# One-component dynamical equation and noise-induced adiabaticity

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The adiabatic theorem addresses the dynamics of a target instantaneous eigenstate of a time-dependent Hamiltonian. We use a Feshbach P-Q partitioning technique to derive a closed one-component integro-differential equation. The resultant equation properly traces the footprint of the target eigenstate. The physical significance of the derived dynamical equation is illustrated by both general analysis and concrete examples. We find an interesting phenomenon showing that a dephasing white noise can enhance and even induce adiabaticity. This phenomenon, distinguishing itself from any artificial control process, may occur in natural physical processes. We also show that particular white noises can shorten the total duration of dynamic processing, such as in adiabatic quantum computing.

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## I. INTRODUCTION

The adiabatic principle is a fundamental concept in quantum mechanics addressing quantum evolution governed by a slowly varying Hamiltonian [1,2]. It states that at any instant the system follows the original stationary and yet time-dependent eigenstate of the instantaneous $H(t)$. Because of its simplicity, the adiabatic principle has a variety of applications in quantum physics. The recent development in quantum information processing has reinforced the importance of the adiabatic principle by its wide-spread applications, such as the quantum adiabatic algorithm [3], fault-tolerance against quantum errors [4], and universal adiabatic and holonomic quantum computation [5–7] based on the Berry's phase [8–10]. The adiabatic principle also has great applications in quantum dynamics control, such as adiabatic passage [11–14], adiabatic gate teleportation [15], and many other protocols (e.g., see [16–21]). In these quantum state engineerings, the choice of initial and the target instantaneous eigenstates is varied depending on the interest of the issues under consideration.

Adiabaticity for a closed system is an idealization. In reality, all experimentally accessible systems are open because of inevitable interactions between systems and their surrounding environments [22,23]. While extensive works have been done in using the closed system adiabaticity combined with external control algorithms [11–14,20], adiabaticity has been theoretically extended into the context of open quantum systems [17,24], where the environmental noises often modify or even ruin a designed adiabatic passage. In special circumstances, noise can even be used to enhance the coherence or entanglement of quantum systems [25]. One example is the stochastic resonance [26,27] taking advantage of optimized noise amplitude; another one is the correlated noises [28] serving as a common bath. Nevertheless, contrary to intuition, we show that an external white noise can be used to enhance adiabaticity and even induce adiabaticity in a nonadiabatic regime. Our approach distinguishes itself from any artificial dynamical decoupling method or the application of the quantum Zeno effect [29]. To put our results into perspective, we present the adiabatic theorem by noting that the theorem essentially addresses the dynamics of one target instantaneous eigenstate or one component of the eigenvectors.

By using the Feshbach P-Q partitioning technique [30], we can derive a simple *one-component* integro-differential equation governing the target instantaneous eigenstate. The derived one-component dynamical equation can signal the onset of the adiabaticity if the integrand appearing in the integro-differential equation has a fast-varying factor, whether natural or engineered, such that the integral in the equation is small (or zero). Therefore the contribution of this term to the dynamics becomes negligible, leading to a general condition of adiabaticity for the system [25]. As to be shown below, a particular type of white noise can effectively induce the desired fast-varying factor so that adiabaticity can be established even when the original Hamiltonian is in a nonadiabatic regime.

The rest of this paper is organized as follows. In Sec. II, we use a Feshbach P-Q partitioning technique to derive a closed one-component integro-differential equation. We apply the equation to two models in the presence of external noisy fields in Sec. III to illustrate the noise-induced adiabaticity. Discussion and conclusion are given in Secs. IV and V, respectively.

## II. ONE-COMPONENT DYNAMICAL EQUATION

Given a time-independent Hamiltonian, the solution to the corresponding Schrödinger equation:

$$i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle, \tag{1}$$

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may be obtained from the instantaneous eigenequation of $H(t)$:

$$H(t)|E_n(t)\rangle = E_n(t)|E_n(t)\rangle, \tag{2}$$

where $E_n(t)$'s and $|E_n\rangle$'s are instantaneous eigenvalues and nondegenerate eigenvectors, respectively. A state at time $t$ can then be expressed as

$$|\psi(t)\rangle = \sum_n \psi_n(t)e^{i\theta_n(t)}|E_n(t)\rangle, \tag{3}$$

where $\theta_n(t) \equiv -\int_0^t E_n(s)ds$ is the dynamical phase. Substituting Eq. (3) into the Schrödinger equation (1), we obtain the following differential equation,

$$\partial_t\psi_m = -\langle E_m|\dot{E}_m\rangle\psi_m - \sum_{n\neq m}\langle E_m|\dot{E}_n\rangle e^{i(\theta_n-\theta_m)}\psi_n, \tag{4}$$

where $\langle E_m|\dot{E}_n\rangle = \frac{\langle E_m|\dot{H}|E_n\rangle}{E_n-E_m}$ $(n \neq m)$ according to Eq. (2). Without loss of generality, the target component can be denoted as $\psi_0$ corresponding to the target eigenstate $|E_0(t)\rangle$ of $H(t)$. In absence of any control approach, the adiabatic theorem is valid when $|\langle E_m|\dot{E}_n\rangle| \ll |E_n - E_m|$. The coefficient of the adiabatic wave function is then $\psi_0(t) = e^{i\beta_0(t)}$, where $\beta_0(t) \equiv i\int_0^t\langle E_0(s)|\dot{E}_0(s)\rangle ds$ is the geometric phase. Physically, the adiabatic theorem asserts that an initial eigenstate $|E_0(0)\rangle$ roughly remains the target instantaneous eigenstate $|E_0(t)\rangle$ at a later time. Equation (4) is also the Schrödinger equation for the vector $|\psi(t)\rangle = [\psi_0,\psi_1,\psi_2,\ldots]'$ with the effective "rotating representation" Hamiltonian. More explicitly, $H_{mn} = -i\langle E_m|\dot{E}_n\rangle e^{i(\theta_n-\theta_m)}$, which contains multiple variables $\psi_m$'s [31].

The adiabatic theorem addresses the dynamics of *one* target component $\psi_0$. Multiple variables are involved when the adiabatic conditions are not satisfied. Thus, it is often desirable to find an exact one-component dynamical equation that can be used to concisely trace the footprint of $\psi_0(t)$. It is shown in this paper that Feshbach P-Q partitioning may provide a very useful approach to realizing our goal.

In general, the state and the effective Hamiltonian in the Schödinger equation (1) can be always partitioned into the following form,

$$|\psi(t)\rangle = \left[\begin{array}{c} P \\ \hline Q \end{array}\right], \quad H = \left(\begin{array}{c|c} h & R \\ \hline W & D \end{array}\right), \tag{5}$$

where $h$ and $D$ correspond to the self-Hamiltonians living in the $P$ subspace and the $Q$ subspace, respectively; and $R$ and $W$ are their mutual correlation terms. Consequently, we have

$$i\partial_t P = hP + RQ, \tag{6}$$

$$i\partial_t Q = WP + DQ. \tag{7}$$

The formal solution to Eq. (7) can be written as

$$Q(t) = -i\int_0^t dsG(t,s)W(s)P(s) + G(t,0)Q(0), \tag{8}$$

where $G(t,s) = \mathcal{T}_{\leftarrow}\{\exp[-i\int_s^t D(s')ds']\}$ is a time-ordered evolution operator. Then we have

$$i\partial_t P(t) = h(t)P(t) - iR(t)\int_0^t dsG(t,s)W(s)P(s) + R(t)G(t,0)Q(0). \tag{9}$$

Assuming that initially $P(0) = 1$ and $Q(0) = 0$, then we have an exact dynamical equation for the $P$ part (any subspace we interested, here $P = \psi_0$):

$$\partial_t P(t) = -ih(t)P(t) - R(t)\int_0^t dsG(t,s)W(s)P(s). \tag{10}$$

In the case when $P = \psi_0$ is one-dimensional, $\psi_0(t)$ satisfies the following one-dimensional integro-differential equation,

$$\partial_t\psi_0(t) = -\langle E_0|\dot{E}_0\rangle\,\psi_0(t) - \int_0^t ds\, g(t,s)\psi_0(s), \tag{11}$$

where $g(t,s) = R(t)G(t,s)W(s)$ is an effective propagator playing a very important role in the analysis of adiabaticity. Here the vector $R \equiv [R_1,R_2,\ldots]$ with $R_m = -i\langle E_0|\dot{E}_m\rangle e^{i(\theta_m-\theta_0)}$, and $W = R^\dagger$. The matrix $D \equiv \sum_{mn} D_{mn}|m\rangle\langle n|$, where $D_{mn} = -i\langle E_m|\dot{E}_n\rangle e^{i(\theta_n-\theta_m)}$ $(m,n \geqslant 1)$. The first term on the right-hand side of Eq. (11) is the same as that in Eq. (4), which corresponds to the Berry's phase that may be switched off in a rotating frame. $|\psi_0(t)|^2$, the probability of finding the eigenstate $|E_0(t)\rangle$ at time $t$, is determined by the accumulation history of the product of the propagator $g(t,s)$ and $\psi_0(s)$.

With the exact dynamical equation (11), a crucial and general adiabatic condition can be cast into the following compact form,

$$\int_0^t ds\, g(t,s)\psi_0(s) = 0. \tag{12}$$

The condition is satisfied when $g(t,s) = 0$ or $g(t,s)$ is factored by a rapid oscillating function [10]. Mathematically, it is easy to understand that the integral of the product of the fast-varying $g(t,s)$ and the slow-varying $\psi_0(s)$ gives rise to a vanishing result. Clearly, the well-known adiabatic condition corresponds to the first-order approximation of this exact result.

Consider an effective two-level system (TLS) or qubit model in the rotating representation,

$$H(t) = \begin{pmatrix} -i\langle E_0|\dot{E}_0\rangle & -i\langle E_0|\dot{E}_1\rangle e^{i\int_0^t E(s)ds} \\ -i\langle E_1|\dot{E}_0\rangle e^{-i\int_0^t E(s)ds} & -i\langle E_1|\dot{E}_1\rangle \end{pmatrix}, \tag{13}$$

where $E \equiv E_0 - E_1$. When the TLS is initially in the eigenstate $|E_0\rangle$, the propagator $g(t,s)$ is given by

$$g(t,s) = -\langle E_0(t)|\dot{E}_1(t)\rangle\langle E_1(s)|\dot{E}_0(s)\rangle e^{\int_s^t(iE-\langle E_1|\dot{E}_1\rangle)ds'}. \tag{14}$$

Equations (11), (12), and (14) are the primary results to be used in analyzing adiabatic dynamics and passages.

### III. NOISE-INDUCED ADIABATICITY

An interesting phenomenon can be observed when a dephasing white noise is added to our one-component equation (11). As a randomly and quickly altered function of time, noise is typically a source of destruction leading to decoherence, but in some cases it may stabilize the quantum features rather than destroy them. We show that a type of noise can indeed induce required adiabaticity. Technically, we will show that noise can render the general adiabaticity condition (12) valid in the same way as the fast-varying function in $g(t,s)$. Crucial to our investigation of this issue is to find an appropriate physical model that can incorporate the required white noise.

One of the simplest physical models is a two-state dephasing model containing a white noise modifying the strength of a Hamiltonian in a random manner. Such a model can be easily obtained if we replace the characteristic energy $J_0$ in a Hamiltonian with $J_0 + c(J,W,t)$. Here $c(J,W,t) \equiv \sum_j c_j \delta(t - t_j)$ is a white noise ($c_j$ is the noise height); more specifically, it is a biased Poissonian white shot noise [32,33] satisfying $M[c(J,W,t)] = JW$, $J = M[c_j]$. Note that $J$ is the noise strength and $W$ measures the average frequency of noise shots (if not specifically mentioned, the "noise" always refers to the white noise throughout this paper). When $W$ goes to infinity, $c(J,W,t)$ becomes a continuous-time white noise denoted as $c(J,t)$. Note that the noise term only rescales the eigenvalues $E_m$'s to $[1 + c(J,t)/J_0]E_m$ but does not change the instantaneous eigenstates. Physically, the noise model considered here naturally arises in many physically interesting settings, such as a rotating spin that is subjected to a random magnetic field. By following the standard steps, we first solve the one-component equation (11) with random noises and do the ensemble average.

The most general time-dependent Hamiltonian of a TSL or qubit may be written as

$$H(t) = J_0\left(a\sigma^x + b\sigma^y + \frac{\omega}{2}\sigma^z\right), \tag{15}$$

where $a$ and $b$ describe the transverse fields and $\omega$ is the longitudinal fields. The instantaneous eigenstates of $H(t)$ can be expressed by

$$\begin{aligned} |E_0\rangle &= e^{-i\beta}\cos\alpha|\uparrow\rangle + \sin\alpha|\downarrow\rangle, \\ |E_1\rangle &= -e^{-i\beta}\sin\alpha|\uparrow\rangle + \cos\alpha|\downarrow\rangle, \end{aligned} \tag{16}$$

where $\beta = \tan^{-1}(b/a)$ and $\alpha = \cos^{-1}\frac{k+\omega}{\sqrt{2k^2+2k\omega}}$ with $k \equiv \pm\sqrt{\omega^2 + 4a^2 + 4b^2}$.

We now consider a simple case with external driving, in which $\omega$ is time independent, $a = \cos(\Omega t)$, and $b = \sin(\Omega t)$ with $\Omega$ is a constant frequency. The propagator for Model A (15) is

$$g(t,s) = \frac{\Omega^2}{k^2}\exp\left\{i\int_s^t [E(s') + \Omega\sin^2\alpha(s')]ds'\right\}, \tag{17}$$

where $E(s') = [J_0 + c(J,s')]k$. The model physically describes a spin-1/2 particle driven by a periodic magnetic field. If $\Omega = \omega$ and in the rotating framework, the original Hamiltonian (15) is $J_0\sigma_x$; and it becomes $[J_0 + c(J,t)]\sigma_x$ in the presence of "noise control", *which is a typical dephasing model and may be accessible experimentally*. When $\Omega$ approaches zero, the standard adiabaticity can be reached, which is shown by the blue dot-dashed curve ($J = 0$) in Fig. 1. Impressively, even when there exists a noise, adiabaticity is improved rather than destructed as shown by the other curves ($J \neq 0$). Thus in the adiabatic regime, it is shown that the stronger the noise is, the better adiabaticity is achieved. It is in stark contrast to our common understanding on how noise affects adiabaticity where noise is a source of disorder or a nuisance. A more important result is follows, that in the nonadiabatic regime,

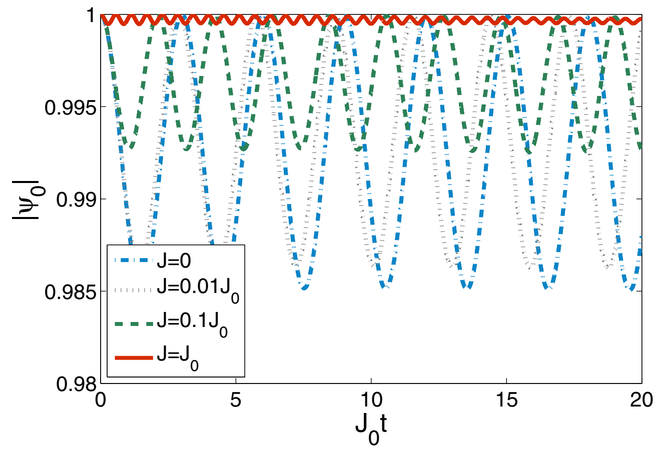


FIG. 1. (Color online) Model A: $|\psi_0(t)|$ for different noise strengths. $\Omega = 0.4J_0$ and $\omega = J_0$ are chosen in the adiabatic regime.

noise can even induce adiabaticity. Strong noise can push a system from a nonadiabatic regime into an adiabatic regime.

Consider the nonadiabatic regime where $\Omega = 5J_0$ and $\omega = 5J_0$. In Fig. 2, the blue dot-dashed curve depicts the noise free term $|\psi_0|$ which strongly oscillates from 1.0 to a minimum value 0.36. The system undergoes transitions between $|E_0\rangle$ and $|E_1\rangle$ constantly. The other curves show that $|\psi_0|$ can be decreased by increasing the noise strength $J$. For a weak noise with $J = 0.01J_0$ shown in the gray dotted curve, the minimum value of $|\psi_0|$ is increased to 0.46. When the noise is moderate ($J = 0.1J_0$), the green dashed curve shows the minimum value attains 0.85. When noise has $J = J_0$, it induces the nearly perfect adiabaticity as shown in the red solid curve.

It is worth emphasizing that the above phenomenon on the noise-induced adiabaticity is a remarkable instance showing that noise without any optimization or autocorrelation can play a positive role in inducing adiabaticity in a very simple system that may arise spontaneously in many contexts in physics. It

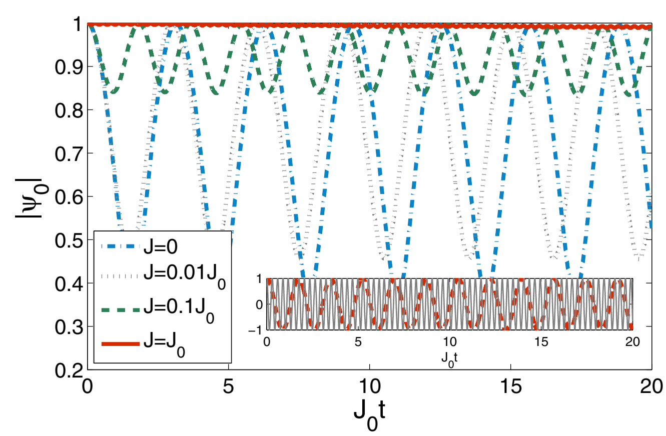


FIG. 2. (Color online) Model A: $|\psi_0(t)|$ for different noise strengths. $\Omega = 5J_0$ and $\omega = 5J_0$ are chosen far from the adiabatic regime. Inset plots the real part of fast-varying factor $e^{-i\int_0^s c(J,s')kds'}$ in gray solid curves and the slowly varying factor $e^{-i\int_0^s [J_0 k + \Omega\sin^2\alpha(s')]ds'}\psi_0(s)$ in red dashed curves. $g(t,s)\psi_0(s)$ is proportional to the product of these two factors.

reveals an interesting observation that the adiabatic process can be realized in quantum open systems in a way that is not seen in a closed quantum system.

Now we turn to Model B with two coupled two-level systems embedded in their individual baths,

$$H = J_0\big(a\,\sigma_1^+\sigma_2^- + a^*\sigma_1^-\sigma_2^+ + B_1\sigma_1^z + B_2\sigma_2^z\big). \qquad (18)$$

Here $c$ is noise-free parameter. $B_1 = B + \omega/4$ and $B_2 = B - \omega/4$, where $B$ is a noise but $\omega$ as a difference between $B_1$ and $B_2$ is noise free. Physically, the two TLSs are subject to a collective noise but to different external fields. When the system state is initially at a single-exciton state: $|\psi(0)\rangle = \mu|\uparrow\downarrow\rangle + \nu|\downarrow\uparrow\rangle$, $|\mu|^2 + |\nu|^2 = 1$, the effective Hamiltonian for this model could be written as $H_{\rm eff} = J_0[(a\,\sigma_1^+\sigma_2^- + {\rm H.c.}) + \omega(\sigma_1^z - \sigma_2^z)/4]$. The corresponding eigenstates of $H_{\rm eff}$ could be also expressed by Eq. (16) if $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are mapped into the two states for a TLS, $|\uparrow\downarrow\rangle \Rightarrow |\uparrow\rangle$ and $|\downarrow\uparrow\rangle \Rightarrow |\downarrow\rangle$. The mappings of operators are $\sigma_1^+\sigma_2^- \Rightarrow \sigma_+$ and $(\sigma_1^z - \sigma_2^z)/2 \Rightarrow \sigma_z$. We use a different group of time-dependent functions: $a = \frac{t}{T}$, and $\frac{\omega}{2} = 1 - \frac{t}{T}$. So the propagator for Model B (18) is

$$g(t,s) = \frac{4}{T^2k^2(t)k^2(s)}\exp\left[i\int_s^t E(s')ds'\right], \qquad (19)$$

where $k(t) = 2\sqrt{T^2 - 2tT + 2t^2}/T$. This model describes a finite time evolution defined by a period $T$. When $T \to \infty$, the system could follow an adiabatic passage from an eigenstate $|\uparrow\downarrow\rangle$ of $H(0) = J_0(\sigma_1^z - \sigma_2^z)/2$ to $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ of $H(T) = J_0(\sigma_1^+\sigma_2^- + {\rm H.c.})$. Of course, one can design different $H(0)$ and $H(T)$ representing different physics at will, yet the passage remains intact in the adiabatic regime.

We here consider nonadiabatic regime where $T = 1/J_0$. In Fig. 3, the blue dot-dashed curve depicts the noise-free $|\psi_0|$; it decays monotonically with time. Again, the other curves show the gradual onset of the adiabaticity induced by noise. The population for the system staying at $|E_0(T)\rangle$ is enhanced by increasing the noise strength. Physically, it means that the while noise sticks the system onto the eigenstate of $H(T)$, hence it induces adiabaticity in the nonadiabatic regime, with a much accelerated evolution time $T$. Figure 3 also shows that creation of adiabaticity does not even require strong noise strength. For instance, for $J = J_0$, $|\psi_0|$ is already maintained as high as above 0.99.

It is interesting to note that this specific system *suffers* from two types of noises. The first type of noise, characterized by $B$, acts on a time-dependent decoherence-free subspace (DFS) [34,35], hence gives rise to no effect on the dynamics. The second noise, embedded in the strength of the Hamiltonian, induces adiabaticity.

## IV. DISCUSSION

Adiabaticity has been shown to be achievable via an external white noise that can significantly modify the integral term contained in Eq. (11). Specifically, suppose that the evolution of $\psi_0$ is considered in a sequence of small intervals, $0 \sim t_1 \sim t_2 \sim t_3 \cdots$. If in each time interval the function $g(t_1,s)\psi_0(s)$ can be decomposed into a fast-varying time-dependent noise function and a slowly varying factor, then the resulting integral will be vanishingly small, which almost fixes $\psi_0(t_1)$ in both module and phase. Figure 4 clearly shows there is no significant deviation of the trajectories of $\psi_0(t)$ from the point of [1,0] on the complex plane in both noise-induced adiabaticity and adiabatic regime compared to that in nonadiabatic regime. Then the value of $\psi_0(t_1)$ will be fed back in the evaluation of $\psi_0(t_2)$, which also ends up with negligible changes. The rest can be done in a similar manner. Our noise model shifts the strength $J_0k(s')$ to $[J_0 + c(J,s')]k(s')$ in the oscillation function $e^{-i\int_0^s E(s')ds'}$ in Eqs. (14), (17), and (19). Insets in Figs. (2) and (3) display the noise-induced fast-varying factor and the rest part in the integrand and show how the fast function washes out the accumulation effect of the slow one.

It should be emphasized that the adiabaticity can also be induced by an external control field that gives rise to a fast-varying factor in $g(t,s)$, such as the artificial phase randomization [36] and weak measurement [37]. Notably, the natural noise requires ensemble average whereas the artificial

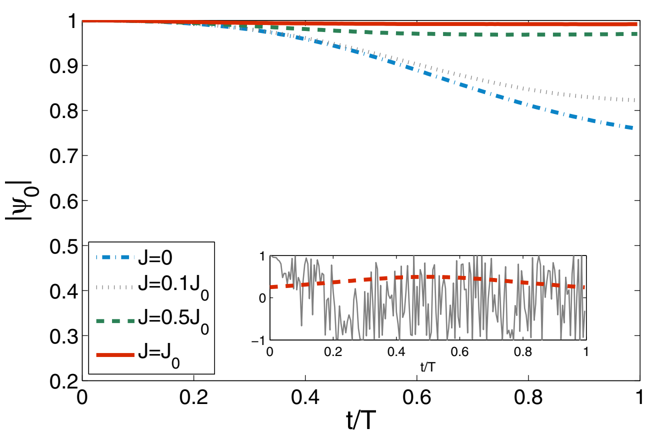


FIG. 3. (Color online) Model B: $|\psi_0(t)|$ for different noise strengths. $T = 1/J_0$ far from the adiabatic regime. Inset plots the real part of fast-varying factor $e^{-i\int_0^s c(J,s')kds'}$ in gray solid curves and the slowly varying factor $e^{-i\int_0^s J_0k(s')ds'}\psi_0(s)/k(s)^2$ in red dashed curves. $g(t,s)\psi_0(s)$ is proportional to the product of these two factors.

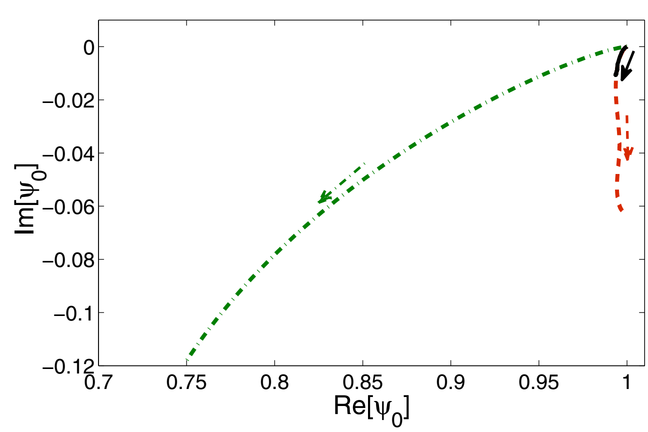


FIG. 4. (Color online) Model B: $\psi_0(t)$ in complex space. Green dot-dashed line for free evolution when $T = 1/J_0$ (nonadiabatic regime); red dashed line for free evolution when $T = 7/J_0$ (adiabatic regime); black solid line for $J = J_0$ under noise control.

control processes have no such a requirement. As such, if an artificial control makes the integral in Eq. (12) vanish, it will ensure the same effect. While on the contrary, even if a natural noise gives rise to a vanishing integral, it does not necessarily lead to an induced adiabaticity since the ensemble average often results in a mixed state rather than a pure instantaneous adiabatic eigenstate.

When the dynamics approaches an adiabatic regime $|\psi_0(t)| \approx 1$, the quantum state evolves on the eigenstate of $H(t)$, $|\psi(t)\rangle \approx |E_0(t)\rangle$. More precisely, the standard stochastic dynamics gives the system density matrix via $\rho(t) = M[|\psi(t)\rangle\langle\psi(t)|]$. $\rho(t) \approx |\psi_0(t)|^2|E_0(t)\rangle\langle E_0(t)| \approx |E_0(t)\rangle\langle E_0(t)|$ holds only if $|\psi_0(t)| \approx 1$. Generally, the dephasing white noise will drive the system to a mixed state where the off-diagonal matrix elements will vanish (whereas adiabaticity induced by control fields time-evolves unitarily).

## V. CONCLUSION

In conclusion, we employ the Feshbach P-Q partitioning technique to derive a one-component integro-differential equation, which naturally gives rise to a general adiabatic condition. Moreover, such a one-component dynamical equation can be used to demonstrate the onset of adiabaticity induced by the white dephasing noise. We work out two examples by analyzing the adiabatic conditions and numerically exhibiting the noise effect on adiabaticity. In addition, we show the significant reduction on the passage time to adiabaticity.

Our results can be applied to many ongoing physical implementations of quantum information and quantum computing protocols such as holonomic and adiabatic quantum computing and the fast energy transfer, where the induced adiabaticity may be embedded in uncontrollable natural noise.

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