual EIT dispersion and the spectral dispersion of group velocity inherent to the light in the system of atoms driven by spatially inhomogeneous control laser field depicted in Fig. 11. Figure 13 shows absorption curves related to the quasistationary light in comparison with slow light and SLP fields. In Fig. 13, the absorption spectrum of the quasistationary light remains still low at the center line.

Finally we note that the same group velocity can be maintained by varying the Rabi frequencies of both control laser fields linearly as shown in Fig. 14. For weak counterpropagating control laser fields $\Omega_- \ll \Omega_+$ the following is obtained from Eq. (6):

$$\widetilde{\hat{P}}_{13}(\omega,z) \cong \chi(\omega;\Omega_+)\{1 - \zeta\cos(2Kz)\} \times \{\widetilde{\hat{\mathcal{E}}}_+(\omega+\Delta,z)e^{i(kz_j)} + \widetilde{\hat{\mathcal{E}}}_-(\omega+\Delta,z)e^{-i(kz_j)}\}, \tag{16}$$

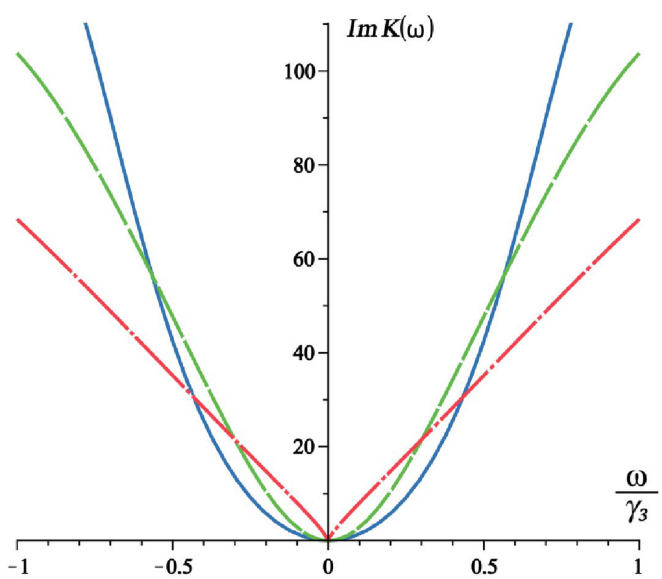

FIG. 13. (Color online) Spectral properties of absorption (c/γ_3) $\mathrm{Im}[K(\omega)]$ for SLP field $\Omega_- = \Omega_+$ (red dashed dotted line), usual EIT slow light $\Omega_- = 0$, $\Omega_+ \neq 0$ (blue solid line) and quasistationary light $\Omega_-/\Omega_+ = 0.66$ (green dashed line).

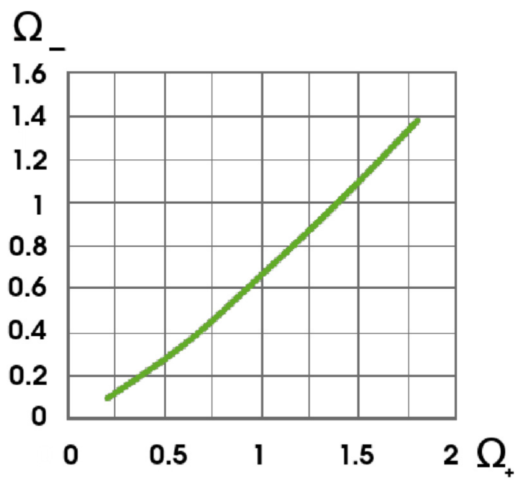

FIG. 14. (Color online) Relation between the two control Rabi frequencies Ω_- and Ω_+ providing the same group velocity of quasistationary light pulse.

where $\chi(\omega;\Omega_+) = \frac{i\mathcal{G}(i\omega+\gamma_2)}{\{(i\omega+\gamma_3)(i\omega+\gamma_2)+\Omega_+^2+\Omega_-^2\}}$ is related to the usual dressed susceptibility of EIT medium in the presence of Ω_+ control field [2]; parameter $\zeta = \frac{2\Omega_+\Omega_-}{\{(i\omega+\gamma_3)(i\omega+\gamma_2)+\Omega_+^2+\Omega_-^2\}}$ determines a modulation depth and coupling between the two coupled light fields.

For the large control field Ω_+ and quite narrow bandwidth of the signal light field $|\delta\omega| \ll \Omega_+$, we get $\zeta \cong \frac{2\Omega_-}{\Omega_+}$. This corresponds to the coupling of two slow light fields $\hat{\mathcal{E}}_+(t,z)$ and $\hat{\mathcal{E}}_-(t,z)$ traveling at the EIT condition determined by $\sqrt{\Omega_+^2+\Omega_-^2} \approx \Omega_+$ control field. Thus the effective formation of a quasistationary light pulse can be realized by adiabatic switching on the second control field $\Omega_-(t)$ up to $\Omega_- \leqslant 0.66\Omega_+$ or to some another magnitude determined by the spectral width of the input light field. After such switching the slowdown group velocity will be also determined by the effective coupling of two interacting fields $\hat{\mathcal{E}}_+(t,z)$ and $\hat{\mathcal{E}}_-(t,z)$ in accordance with $v_+\sqrt{1-\Omega_-^2/\Omega_+^2}$ as discussed above.

V. CONCLUSION

We analyzed stationary and quasistationary light pulses in a Λ-type cold atomic medium. By developing a model based on Maxwell-Bloch equations for slowly varied amplitudes [22–24,26] we derived quite a simple analytical expression for the dispersion relation of stationary and quasistationary light field modes. By using the dispersion relation we performed a detailed analysis of basic spectral properties of SLP and quasistationary light pulses. Herein, we revealed quite a narrow spectral range of SLP and observed a strong influence of the atomic decoherence on the spatial splitting of SLP. We also predicted a promising regime for the generation of quasistationary light pulses characterized by weaker spatial spreading in the presence of the control laser fields with appropriate relative intensities. Overall, the performed analysis provided a clearer physical picture of the basic properties of SLP in a cold atomic medium, and a proposed scheme of quasistationary light pulse control for the application of enhanced nonlinear interactions of weak light pulses at the EIT condition.

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