Induced effects of the Dzyaloshinskii-Moriya interaction on the thermal entanglement in spin-1/2 Heisenberg chains

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The nearest-neighbor spins in the one-dimensional spin-1/2 XX model with the added Dzyaloshinskii-Moriya (DM) interaction are entangled at zero temperature. In the presence of a transverse magnetic field (TF) they remain entangled up to a quantum critical field, h_c. Using the fermionization technique, we have studied the mutual effect of the DM interaction and a TF on the thermal entanglement (TE) in this model. The critical temperature where the entanglement disappears is specified. It is found that the TE in the finite-temperature neighborhood of the quantum critical field shows a scaling behavior with the critical exponent equal to the critical gap exponent. We also argue that thermodynamical properties like the specific heat and the magnetocaloric effect (instead of the usual internal energy and the magnetization) can detect the mentioned quantum entanglement in solid systems. In addition, we suggest a tactic to find all critical temperatures, which is based on the derivative of the entanglement witness with respect to the temperature.

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I. INTRODUCTION

Quantum entanglement is one of the most important predictions of modern quantum mechanics and indeed a valuable resource in quantum-information processing [1–3]. In fact entanglement is a unique quantum property of any nonlocal superposition state of two or more quantum systems. Much effort is devoted to describing the nature of the entanglement [4,5].

A kind of innate entanglement, the so-called the thermal entanglement (TE), is of particular interest and demonstrates that nonlocal correlations persist even in the thermodynamic limit [6,7]. It is believed that a connection between the quantum-information theory and condensed matter physics can be made by the study of TE, zero-temperature entanglement, and the relation between quantum-phase transitions. Since TE can be inferred from macroscopic variables that have been experimentally detected [8–10] much research has been dedicated to quantifying TE.

One-dimensional spin-1/2 systems are a special and practical category in which to study the TE phenomenon [11–25]. The ground state of the spin-1/2 antiferromagnetic XX chain is in the Luttinger liquid phase where the nearest-neighbor spins are entangled. By increasing temperature, the TE is reduced and will be zero at a critical temperature (T_c) which is independent of a transverse magnetic field (TF). For values of the TF larger than the quantum critical field, there is no pairwise entanglement at zero temperature. In this case, adding temperature creates pairwise entanglement at a low-temperature interval [21]. In addition, the pairwise quantum discord has also been studied recently [23]. It has been shown how quantum discord can be increased with temperature as the TF is varied. The effect of a staggered magnetic field on the TE of the spin-1/2 XX model has also been investigated [26] and it has been found that an alternating magnetic field suppresses TE.

Fundamentally, the magnetic behavior is determined by the Heisenberg model of interaction. In addition, the Dzyaloshinskii-Moriya (DM) interaction [27,28], which is arises from spin-orbit coupling, describes the superexchange between the interacting spins and it is believed that it can generate many surprising characteristics such as canting [29] or the induced gap in the one-dimensional (1D) spin-1/2 isotropic Heisenberg model [30]. Some antiferromagnetic systems are expected to be described by DM interaction, such as $Cu(C_6D_5COO)_2 3D_2O$ [31,32], Yb_4As_3 [33–35], $BaCu_2Si_2O_7$ [36], $\alpha-Fe_2O_3$, $LaMnO_3$ [37], $CuSe_2O_5$ [38,39], Cs_2CuCl_4 [40], and $K_2V_3O_8$ [41], which exhibit unusual and interesting magnetic properties due to quantum fluctuations in the presence of an applied magnetic field [37,42,43]. La_2CuO_4 also belongs to the class of DM antiferromagnets, which is a parent compound of high-temperature superconductors [44]. This has stimulated extensive investigation of the properties which are created from the DM interaction. On the other hand, for an explanation of the electric polarization behavior in multiferroic materials [45], an important and sufficient mechanism which is based on the DM interaction is proposed [46,47].

The induced effects of the DM interaction on TE is investigated only for two-qubit spin-1/2 XX chains [48–52]. It is believed that two- or three-qubit systems are not large enough to reveal interesting correlation properties in condensed matter physics. Also, it is possible that some of the correlation phenomena are exclusively associated with the fact that only two qubits are considered. Moreover, as we have mentioned there are a large number of quasi-1D antiferromagnetic compounds in which low-temperature behavior has been studied experimentally. These compounds are very good candidates to study the effect of the DM interaction on TE. Therefore, in this paper we consider an infinite 1D spin-1/2 XX model with added DM interaction in a TF. Using the Jordan-Wigner transformation we find an analytical solution for the TE between NN spins in the thermodynamic limit. In the absence of the TF, despite the fact that the DM interaction does not affect the amount of entanglement between the NN spins at the zero temperature, it can sufficiently affect it at the finite temperature. In the presence of the TF, we show that depending on the value of the magnetic field, one or two critical temperatures can be found. In addition, an entanglement witness equivalent to the difference between the total energy (U) and the magnetic energy ($-hM$) is defined. The parameter regions in which entanglement can be detected in the solid state system are determined using an entanglement witness. It is also argued that the derivative of the witness with respect to the temperature can be used to detect the intermediate temperature interval where the revival phenomenon happens. Indeed, we suggest this technique to observe this phenomenon experimentally.

The paper is organized as follows. In the forthcoming section we introduce the model and map it onto a pure 1D spin-1/2 XX model in a TF. In Sec. III, we present our exact analytical results on the thermal behavior of the entanglement between NN spins. In Sec. IV, we introduce an entanglement witness and explain how one can detect the quantum entanglement in the solid state systems. We conclude and summarize our results in Sec. V.

II. THE MODEL

We start our investigation with the 1D spin-1/2 XX model with added DM interaction in a TF where the Hamiltonian is written as

$$\mathcal{H} = J\sum_{j=1}^{N}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right) + \overrightarrow{D}\sum_{j=1}^{N}(\overrightarrow{S}_j \times \overrightarrow{S}_{j+1}) - h\sum_{j=1}^{N} S_j^z. \quad (1)$$

where S_j is the spin-1/2 operator on the jth site, J denotes the exchange coupling constant, h is the TF, and $\overrightarrow{D}$ is known as the DM vector. By considering the uniform DM vector as $\overrightarrow{D} = D\hat{z}$ and doing the rotation [53–56] around the z axis as $S_j^{\pm} \longrightarrow S_j^{\pm}\exp(\mp i\alpha)$, where $\tan\alpha = \frac{-D}{J}$, the Hamiltonian is transformed to the following 1D spin-1/2 XX model in a TF:

$$\mathcal{H} = \tilde{J}\sum_{j=1}^{N}\left(\widetilde{S}_j^x\widetilde{S}_{j+1}^x + \widetilde{S}_j^y\widetilde{S}_{j+1}^y\right) - h\sum_{j=1}^{N}\widetilde{S}_j^z, \quad (2)$$

with an effective exchange, $\tilde{J} = \sqrt{J^2 + D^2}$. It is known that at zero temperature, $T = 0$, the ground state of the system is in the Luttinger liquid (LL) phase [57]. By increasing the TF from zero up to the critical TF, $h_c = \tilde{J}$, the ground state remains in the LL phase where a quantum phase transition into the ferromagnetic phase with saturation magnetization along the TF will happen. At zero temperature, in the absence of the TF, NN are entangled and by increasing the TF the concurrence decreases and is equal to zero at the critical TF h_c. In the saturated ferromagnetic phase, spins clearly are not entangled.

Theoretically, the energy spectrum is needed to investigate the thermodynamic properties of the model. In this respect, we implement the Jordan-Wigner transformation to fermionize the transformed model [Eq. (2)]. Using the Jordan-Wigner transformation

$$\widetilde{S}_j^z = a_j^\dagger a_j - \frac{1}{2},$$
$$\widetilde{S}_j^+ = a_j^\dagger \exp\left(i\pi\sum_{l<j} a_l^\dagger a_l\right), \quad (3)$$
$$\widetilde{S}_j^- = a_j \exp\left(-i\pi\sum_{l<j} a_l^\dagger a_l\right),$$

the transformed Hamiltonian is mapped onto a 1D model of noninteracting spinless fermions:

$$H_f = \frac{Nh}{2} + \widetilde{J}\sum_j (a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) - h\sum_j a_j^\dagger a_{j+1}. \quad (4)$$

By performing a Fourier transformation into the momentum space as $a_j = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{-ikj} a_k$, the diagonalized Hamiltonian is given by

$$\mathcal{H} = \sum_{k=-\pi}^{\pi}\varepsilon(k) a_k^\dagger a_k, \quad (5)$$

where $\varepsilon(k)$ is the dispersion relation

$$\varepsilon(k) = \widetilde{J}\cos(k) - h. \quad (6)$$

III. THERMAL ENTANGLEMENT

We confine our interest to the entanglement between two sites, which is measured by the concurrence. The concurrence between two spins at sites i and j in the ground state and at a finite temperature can be achieved from the corresponding reduced density matrix $\rho_{i,j}$, which in the standard basis $(|11\rangle,|10\rangle,|01\rangle,|00\rangle)$ can be expressed as [21]

$$\rho_{i,j} = \begin{pmatrix} \langle P_i^\uparrow P_j^\uparrow\rangle & \langle P_i^\uparrow \sigma_j^-\rangle & \langle \sigma_i^- P_j^\uparrow\rangle & \langle \sigma_i^-\sigma_j^-\rangle \\ \langle P_i^\uparrow \sigma_j^+\rangle & \langle P_i^\uparrow P_j^\downarrow\rangle & \langle \sigma_i^-\sigma_j^+\rangle & \langle \sigma_i^- P_j^\downarrow\rangle \\ \langle \sigma_i^+ P_j^\uparrow\rangle & \langle \sigma_i^+\sigma_j^-\rangle & \langle P_i^\downarrow P_j^\uparrow\rangle & \langle P_i^\downarrow \sigma_j^-\rangle \\ \langle \sigma_i^+\sigma_j^+\rangle & \langle \sigma_i^+ P_j^\downarrow\rangle & \langle P_i^\downarrow \sigma_j^+\rangle & \langle P_i^\downarrow P_j^\downarrow\rangle \end{pmatrix},$$

where $P^\uparrow = \frac{1}{2}(1+\sigma^z)$ and $P^\downarrow = \frac{1}{2}(1-\sigma^z)$. The brackets symbolize the ground state and thermodynamic average values at zero temperature and a finite temperature, respectively, and σ^x, σ^y, and σ^z are Pauli matrices [21]. The concurrence between two spins is given by $C_j = \max(0,\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$, where λ_i is the square root of the eigenvalue of $R = \rho_{j,j+1}\tilde{\rho}_{j,j+1}$ and $\tilde{\rho}_{j,j+1} = (\sigma_j^y \otimes \sigma_{j+1}^y)\rho^*(\sigma_j^y \otimes \sigma_{j+1}^y)$. By applying the Jordan-Wigner transformation, the reduced density matrix can be written as

$$\rho_{i,j} = \begin{pmatrix} X_j^+ & 0 & 0 & 0 \\ 0 & Y_j^+ & Z_j^* & 0 \\ 0 & Z_j & Y_j^- & 0 \\ 0 & 0 & 0 & X_j^- \end{pmatrix},$$

where $X_j^+ = \langle n_j n_{j+1}\rangle (n_j = a_j^\dagger a_j)$, $Y_j^+ = \langle n_j(1-n_{j+1})\rangle$, $Y_j^- = \langle n_{j+1}(1-n_j)\rangle$, $Z_j = \langle a_j^\dagger a_{j+1}\rangle$, and $X_j^- = \langle 1 - n_j -$

$n_{j+1} + n_j n_{j+1}\rangle$. Thus the concurrence is transformed into

$$C_j = \max\{0, 2(|Z_j| - \sqrt{X_j^+ X_j^-})\}, \tag{7}$$

where

$$Z_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{ik}}{1+e^{\beta\varepsilon(k)}}dk, \tag{8}$$

$$n_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1+e^{\beta\varepsilon(k)}}dk, \tag{9}$$

where $\beta = \frac{1}{k_B T}$ and the Boltzmann constant is taken as $k_B = 1$. One should note that the Fermi distribution function is $f(k) = \frac{1}{1+e^{\beta\varepsilon(k)}}$. Using the solution of the retarded Green's function [58], X_j^+ approximately is obtained as $X_j^+ = \langle n_j\rangle^2 - Z_j^2$.

In following, we investigate the concurrence between NN spins for different values of J, D, and h and depict the behavior of concurrence with respect to each of the above parameters. The thermal behavior of the concurrence between NN spins in the pure spin $= 1/2$ Heisenberg XX model ($D = 0$) in a TF has been studied [21]. It has been found that the TE reduces by increasing temperature and will be zero at a critical temperature which has been shown to be independent of the TF. On the other hand, for the values of the TF that are more than the quantum critical TF, the amount of concurrence will be retrieved so that the revival phenomenon can happen for these values of the TF.

Now, we try to depict a physical picture of the DM effect on the thermal behavior of the concurrence in the introduced model. In Fig. 1, we present our analytical results on the behavior of the TE of the model in the absence of the DM interaction [panels (a) and (b)] and in the presence of the DM interaction [panels (c) and (d)]. It can be clearly seen that, for values of the TF less than the quantum critical point [Figs. 1(a) and 1(c)], the TE decreases with increasing the temperature and vanishes at a field-independent critical temperature (T_c). From the physical point of view, the number of excited states involved depends on temperature, where more states are added as the temperature is raised. This mixing of excited states with the ground state acts as a destructive noise that reduces the amount of entanglement contained in the system. When the temperature reaches a certain value, which varies based on the system's characteristics and parameter values, the amount of noise created by the excited states due to thermal fluctuations is sufficient to turn the system into a disentangled state. This temperature is known as the critical temperature, where below it the system is guaranteed to be entangled. In principle, at this temperature all quantum correlations will be destroyed by classical thermal fluctuations. Therefore at $T = T_c$, $C_{\rm Th} = 0$ and one can derive [21]

$$\langle n_j\rangle - \langle n_j\rangle^2 = -\sqrt{2}Z_j - Z_j^2. \tag{10}$$

In the absence of the TF, $\langle n_j\rangle = 1/2$ at any temperature. Thus the critical temperature can be found by solving the following equation:

$$\frac{\sqrt{2}-1}{2} = \frac{\widetilde{J}}{\pi T_c}\int_0^1 \frac{\sqrt{1-x^2}}{1+\cosh(\widetilde{J}x/T_c)}dx, \quad x = \cos(k), \tag{11}$$

which indicates that the critical temperature in the absence of the TF is related to the DM interaction as

$$T_c \simeq 0.48J\sqrt{1+D^2/J^2}, \tag{12}$$

which is smaller than the critical temperature in the two-qubit systems [48,50,51].

On the other hand for values of the TF greater than the quantum critical point [Figs. 1(b) and 1(d)], NN spins are not entangled at $T = 0$. By increasing the temperature from zero, NN spins remain unentangled up to the first critical temperature $T_{c_1}(h)$. As soon as the temperature increases from T_{c_1}, the TE regains and takes a maximum value and then decreases and reaches zero at the second critical temperature T_{c_2}. The existence of the second critical temperature is completely natural, since sufficiently large thermal fluctuations will destroy all classical and quantum correlations. It is seen that the amount of T_{c_1} increases when the external TF increases, but T_{c_2} is almost field independent. Therefore, the width of the temperature interval within which the NN spins become entangled gets smaller by increasing the TF. We have calculated numerically the width of this entangled region as a function of $h - h_c$ and the results show a linear scaling behavior as

$$T_{c_2} - T_{c_1} = 0.932 - 0.381 \times (h - h_c),\ h \geqslant h_c. \tag{13}$$

In addition, the maximum value of the entanglement in the mentioned temperature interval behaves as

$$C_{\rm Th}^{\max} = 0.334 - 0.017 \times (h - h_c)^2,\ h \geqslant h_c. \tag{14}$$

At the quantum critical TF $h = h_c$ and zero temperature, the system is at the quantum critical point and the entanglement is zero ($C_{\rm th} = 0$). There are many studies on the behavior of entanglement close to the quantum phase transition point. In a study of the role of temperature on the quantum properties

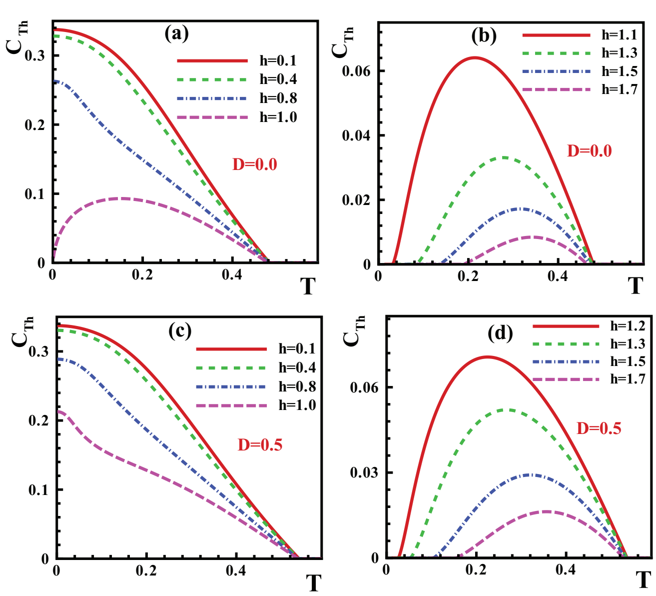

FIG. 1. (Color online) The TE between NN spins as a function of the temperature for values of the TF less than the quantum critical point (a) $D = 0$ and (c) $D = 0.5$ and larger than the quantum critical point (b) $D = 0$ and (d) $D = 0.5$.

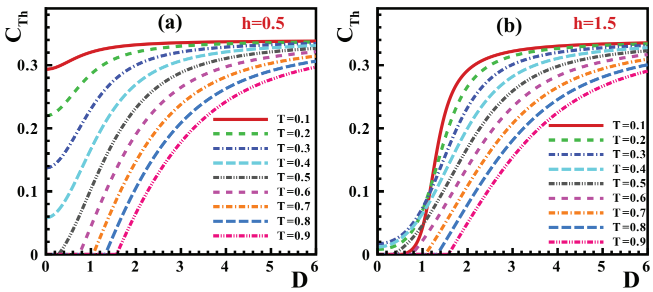

FIG. 2. (Color online) The thermal entanglement between NN spins as a function of the DM interaction for different values of temperature and the TF: (a) $h = 0.5$, (b) $h = 1.5$.

of entanglement [59], it is suggested that the entanglement sensitivity to thermal and to quantum fluctuations obeys universal finite-temperature scaling laws. In the following, we study the scaling behavior of TE in a finite-temperature neighborhood of h_c. One should note that the TE is not diverging at the quantum critical point but it is affected by the quantum criticality. We analyzed our analytical results and found that as soon as the temperature increased from zero, the TE between NN spins increased from zero and showed a scaling behavior as

$$C_{\text{th}} \propto T^{\varepsilon}, \tag{15}$$

with the critical exponent $\varepsilon = 0.70 \pm 0.04$. It is surprising that the mentioned critical exponent is almost the same as the critical exponent of the energy gap ($\varepsilon = 2/3$) [60,61] in the vicinity of this critical TF.

To have a deep insight into the nature of the system, we continue our study through the case of a fixed TF. In this case one can find the induced effects of the DM interaction on the quantum correlations of the spin-1/2 Heisenberg XX model at a finite temperature. We have presented our results in Fig. 2 for the exchange $J = 1$ and different values of the temperature. As is seen from Fig. 2, an increase in the DM interaction leads to an increase in the amount of the TE until it reaches its saturation value ($\simeq$1/3). Moreover, at low temperatures, this saturation value can be achieved at small values of the DM interaction, while as the temperature increases reaching this saturation value happens at larger values of the DM interaction due to the classical thermal fluctuations. The increasing of the TE with the DM interaction at a fixed temperature is a consequence of the fact that, when the DM interaction is turned on, the low-lying excited states tend to be more correlated. Note that the increasing behavior of the TE with respect to the DM interaction at low temperatures is field independent.

IV. ENTANGLEMENT WITNESS

In recent years, the realization that entanglement can also affect macroscopic properties of bulk solid-state systems has increased the interest in characterizations of entanglement in terms of macroscopic thermodynamical [8,9,62] observables. The entanglement witness is called an observation which can distinguish between entangled and separable states in quantum physics [63]. In principle, an entanglement witness has a positive expectation value for separable states and a negative one for some specific, entangled states. From an experimental point of view, several methods for the detection of entanglement using witness operators have been proposed [64]. As a result of these studies, entanglement witnesses have been obtained in terms of expectation values of thermodynamical observables such as internal energy, magnetization, and magnetic susceptibility.

Here, we define the entanglement witness as [26,65]

$$W = \frac{1}{\beta N}\frac{\partial \ln Z}{\partial \widetilde{J}} = \frac{1}{N}\sum_{j=1}^{N}\left(\left\langle \widetilde{S}_j^x \widetilde{S}_{j+1}^x\right\rangle + \left\langle \widetilde{S}_j^y \widetilde{S}_{j+1}^y\right\rangle\right)$$
$$= \frac{U + hM}{N\widetilde{J}}, \tag{16}$$

where $U = \langle H\rangle$ and $M = \sum_{j=1} \widetilde{S}_j^z$ are the total energy and the magnetization, respectively. Our witness is physically equivalent to the difference between the total energy (U) and the magnetic energy ($-hM$). If

$$\frac{|U + hM|}{N\widetilde{J}} > 0.25, \tag{17}$$

then the system is in an entangled state. In the absence of the magnetic field, the magnetization is zero and the thermodynamic witness reduces to $\frac{|U|}{N\widetilde{J}} > 0.25$. In this case the concurrence is given by $\max\{0, \frac{|U|}{N\widetilde{J}} - 0.25\}$.

Applying the fermionized operators, the entanglement witness is obtained as

$$W = \left|\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos k}{1 + e^{\beta\varepsilon(k)}}dk\right|. \tag{18}$$

Using this equation we have determined the parameter regions where entanglement can be detected in the solid state systems. Results are presented in Figs. 3(a) and 3(b). In the absence of the magnetic field [Fig. 3(a)], by adding the DM interaction, the critical temperature decreases and in the region $D \succeq 0.8$ the detection of the entanglement in the spin-1/2 XX Heisenberg solid state system from the entanglement witness $\frac{|U+hM|}{N\widetilde{J}}$ is impossible. The effect of the TF on the critical temperature is shown in Fig. 3(b). In the absence of the DM interaction, the critical temperature decreases with increasing the TF and cannot be determined experimentally for values of the TF $h \succeq 0.7$ in complete agreement with the results of Ref. [62]. In

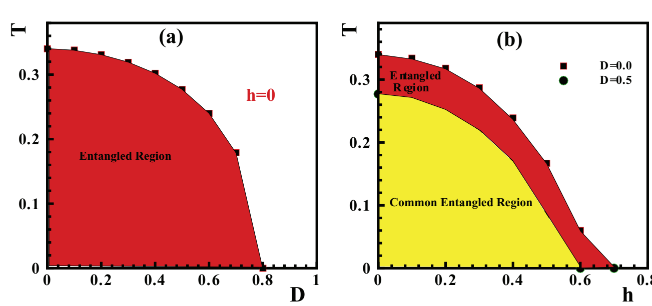

FIG. 3. (Color online) The parameter region of temperature T and (a) the DM interaction ($h = 0$) and (b) the magnetic field ($D = 0,0.5$). The thermodynamic witness $\frac{|U+hM|}{N\widetilde{J}}$ is more than 0.25 at temperatures less than the critical line which detects entanglement in the system.

addition, applying the DM interaction, the critical temperature decreases and, for example, in the region of the TF, $h(D = 0.5) \succeq 0.6$, is impossible to detect experimentally by focusing on the witness.

Comparing the results of TE [Eq. (7)] and the witness [Eq. (18)], some disagreement is seen. First, Eq. (12) shows that the critical temperature in the absence of the TF should be increased with increasing the DM interaction, which was not observed by measuring the thermodynamic witness. Second, the TE shows that the system will be entangled in an intermediate region of temperature ($T_{c_1} < T < T_{c_2}$) for fields larger than the quantum critical point, where the detection of this phenomenon is impossible by measuring the thermodynamic witness. In the following, we propose a tactic to resolve the mentioned disagreement.

With regard to the critical phenomena, it is known that the derivative of the response functions with respect to the control parameter can provide very useful and interesting results about the critical points. Inspired by this subject, instead of focusing on the witness we focus on the derivative of the witness with respect to the temperature:

$$\frac{dW}{dT} = \frac{\partial}{\partial T}\left\{\left|\frac{1}{N}\sum_{j=1}^{N}\left(\langle\widetilde{S}_j^x\widetilde{S}_{j+1}^x\rangle + \langle\widetilde{S}_j^y\widetilde{S}_{j+1}^y\rangle\right)\right|\right\} = \frac{\partial}{\partial T}\left(\frac{|U + hM|}{N\widetilde{J}}\right). \tag{19}$$

We know that the specific heat is defined as

$$C_v = \frac{\partial U}{\partial T}, \tag{20}$$

and on the other hand the derivative of the magnetization with respect to the temperature is known as the magnetocaloric effect:

$$-\left(\frac{\partial M}{\partial T}\right)|_h = (\delta Q/\delta h)/T, \tag{21}$$

where δQ is the amount of heat created or absorbed by the solid state sample for a field change δh due to the magnetocaloric effect. Thus, the derivative of our witness with respect to the temperature is physically equivalent to the difference between the specific heat (C_v) and the magnetocaloric effect $[-(\frac{\partial M}{\partial T})|_h]$.

Figure 4 shows the dW/dT as a function of temperature for the DM interaction $D = 0.5$ and different values of the TF: (a) $h = 0.5 < h_c$ and (b) $h = 1.5 > h_c$. As it is seen, the derivative of the witness with respect to the temperature shows one or two extrema at certain temperatures for values of the TF less or more than the quantum critical field. The same results are found for other values of the DM interaction. As a new approach, we suggest these certain temperatures as the critical temperatures at which to observe the entanglement in solid state systems.

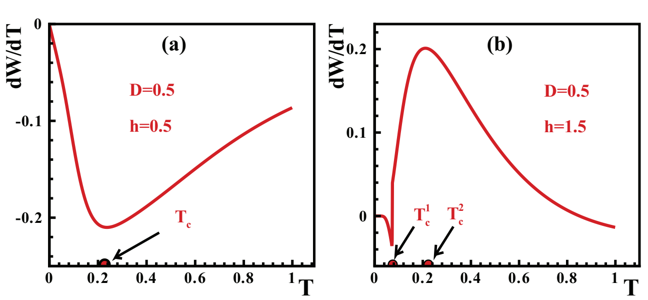

FIG. 4. (Color online) The derivative of our witness with respect to the temperature, dW/dT, as a function of the temperature for thre DM interaction $D = 0.5$ and different values of the TF: (a) $h = 0.5 < h_c$ and (b) $h = 1.5 > h_c$.

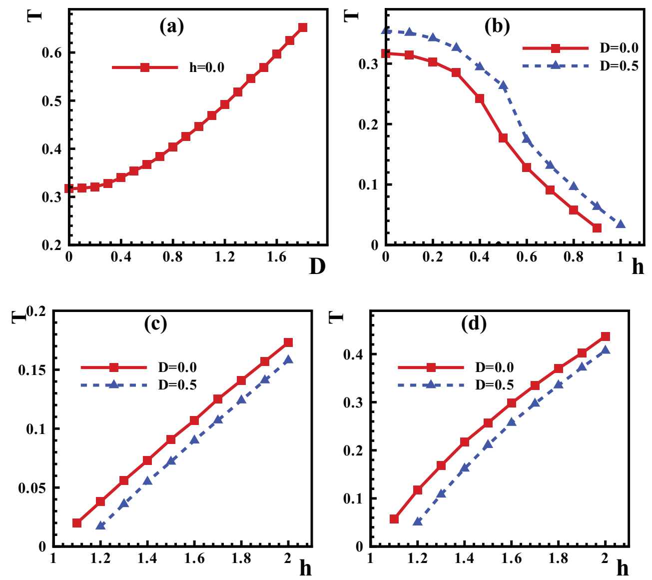

FIG. 5. (Color online) The parameter region of first critical temperature T and (a) the DM interaction ($h = 0$) and (b) the TF ($D = 0$ and 0.5). Panels (c) and (d) show the results on the first and second critical temperatures for values of the TF larger that the quantum critical field.

Using the mentioned approach, we have calculated the critical temperatures and the results are presented in Fig. 5. It can be clearly seen from Fig. 5(a) that, in the absence of the TF, the critical temperature at which the system is entangled below it increases by the increase in the DM interaction in complete agreement with TE [Eq. (12)]. The effect of the TF is studied in Figs. 5(b)–5(d). From Fig. 5(b) it can be seen that the critical temperature decreases with increasing the TF and vanishes exactly at the quantum critical field. The value of the critical temperature increases by increasing the DM interaction in the presence of the TF, which is also in agreement with the TE. As soon as the TF becomes larger than the quantum critical point, the first (T_{c_1}) and second (T_{c_1}) critical temperatures are observed. The first critical temperature, T_{c_1}, increases by increasing the TF [Fig. 5(c)], in complete agreement with the TE. But, the second one [Fig. 5(d)] also increases by the TF, which is not in agreement with the TE. Anyway, we believe that the increasing behavior must be related to increases of the energy gap in the saturated ferromagnetic phase.

V. CONCLUSION

We considered the 1D spin-1/2 XX model with added Dzyaloshinskii-Moriya interaction in a TF. At zero temperature, it has been found that the NN spins are entangled in the absence of the DM interaction and the TF. Adding the DM interaction does not affect the amount of entanglement between NN spins. However, since the TF causes a quantum phase transition into a saturated ferromagnetic phase, the entanglement between NN spins decreases with increasing the TF and will be zero at the quantum critical field $h = h_c(D)$.

In this work, we study the temperature dependence of the entanglement between NN spins using the fermionization technique. It is found that the TE in the region $h < h_c(D)$ decreases by thermal fluctuations and will be zero at a field-independent critical temperature $T = T_c(D)$, which shows that at this critical temperature all quantum correlations will be destroyed by classical thermal fluctuations. The critical temperature T_c increases by increasing the DM interaction. It is inferred that in the absence of the TF, although the DM interaction does not affect the amount of entanglement between NN spins at zero temperature, it can sufficiently affect it at a finite temperature.

At the critical field $h = h_c(D)$ and zero temperature, the system is at the quantum critical point and the entanglement is zero. The scaling behavior of the TE in a finite-temperature neighborhood of h_c is studied by analyzing our exact results. It is found that as soon as the temperature increases from zero, the TE increases and shows a scaling behavior as $C_{\text{th}} \propto T^{\varepsilon}$ with the critical exponent $\varepsilon = 0.70 \pm 0.04$, the same as the critical exponent of the energy gap in the vicinity of this critical field.

For values of the TF greater than the quantum critical point $h > h_c(D)$, by increasing the temperature from zero, NN spins remain unentangled up to a first critical temperature T_{c_1}. As soon as the temperature increases from T_{c_1}, the TE regains and takes a maximum value and then decreases and reaches zero at the second critical temperature T_{c_2}. The existence of the second critical temperature is completely natural, since sufficiently large thermal fluctuations will destroy all classical and quantum correlations.

Finally, an entanglement witness equivalent to the difference between the total energy (U) and the magnetic energy ($-hM$) is defined. Using an entanglement witness, the parameter regions where entanglement can be detected in the solid state system are determined. In the regions of $D(h = 0) \succeq 0.8$ and $h(D = 0) \succeq 0.7$, it is impossible to detect the entanglement in the XX Heisenberg solid state system. Generally, applying the DM interaction reduces the area of the entangled region of the spin-1/2 XX model in the TF. By comparing the results of the TE and our witness, some disagreements are presented. First, Eq. (12) shows that the critical temperature in the absence of the magnetic field should be increased by increasing the DM interaction, which was not observed by measuring the thermodynamic witness. Second, TE shows that the system will be entangled in an intermediate region of temperature ($T_{c_1} < T < T_{c_2}$) for fields larger than the quantum critical point, in which the detection of this phenomenon by measuring the thermodynamic witness is impossible. To resolve these disagreements, we have suggested focusing on the derivative of the witness with respect to the temperature which is physically equivalent to the difference between the specific heat (C_v) and the magnetocaloric effect $[-(\frac{\partial M}{\partial T})|_h]$. However, we suggest that it is possible to detect all critical temperatures from an experimental point of view if one focuses on the derivative of the witness with respect to the temperature.

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