Quantum wireless multihop communication based on arbitrary Bell pairs and teleportation

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Communication in quantum wireless multihop networks is useful in global quantum networks. We propose a scheme for faithful quantum communication in quantum wireless multihop networks, by performing quantum teleportation between two distant nodes which do not initially share entanglement with each other. The required entanglement among intermediate nodes is established through entanglement swapping based on arbitrary types of Bell pairs. All the measurement outcomes and the types of Bell pairs are sent to the destination node independently. The initial quantum state can be finally recovered by corresponding local operations. Our scheme can reduce end-to-end communication delay by using simultaneous measurements in contrast to the scheme based on sequential entanglement swapping.

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I. INTRODUCTION

Quantum physics provides completely new methods to create, transmit, manipulate, and store information. Hence in recent years a great interest has been aroused in quantum information science, such as quantum communication [1] and quantum computation [2]. Quantum communication is necessary for any future quantum networks, such as quantum Internet [3,4] and quantum wireless networks [5–7], irrespective of whether such communication is over distances of centimeters or thousands of kilometers [8–10].

Quantum teleportation [11] is usually utilized to achieve quantum communication. It employs a special form of bipartite maximally entangled state known as the Bell pair, where two physically separated quantum systems, such as electrons or photons, share a nonlocal correlation that Einstein famously referred as "spooky action at a distance" [12]. Although long-distance quantum teleportation has been realized in experiment [13–15] since the first experimental realization in 1997 [16], the point-to-point communication distance is limited due to losses in quantum channels. This limitation can be overcome by entanglement swapping [17,18], which aims to establish entanglement between two distant nodes via intermediate nodes. Entanglement swapping is of practical importance in quantum networks and is the basis of quantum repeaters [19,20].

Quantum communication schemes are important for an efficient quantum network [3]. Wireless technology has supplied more flexible and inexpensive ways in the classical communication field, while due to the quantum features of quantum networks, quantum wireless communication requires new communication mechanisms [6]. Cheng *et al.* [5] first proposed a quantum routing mechanism in a hierarchical network architecture to teleport a quantum state from one node to another even though they do not share Bell pairs mutually. Then Yu *et al.* [21] proposed a routing protocol for a wireless *ad hoc* quantum communication network and a distributed wireless network [7].

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In this paper, we propose a quantum communication scheme enabling quantum teleportation between two nodes which do not initially share entanglement mutually. The required Bell pairs are distributed to them by entanglement swapping via intermediate nodes. Here we consider a practical case where the Bell pairs among the intermediate nodes do not need to be the same type. Moreover, in contrast to the sequential entanglement swapping scheme commonly used in quantum repeaters, our scheme employs simultaneous Bell measurements in all intermediate nodes where the measurement results and the Bell pair types are sent to the destination node independently. While in the sequential entanglement swapping scheme, the Bell measurement results are sent to the upstream node and thus each intermediate node has to wait for the classical information transmission before performing entanglement swapping. These sequential operations introduce large delay to the whole quantum state communication process and, furthermore, the neighbor nodes have to build communication channels to exchange classical information. We show the simultaneous swapping scheme utilized in our paper can reduce the end-to-end communication delay compared with the sequential entanglement swapping scheme. In addition, the classical communication channels are more flexible and are not indispensable between neighbor nodes. The local operations to recover the initial quantum state are only required at the destination node. We make detailed calculations on the recovering operations corresponding to all combinations of measurement results and Bell pair types.

The rest of the paper is organized as follows. In Sec. II, the general multihop communication system is introduced. Teleportation is investigated in detail from one-hop to multihop cases and a state discriminant vector P is introduced in Sec. III. In Sec. IV, two methods are proposed to determine P and are applied to more general cases. Discussions and conclusions are drawn in Secs. V and VI.

II. QUANTUM WIRELESS MULTIHOP NETWORK SYSTEM

A quantum wireless multihop network (QWMN), as shown in Fig. 1, is composed of spatially separated wireless quantum

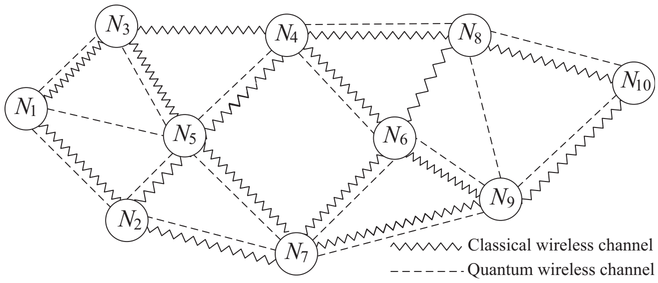

FIG. 1. Quantum wireless multihop network model.

nodes (WQNs). Most of them are distributed mobile quantum devices with both quantum and wireless communication capabilities. They can store quantum particles and process information. The data transmitted in QWMNs are mostly the quantum bits (qubits) instead of the classical bits (c-bits). Qubit is the unit of quantum information, which is represented as a linear combination of computational basis states $|0\rangle$ and $|1\rangle$. Generally, we use $|\chi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ to model the qubit, where α and β are complex probability amplitudes satisfying $|\alpha|^2 + |\beta|^2 = 1$.

In this paper, quantum teleportation is used to transfer a quantum state from one node to another with communication channels between them. There are two kinds of communication channels: classical wireless channels for transmitting c-bits and quantum wireless channels for teleporting qubits. In Fig. 1, the wave and dashed lines represent classical and quantum wireless channels, respectively.

Two nodes are called one-hop neighbors if they both fall within the mutual communication radius and thus can communicate with each other. Classical wireless channels exist between them. Two nodes which share at least one Bell pair are called quantum neighbors. Quantum wireless channels exist between them. Four types of Bell pairs used in quantum communication are defined as [22]

$$
\begin{aligned}
|\beta_{00}\rangle &\equiv |\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2},\\
|\beta_{01}\rangle &\equiv |\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2},\\
|\beta_{10}\rangle &\equiv |\Phi^-\rangle = (|00\rangle - |11\rangle)/\sqrt{2},\\
|\beta_{11}\rangle &\equiv |\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}.
\end{aligned} \tag{1}
$$

The Bell pairs can be shared by nodes in advance. They can be transformed into each other through unitary operations with *Pauli* matrices such as

$$
\begin{aligned}
&|\beta_{01}\rangle = X \otimes I\,|\beta_{00}\rangle\,, \quad |\beta_{10}\rangle = Z \otimes I\,|\beta_{00}\rangle\,,\\
&|\beta_{11}\rangle = iY \otimes I\,|\beta_{00}\rangle = ZX \otimes I\,|\beta_{00}\rangle\,,
\end{aligned} \tag{2}
$$

where $X = [\begin{smallmatrix}0 & 1\\ 1 & 0\end{smallmatrix}]$, $Y = [\begin{smallmatrix}0 & -i\\ i & 0\end{smallmatrix}]$, and $Z = [\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}]$ are *Pauli* matrices and $I = [\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}]$ is the identity matrix.

The entanglement is a precious resource in QWMNs and consumed in every teleportation process. It is impossible for one node to share Bell pairs with every WQN in the network. If there is no direct Bell pair shared between the source and destination nodes, entanglement swapping can be used to establish entanglement between them. Measurement outcomes and Bell pair types are transmitted in classical wireless channels. Two nodes could transfer quantum information if and only if both quantum and classical wireless channels, direct or hop by hop, exist simultaneously between them. A routing protocol establishes both classical and quantum communication paths in distributed way. The nodes on quantum paths are used to establish quantum channels and the nodes on classical communication paths are used to transmit classical information to the destination node for each node on quantum paths. Therefore, these two paths can be different. As shown in Fig. 1, for example, no direct quantum and classical channels exist between N_1 and N_4. However, with the help of the intermediate nodes N_5 and N_3, we could establish quantum and classical channels between them. For example, a quantum path can select the path $N_1 \rightarrow N_5 \rightarrow N_4$. N_1 can communicate with N_4 through $N_1 \rightarrow N_3 \rightarrow N_4$, while N_5 can communicate with N_4 directly. The nodes on quantum paths can transmit classical information to the destination node in different communication paths. The routing mechanism is not discussed in this paper and the paths are assumed to have been selected appropriately according to the network conditions and metrics using existing routing mechanisms in wireless networks and quantum networks [23–26]. Moreover, besides the multihop network our scheme can also function in a network where each node can send classical communication only to the quantum teleportation destination node.

III. QUANTUM TELEPORTATION BASED ON ARBITRARY TYPES OF BELL PAIRS

Quantum teleportation is widely used in many applications to transmit quantum states from one node to another separate node. With Bell pairs shared between two nodes, the sender performs the Bell measurement on its particle from the entangled pair and the qubit to be teleported. Then the sender sends the measurement results to the receiver in the classical channel. The receiver performs result-dependent unitary operations on its particle to recover the quantum state. The receiver would know nothing about the initial state and the teleportation would fail without the classical information.

A. One-hop quantum teleportation based on different types of Bell pairs

We first review the process of the standard quantum teleportation. The basic quantum circuit [22] for teleportation is given in Fig. 2. *Alice* first sends the qubit state to be teleported $|\chi\rangle$ and the particle shared from Bell pairs to the controlled-NOT (CNOT) gate, followed by a Hadamard (H) gate

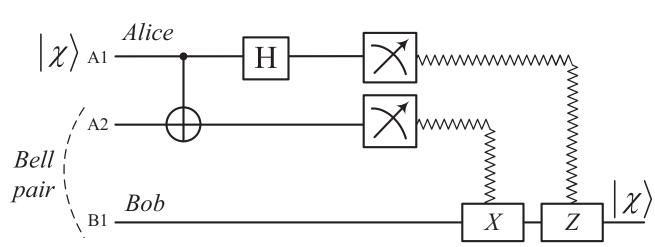

FIG. 2. Basic quantum circuit for quantum teleportation.

on the control state and two projection measurements on the computation basis states on the two particles. The CNOT gate, H gate, and the projection measurements together constitute the Bell measurement. The wave line denotes the classical information reported to the receiver through the classical wireless channel.

Assume *Alice* intends to send a qubit $|\chi\rangle$ to *Bob* and they share the Bell pair $|\beta_{11}\rangle = (|01\rangle - |10\rangle)_{A2B1}/\sqrt{2}$ in advance. The subscripts (e.g., $A2$, $B1$.) stand for the ordinal number at the owner's side. For example, $A2$ represents *Alice*'s second qubit. The state of the initial three-qubit system $|\Upsilon_{11}\rangle$ can be written as

$$\begin{aligned} |\Upsilon_{11}\rangle &= |\chi\rangle_{A1} \otimes |\beta_{11}\rangle_{A2B1} \\ &= (\alpha\,|0\rangle + \beta\,|1\rangle)_{A1} \otimes \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{A2B1} \\ &= \frac{1}{\sqrt{2}}[\alpha\,(|001\rangle - |010\rangle) + \beta\,(|101\rangle - |110\rangle)]_{A1A2B1}. \end{aligned} \tag{3}$$

After *Alice* sends her qubits through a CNOT gate and an H gate, the entire system state turns into

$$\begin{aligned} |\Upsilon'_{11}\rangle = \tfrac{1}{2}[&-|11\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) \\ &+ |10\rangle \otimes (\alpha|1\rangle + \beta|0\rangle) - |01\rangle \otimes (\alpha|0\rangle - \beta|1\rangle) \\ &+ |00\rangle \otimes (\alpha|1\rangle - \beta|0\rangle)]_{A1A2B1}. \end{aligned} \tag{4}$$

Then *Alice* performs projection measurements on her two qubits with the basis states $|11\rangle$, $|10\rangle$, $|01\rangle$, and $|00\rangle$. The measurement outcomes indicate that *Bob*'s qubit state changes to one of the four possible states accordingly: $\alpha\,|0\rangle + \beta\,|1\rangle$, $\alpha\,|1\rangle + \beta\,|0\rangle$, $\alpha\,|0\rangle - \beta\,|1\rangle$, and $\alpha\,|1\rangle - \beta\,|0\rangle$, which can be transformed from the original state $|\chi\rangle$ by *Pauli* operators, and hence the system state can be rewritten as

$$\begin{aligned} |\Upsilon'_{11}\rangle = \tfrac{1}{2}\,(&-|11\rangle \otimes |\chi\rangle + |10\rangle \otimes X\,|\chi\rangle \\ &- |01\rangle \otimes Z\,|\chi\rangle + |00\rangle \otimes XZ\,|\chi\rangle)_{A1A2B1}. \end{aligned} \tag{5}$$

Therefore, according to the two c-bits measurement result information sent by *Alice*, *Bob* can perform local unitary operations on his qubit to recover the original qubit state. For example, when the measurement result *Bob* gets is 10, his qubit state is $X\,|\chi\rangle$ and thus the X unitary operation is needed.

We should note that the minus signs before states $|11\rangle$ and $|01\rangle$ are global phases for the final states which have no affections to the teleportation results. Hence, in the following paper, we ignore them when we refer the measurement results and the final qubit states.

With analogy calculations we can obtain the final system states in one-hop quantum teleportation when the other three types of Bell pairs are shared. In Table I, measurement outcomes are listed out corresponding to the four possible qubit states in the receiver node for different types of Bell pairs shared. The relation between measurement outcomes and possible qubit states varies according to the types of Bell pairs shared. Consequently, different quantum unitary operations should be applied when the types of the Bell pairs shared between two nodes are different. For example, when the measurement outcome is 11, if $|\beta_{11}\rangle$ is shared, *Bob* does not need to do anything, while if $|\beta_{00}\rangle$ is shared, *Bob* then needs to apply both X and Z unitary operations to recover the original state.

TABLE I. Relation between measurement outcomes and possible qubit states.

	$\|\beta_{00}\rangle$	$\|\beta_{01}\rangle$	$\|\beta_{10}\rangle$	$\|\beta_{11}\rangle$
$\|\chi\rangle$	00	01	10	11
$X\,\|\chi\rangle$	01	00	11	10
$Z\,\|\chi\rangle$	10	11	00	01
$XZ\,\|\chi\rangle$	11	10	01	00

In the complex network environment, all shared Bell pairs are not guaranteed to be of the same type. Therefore, the Bell pair types shared in every hop are also essential for faithful teleportation and are crucial for determining the corresponding unitary operations to recover the final qubit state. Consequently, not only the measurement outcomes but also the Bell pair types should be sent to the destination node as the classical information. Thus, after doing the measurements simultaneously, four c-bits classical information should be reported independently from each node on path: two c-bits measurement outcomes and two c-bits Bell pair types.

B. Two-hop quantum teleportation

In QWMN, there is usually no direct Bell pair shared between the source and destination nodes. In this case, quantum communication is feasible in the multihop way, where entanglement swapping (ES) is used to distribute entangled particles to two nodes which have never interacted with each other before. For detail, as shown in Fig. 3, assume *Alice* wants to teleport a quantum state to *Bob*. They have no direct Bell pair shared, but the intermediate node *Candy* shares one Bell pair with *Alice* and another with *Bob*. When *Candy* performs Bell measurements on her two particles and sends measurement results to *Alice* and *Bob*, the remaining two particles at *Alice* and *Bob* sides become entangled. In this way, a quantum channel between *Alice* and *Bob* is established.

Figure 4 shows the quantum circuit for two-hop quantum teleportation, where compared with the basic one-hop one both *Alice* and *Candy* are required to transmit classical information to *Bob* through classical wireless channels. After collecting the information, *Bob* would apply the proper unitary operation(s) to recover the original state.

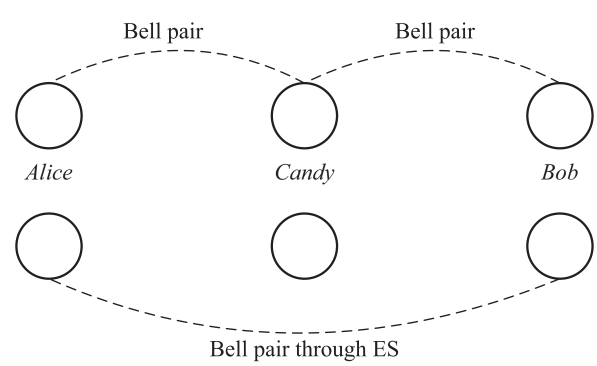

FIG. 3. Two-hop teleportation.

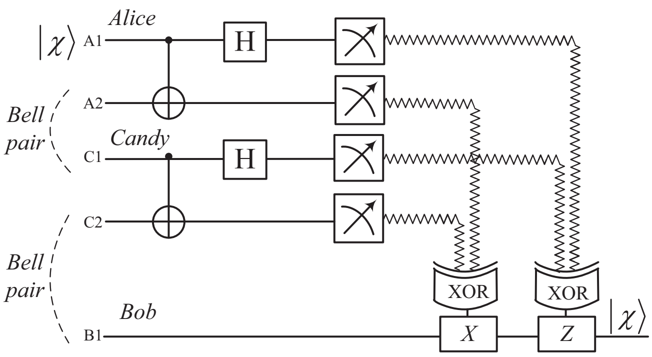

FIG. 4. Quantum circuit for two-hop communication.

We consider a practical network where the Bell pairs shared are not the same type. Here, for analysis in detail, we assume that *Candy* shares $|\beta_{00}\rangle$ and $|\beta_{11}\rangle$ with *Alice* and *Bob*, respectively. Then the four-qubit state of Bell pair channels has the form

$$|\varphi\rangle = |\beta_{00}\rangle_{A2C1} \otimes |\beta_{11}\rangle_{C2B1}. \tag{6}$$

When *Candy* sends her two qubits through a CNOT gate and her first qubit through an H gate, then we have

$$|\varphi\rangle \to \tfrac{1}{2}(|00\rangle_{C1C2} \otimes |\beta_{11}\rangle_{A2B1} - |01\rangle_{C1C2} \otimes |\beta_{10}\rangle_{A2B1} + |10\rangle_{C1C2} \otimes |\beta_{01}\rangle_{A2B1} - |11\rangle_{C1C2} \otimes |\beta_{00}\rangle_{A2B1}). \tag{7}$$

The 2 c-bits measurement outcome information transmitted by *Candy* indicates the type of the Bell pair *Alice* and *Bob* now share. With this Bell pair, *Alice* and *Bob* can perform quantum teleportation as normal. The entire five-qubit system state is written as

$$|\Upsilon_{0011}\rangle = |\chi\rangle_{A1} \otimes |\beta_{00}\rangle_{A2C1} \otimes |\beta_{11}\rangle_{C2B1}. \tag{8}$$

After *Alice* and *Candy* both perform CNOT gates and H gates, the entire system becomes

$$\begin{aligned} |\Upsilon'_{0011}\rangle = \tfrac{1}{4}[&(|0110\rangle - |1001\rangle - |1100\rangle - |0011\rangle)_{A1A2C1C2} \\ &\otimes (\alpha\,|0\rangle + \beta\,|1\rangle)_{B1} \\ &+ (|1000\rangle + |1101\rangle + |0010\rangle - |0111\rangle)_{A1A2C1C2} \\ &\otimes (\alpha\,|1\rangle + \beta\,|0\rangle)_{B1} \\ &+ (|1110\rangle - |1011\rangle - |0001\rangle - |0100\rangle)_{A1A2C1C2} \\ &\otimes (\alpha\,|0\rangle - \beta\,|1\rangle)_{B1} \\ &+ (|0000\rangle + |0101\rangle + |1010\rangle - |1111\rangle)_{A1A2C1C2} \\ &\otimes (\alpha\,|1\rangle - \beta\,|0\rangle)_{B1}]. \end{aligned} \tag{9}$$

In Eq. (9), all the 16 possible measurement outcomes are classified into four groups leading *Bob*'s qubit to four possible states. For example, when the outcomes are contained in the group of 0000, 1111, 0101, and 1010, *Bob*'s state should be $\alpha\,|1\rangle - \beta\,|0\rangle$ and both X and Z unitary operations are needed to recover the original state. Introducing the logical relation, if and only if the logic algebra expression $\overline{A1 \oplus C1} \cdot \overline{A2 \oplus C2} = 1$, *Bob*'s qubit state is $\alpha\,|1\rangle - \beta\,|0\rangle$. Here the original subscripts $A1$, $A2$, $C1$, and $C2$ are taken to denote the measurement outcomes 0 and 1. The signals "$\oplus$," "$\cdot$," and "–" represent logic exclusive or (XOR), AND, and negation, respectively. The logic algebra expressions show the logical relation of every group of measurement outcomes, which can be seen as the discriminant of the final qubit state. Thus we can express *Bob*'s qubit state as

$$\begin{aligned} |\Lambda\rangle = &(A1 \oplus C1 \cdot A2 \oplus C2) \times (\alpha|0\rangle + \beta|1\rangle)_{B1} \\ &+ (A1 \oplus C1 \cdot \overline{A2 \oplus C2}) \times (\alpha|1\rangle + \beta|0\rangle)_{B1} \\ &+ (\overline{A1 \oplus C1} \cdot A2 \oplus C2) \times (\alpha|0\rangle - \beta|1\rangle)_{B1} \\ &+ (\overline{A1 \oplus C1} \cdot \overline{A2 \oplus C2}) \times (\alpha|1\rangle - \beta|0\rangle)_{B1}. \end{aligned} \tag{10}$$

We can see that only one logical algebra expression can equal to 1, so that *Bob*'s final qubit state must be one of the four possible states. For instance, when the measurement outcome is 0110, $A1 = 0$, $A2 = 1$, $C1 = 1$, and $C2 = 0$. Substituting them into Eq. (10), we could obtain the final qubit state $|\Lambda\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. We define L_{00}, L_{01}, L_{10}, and L_{11} to denote the logic algebra expressions

$$\begin{aligned} L_{00} &= \overline{A1 \oplus C1} \cdot \overline{A2 \oplus C2}, \quad L_{01} = \overline{A1 \oplus C1} \cdot A2 \oplus C2, \\ L_{10} &= A1 \oplus C1 \cdot \overline{A2 \oplus C2}, \quad L_{11} = A1 \oplus C1 \cdot A2 \oplus C2. \end{aligned} \tag{11}$$

Then Eq. (10) can be rewritten as

$$|\Lambda\rangle = [L_{11}, L_{10}, L_{01}, L_{00}] \begin{bmatrix} |\chi\rangle \\ X\,|\chi\rangle \\ Z\,|\chi\rangle \\ XZ\,|\chi\rangle \end{bmatrix}. \tag{12}$$

The vector $[L_{11}, L_{10}, L_{01}, L_{00}]$ is the state discriminant vector to the possible qubit state vector $[|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T$. For different Bell pair types, we assume the possible qubit state vector is fixed. The state discriminant vector would vary on the sequence of the four logic algebra expressions in different Bell pair type cases.

C. State discriminant vectors for arbitrary Bell pair combinations

We first consider the affection of the Bell pair sequence on the state discriminant vector by making *Candy* share $|\beta_{11}\rangle$ and $|\beta_{00}\rangle$ with *Alice* and *Bob*, respectively. Compared with Eq. (6), the two Bell pairs are identical but with swapped sequence. After *Candy* performs the CNOT and H gates, the four-qubit state of Bell pairs evolves as follows:

$$\begin{aligned} |\varphi\rangle = &|\beta_{11}\rangle_{A2C1} \otimes |\beta_{00}\rangle_{C2B1} \\ &\to |00\rangle_{C1C2} \otimes |\beta_{11}\rangle_{A2B1} + |01\rangle_{C1C2} \otimes |\beta_{10}\rangle_{A2B1} - |10\rangle_{C1C2} \\ &\quad \otimes |\beta_{01}\rangle_{A2B1} - |11\rangle_{C1C2} \otimes |\beta_{00}\rangle_{A2B1}. \end{aligned} \tag{13}$$

We can see that Eqs. (7) and (13) have the same relation between measurement outcomes and possible Bell pairs ignoring the global phase factors. Therefore, *Bob*'s final qubit state has the same form according to the measurement outcomes as shown in Eq. (12). We can similarly examine all the other Bell pair combinations, and can reach the conclusion that the sequence of the Bell pairs on the paths has no affection on the state discriminant vectors for *Bob*'s final qubit state. We

TABLE II. State discriminant vectors P in the two-hop case for different Bell pair groups.

	$\lvert\beta_{00}\rangle_{A2C1}$	$\lvert\beta_{01}\rangle_{A2C1}$	$\lvert\beta_{10}\rangle_{A2C1}$	$\lvert\beta_{11}\rangle_{A2C1}$
$\lvert\beta_{00}\rangle_{C2B1}$	P_1	P_2	P_3	P_4
$\lvert\beta_{01}\rangle_{C2B1}$	P_2	P_1	P_4	P_3
$\lvert\beta_{10}\rangle_{C2B1}$	P_3	P_4	P_1	P_2
$\lvert\beta_{11}\rangle_{C2B1}$	P_4	P_3	P_2	P_1

define P as the state discriminant vector, which is one of the four possible values P_k ($k = 1, 2, 3, 4$) shown as follows:

$$\begin{aligned} P_1 &= [L_{00},L_{01},L_{10},L_{11}], \quad P_2 = [L_{01},L_{00},L_{11},L_{10}], \\ P_3 &= [L_{10},L_{11},L_{00},L_{01}], \quad P_4 = [L_{11},L_{10},L_{01},L_{00}]. \end{aligned} \tag{14}$$

Bob's qubit state can be then written as

$$|\Lambda\rangle = P \cdot [|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T. \tag{15}$$

We give the state discriminant vectors P for different Bell pair groups in the two-hop case corresponding to $[|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T$ in Table II. The rows represent the types of Bell pair shared between the first hop (*Alice* and *Candy*) with the subscript $A2C1$, while the columns show the shared Bell pair between the second hop (*Candy* and *Bob*) with the subscript $C2B1$. We can see that P is symmetrical because the sequence of the Bell pair has no influence on the state discriminant vector and the final qubit state.

D. Multihop quantum teleportation

The above calculations can be generalized to multihop quantum teleportation straightforwardly. For the n-hop case, we redefine the logic algebra expressions L_{00}, L_{01}, L_{10}, and L_{11} as

$$\begin{aligned} L_{00} &= \overline{\oplus_{i=1}^n N_i^1} \cdot \overline{\oplus_{i=1}^n N_i^2}, \quad L_{01} = \overline{\oplus_{i=1}^n N_i^1} \cdot \oplus_{i=1}^n N_i^2, \\ L_{10} &= \oplus_{i=1}^n N_i^1 \cdot \overline{\oplus_{i=1}^n N_i^2}, \quad L_{11} = \oplus_{i=1}^n N_i^1 \cdot \oplus_{i=1}^n N_i^2, \end{aligned} \tag{16}$$

where N_i^1 and N_i^2 denote the measurement outcomes of the first and second particles in the ith node, respectively, and $\oplus_{i=1}^n N_i^1 = N_1^1 \oplus N_2^1 \cdots \oplus N_n^1$, (the same for $\oplus_{i=1}^n N_i^2$).

Substituting Eq. (16) into Eq. (14), *Bob*'s qubit state can then be expressed as Eq. (15). This expression can also be applied to the one-hop case. Take $|\beta_{00}\rangle$ as Bell pair shared, for example. With the one-hop outcomes N_1, *Bob*'s qubit state could be written as

$$\begin{aligned} |\Lambda\rangle &= (\overline{A1} \cdot \overline{A2}) \times (\alpha|0\rangle + \beta|1\rangle)_B \\ &\quad + (\overline{A1} \cdot A2) \times (\alpha|1\rangle + \beta|0\rangle)_B \\ &\quad + (A1 \cdot \overline{A2}) \times (\alpha|0\rangle - \beta|1\rangle)_B \\ &\quad + (A1 \cdot A2) \times (\alpha|1\rangle - \beta|0\rangle)_B \\ &= P_1 \cdot [|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T. \end{aligned} \tag{17}$$

We give the state discriminant vector P corresponding to $[|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T$ in Table III. We can see the results coincide with those given by Table I.

In the n-hop case, there would be 4^n possible measurement outcomes. We need an effective way to determine the state discriminant vector P in the n-hop case with reasonable complexity. In the next section, we present two methods to determine the specific P corresponding to different cases.

TABLE III. State discriminant vectors P in the one-hop case with arbitrary Bell pair types.

	$\lvert\beta_{00}\rangle$	$\lvert\beta_{01}\rangle$	$\lvert\beta_{10}\rangle$	$\lvert\beta_{11}\rangle$
P	P_1	P_3	P_2	P_4

IV. GENERAL QUANTUM COMMUNICATION SCHEME

In this section we first give two methods for calculating the state discriminant vector P, and then describe the whole scheme followed by an example.

A. Determine P through derivation

The derivation for the discriminant vector P may be deduced by finding the discipline among the results in Table II. Let us reconsider the two-hop case shown in Fig. 4, where the Bell pair group is $|\beta_{00}\rangle$ and $|\beta_{11}\rangle$. The initial system state given by Eq. (8) can be rewritten as

$$|\Upsilon_{0011}\rangle = |\chi\rangle_{A1} \otimes |\beta_{00}\rangle_{A2C1} \otimes [(ZX \otimes I)|\beta_{00}\rangle]_{C2B1}. \tag{18}$$

After the two CNOT gates and H gates the system state becomes

$$\begin{aligned} |\Upsilon'_{0011}\rangle = &[(H \otimes I)U]_{A1A2} \otimes [(H \otimes I)U(I \otimes ZX)]_{C1C2} \\ &\otimes I_{B1}|\chi\rangle_{A1} \otimes |\beta_{00}\rangle_{A2C1} \otimes |\beta_{00}\rangle_{C2B1}, \end{aligned} \tag{19}$$

where U and H represent the CNOT and H operators given by

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{20}$$

By inserting the identity operator in the Hilbert space of qubits $C1$ and $C2$, $I_{C1C2} = U_{C1C2}(H_{C1} \otimes I_{C2})(H_{C1} \otimes I_{C2})U_{C1C2}$ in Eq. (19), it can be rewritten as

$$\begin{aligned} |\Upsilon'_{0011}\rangle = &[(H \otimes I)U]_{A1A2} \\ &\otimes [(H \otimes I)U(I \otimes ZX)U(H \otimes I)(H \otimes I)U]_{C1C2} \\ &\otimes I_{B1}|\chi\rangle_{A1} \otimes |\beta_{00}\rangle_{A2C1} \otimes |\beta_{00}\rangle_{C2B1} \\ = &(I \otimes I)_{A1A2} \otimes T_{C1C2}^{11} \otimes I_{B1}|\Upsilon'_{0000}\rangle, \end{aligned} \tag{21}$$

where

$$\begin{aligned} |\Upsilon'_{0000}\rangle = &[(H \otimes I)U]_{A1A2} \otimes [(H \otimes I)U]_{C1C2} \otimes I_{B1} \\ &\times |\chi\rangle_{A1} \otimes |\beta_{00}\rangle_{A2C1} \otimes |\beta_{00}\rangle_{C2B1}, \end{aligned} \tag{22}$$

is the system state after two CNOT and H gates when the Bell pair group is $|\beta_{00}\rangle$ and $|\beta_{00}\rangle$, and

$$\begin{aligned} T_{C1C2}^{11} &= [(H \otimes I)U(I \otimes ZX)U(H \otimes I)]_{C1C2} \\ &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{23}$$

can be seen as the transformation operator on qubits $C1$, $C2$, for transforming $|\Upsilon'_{0000}\rangle$ to $|\Upsilon'_{0011}\rangle$. To obtain the correction operation to the state discriminant vector P_1 for the Bell group $|\beta_{00}\rangle$, $|\beta_{00}\rangle$, we first calculate the transformation operator T^{11}_{C1C2} on the computational basis state, having

$$\begin{aligned} T^{11}|00\rangle &= -|11\rangle, \quad T^{11}|01\rangle = |10\rangle, \\ T^{11}|10\rangle &= -|01\rangle, \quad T^{11}|11\rangle = |00\rangle. \end{aligned} \tag{24}$$

Comparing the above transformation results with the logic algebra expressions given by Eqs. (11) and (14) we can infer the correction operations are $L_{00} \to L_{11}$, $L_{01} \to L_{11}$, $L_{10} \to L_{01}$, and $L_{11} \to L_{00}$ or, equivalently, $P_1 \to P_4$. Then we can define a correction matrix C_3 to the state discriminant vector P_1 satisfying

$$P_4 = P_1 C_3, \quad \text{with} \quad C_3 \equiv \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

With analogy calculations, we may find when Bell pair groups are $|\beta_{00}\rangle$, $|\beta_{01}\rangle$ and $|\beta_{00}\rangle$, $|\beta_{10}\rangle$ the transformation operators from $|\Upsilon'_{0000}\rangle$ to $|\Upsilon'_{0001}\rangle$ and $|\Upsilon'_{0010}\rangle$ are

$$T^{01}_{C1C2} = [(H \otimes I)U(I \otimes X)U(H \otimes I)]_{C1C2}, \tag{26}$$

$$T^{10}_{C1C2} = [(H \otimes I)U(I \otimes Z)U(H \otimes I)]_{C1C2}, \tag{27}$$

respectively. The corresponding correction matrices to the state discriminant vector P_1 can be found as

$$C_1 \equiv \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad C_2 \equiv \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{28}$$

which satisfy $P_2 = P_1 C_1$ and $P_3 = P_1 C_2$. For completion, we define

$$C_0 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{29}$$

as the correction matrix for P_1 itself. Then, as Eq. (15), we can write *Bob*'s qubit state,

$$|\Lambda_{0011}\rangle = P_1 C_0 C_3 \cdot [|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T, \tag{30}$$

with the discriminant vector $P = P_4 = P_1 C_0 C_3$.

It can be easily verified that the four matrices C_0, C_1, C_2, and C_3, are commutative, which is in accordance with our previous statement that the sequence of Bell pair types does not affect the state discriminant vector P. Therefore, Bell pair types and corresponding numbers would determine the state discriminant vector P. The type of Bell pairs in each hop is transmitted to the receiver together with the measurement outcomes. The number of Bell pairs shared along the path is equal to the number of quantum hops. Therefore, when the system comes to the n-hop case, the system state with n Bell pairs is

$$\begin{aligned} |\Upsilon\rangle &= |\chi\rangle_{A1} \otimes |\beta_{00}\rangle^{\otimes a_0} \otimes |\beta_{01}\rangle^{\otimes a_1} \otimes |\beta_{10}\rangle^{\otimes a_2} \otimes |\beta_{11}\rangle^{\otimes a_3} \\ &= I \otimes (I \otimes I)^{\otimes a_0} \otimes (X \otimes I)^{\otimes a1} \otimes (Z \otimes I)^{\otimes a_2} \\ &\quad \otimes (ZX \otimes I)^{\otimes a_3} |\chi\rangle_{A1} |\beta_{00}\rangle^{\otimes n}. \end{aligned} \tag{31}$$

where a_0, a_1, a_2, and a_3 denote the numbers of the four types of Bell pairs shared on quantum paths and their sum is equal to quantum hop count n. Then *Bob*'s qubit state can be written as

$$\begin{aligned} |\Lambda\rangle &= P_1 C_0^{a_0 \bmod 2} C_1^{a_1 \bmod 2} C_2^{a_2 \bmod 2} C_3^{a_3 \bmod 2} \\ &\quad \times [|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T \\ &= P \cdot [|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T, \end{aligned} \tag{32}$$

with the state discriminant vector

$$P = P_1 \cdot \prod_{i=0}^{3} C_i^{a_i \bmod 2}. \tag{33}$$

Note that modulo-2 arithmetic is applied on the type numbers to reduce the matrix calculation complexity since the operators have the properties ${C_0}^2 = {C_1}^2 = {C_2}^2 = {C_3}^2 = I_4$, where I_4 is the identity matrix in four-dimensional space.

B. Determine the parameter P through finite state machine

We have found that P_1, P_2, P_3, and P_4 can transform into each other with correction matrices defined by Eqs. (25), (28), and (29). The transformations of state discriminant vectors can form an finite state machine (FSM) as shown in Fig. 5. The circles denote the four state discriminant vectors, and arrow lines represent the transformations of the state discriminant vectors under the effect of the Bell pairs marked on the arrow lines. With the help of the FSM, the vector P in Eq. (15) can be determined by the Bell pairs shared between intermediate nodes along the path. Moreover, the state transforms back when there are an even number of Bell pairs with the same type. This property is analogous to modulo-2 arithmetic in Eq. (32).

Without loss of generality, assume P_1 as the initial state discriminant vector. Take the Bell pair group $|\beta_{00}\rangle$, $|\beta_{11}\rangle$, and $|\beta_{10}\rangle$ as an example to illustrate the function of the FSM.

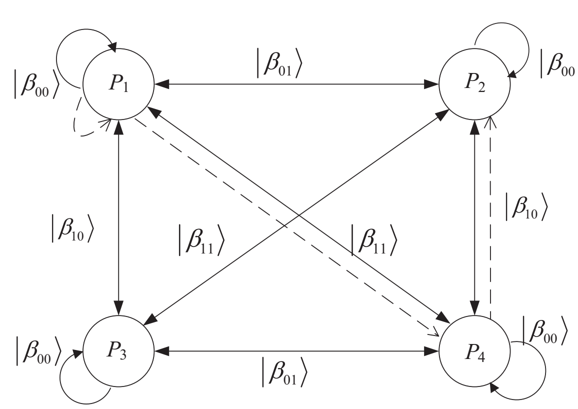

FIG. 5. Finite state machine model to obtain state discriminant vectors.

It is a three-hop communication with parameter $n = 3$. The vector starts from P_1. Under the effect of $|\beta_{00}\rangle$, the vector remains unchanged. Then it changes to P_4 because of the Bell pair $|\beta_{11}\rangle$ and finally goes to P_2. This process is marked on Fig. 5 with dashed arrow lines. Hence we get $P = P_2 = [L_{01}, L_{00}, L_{11}, L_{10}]$; the final qubit state is

$$|\Lambda\rangle = P_2 \cdot [|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T. \tag{34}$$

Substituting the measurement outcomes into Eq. (34), we can obtain the final qubit state and the corresponding unitary operations for recovering the original quantum state.

C. Description of the whole scheme

The whole scheme for communication in multihop networks based on arbitrary Bell pairs and teleportation is shown in Fig. 6. In the QWMN, the nodes on quantum paths share Bell pairs with adjacent nodes. They also have classical paths connecting the destination node. First, every node on quantum paths except the destination node performs measurements on its two qubits independently. The measurements do not depend on any information from other nodes and can be carried out immediately after the nodes are selected on quantum paths. After measurements, each of the source and intermediate nodes sends four c-bits information to the destination node, including two c-bits measurement outcomes and two c-bits Bell pair types shared with its upstream node. The transmissions of classical information are in a wireless hop-by-hop way. The wireless communication paths can be different from quantum paths. Finally, when the destination node has gathered all classical information reported, it would do the following three steps.

(1) Perform XOR on measurement outcomes of corresponding particles to calculate the logical relation among them.

(2) Determine the state discriminant vector P through two methods introduced above.

(3) Via the state discriminant vector P determine the final qubit state and corresponding unitary operations.

The above three steps at the destination node are integrated together into a generalized formula given by Eq. (32). It can be seen as a black box, the input of which are measurement outcomes and Bell pair types, and the output of which is the required unitary operations. Finally, the corresponding unitary operations are performed to reconstruct the original state.

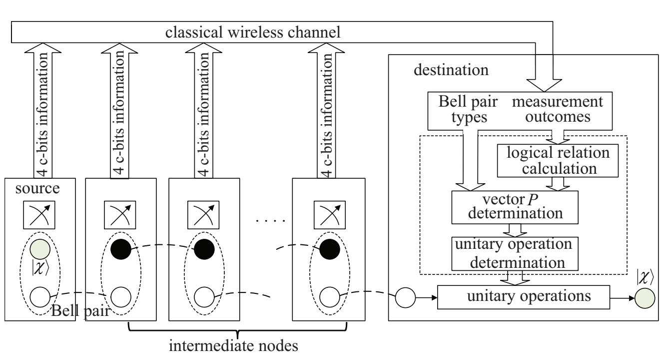

FIG. 6. (Color online) Scheme for quantum communication in QWMN.

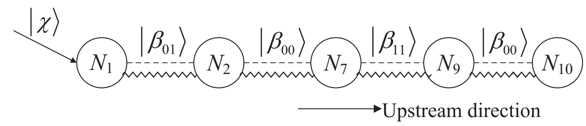

FIG. 7. Example of multihop quantum communication. (The selected path on topology of Fig. 1.)

D. Example of communication in QWMN

An example is presented to better illustrate the whole process of communication in the QWMN. As shown in Fig. 1, assume the source node N_1 intends to transmit a qubit state $|\chi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ to the destination node N_{10}. However, there is no available Bell pair shared between N_1 and N_{10}. To achieve successful transmission, a multihop path has been selected according to specific metrics. We choose the path $N_1 \to N_2 \to N_7 \to N_9 \to N_{10}$ for simplicity, where the quantum and classical paths are assumed to be the same. We model it as a line of nodes shown in Fig. 7.

The direction to the destination node is defined as upstream direction. Along this direction, the Bell pair $|\beta_{01}\rangle$, $|\beta_{00}\rangle$, $|\beta_{11}\rangle$, and $|\beta_{00}\rangle$ are shared between adjacent nodes as marked above dashed lines in Fig. 7. There are also classical wireless channels available so that every node can report classical information to the destination node N_{10}. When the communication starts, the source and intermediate nodes measure their two particles which are entangled with adjacent nodes. Each of the three intermediate nodes and the source node produces two c-bits measurement outcomes, one c-bit for the first particle and the other for the second one. Then these four nodes transmit these two c-bits measurement outcomes and two c-bits Bell pair types to the destination node independently through classical wireless channels hop by hop. After the destination node N_{10} has successfully collected all information from these four nodes, it calculates the logical relation among measurement outcomes. Assume the measurement outcomes on N_1^1, N_1^2, N_2^1, N_2^2, N_7^1, N_7^2, N_9^1, N_9^2 are 0, 1, 0, 0, 1, 1, 1, 0, so that $P_1 = [1,0,0,0]$. Then substituting $a_0 = 2$, $a_1 = 1$, $a_2 = 0$, $a_3 = 1$, and $n = 4$ into Eq. (33), we obtain

$$\begin{aligned}
P &= P_1 \cdot \prod_{i=0}^{3} C_i^{a_i \bmod 2} \\
&= P_1 \cdot \begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix}^{2 \bmod 2} \begin{bmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0 \end{bmatrix}^{1 \bmod 2} \\
&\quad \times \begin{bmatrix} 0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0 \end{bmatrix}^{0 \bmod 2} \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0 \end{bmatrix}^{1 \bmod 2} \\
&= [1,0,0,0] \cdot \begin{bmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0 \end{bmatrix} \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0 \end{bmatrix} \\
&= [0,0,1,0] = P_3.
\end{aligned} \tag{35}$$

Then substituting P into Eq. (15), we obtain the final qubit state at the node of N_{10}

$$
\begin{aligned}
|\Lambda\rangle &= [0,0,1,0]\cdot[|\chi\rangle, X|\chi\rangle, Z|\chi\rangle, XZ|\chi\rangle]^T \\
&= Z|\chi\rangle = \alpha|0\rangle - \beta|1\rangle .
\end{aligned} \tag{36}
$$

Therefore, the required unitary operation to recover the state $|\chi\rangle$ should be Z. Until then, the whole process of quantum multihop communication has been completed. The original state has been teleported to the destination N_{10} from the source N_1 successfully.

We can also determine P through the state transformation with FSM depicted in Fig. 5. With the FSM, we can also obtain the same result $P = P_3 = [0,0,1,0]$ and thus the same final qubit state and the required operations.

V. DISCUSSIONS

In our scheme, a simultaneous and independent entanglement swapping method is utilized. The source and intermediate nodes perform measurements and do not depend on any specific sequence. Then each node sends four c-bits information to the destination node independently, including two c-bits measurement outcomes and two c-bits Bell pair types. After gathering all this information, the destination node determines the corresponding unitary operations and recovers the qubit state. For discussion, we refer to our scheme as scheme A.

As we referred to in Sec. I, the commonly used method in quantum repeaters is the sequential entanglement swapping scheme, which is different from our scheme. We refer to it as scheme B. The intermediate nodes on quantum paths perform entanglement swapping sequentially. It is not necessary for the destination node to know all the Bell pair types in the network. Starting from the intermediate node closest to the source node, the intermediate node performs entanglement swapping one by one and sends two c-bits Bell pair types created to its upstream node. The quantum channels from the source node to the destination node are established by these sequential entanglement swapping operations. Then the source node teleports the quantum state and transmits two c-bits measurement outcomes to the destination node for initial state recovery. For the example shown in Fig. 7, the scheme B works as follows.

(1) N_2 performs entanglement swapping with N_7, creating a quantum channel between N_1 and N_7. Because N_2 knows the state of its shared entanglement with N_1 and N_7, only two c-bits need to be communicated from N_2 to N_7.

(2) N_7 performs entanglement swapping with N_9, transmitting two c-bits to N_9, creating entanglement between N_9 and N_1.

(3) N_9 performs entanglement swapping with N_{10}, transmitting two c-bits to N_{10}, creating entanglement between N_1 and N_{10}. Thus N_{10} knows that the entanglement swapping is completed.

(4) N_1 teleports its qubit to N_{10}, transmitting two c-bits measurement outcomes to N_{10} (hopping through N_2, N_7, and N_9).

In this section, we discuss the performance of our scheme and make comparisons on our scheme A and the scheme B. Two performance metrics are adopted including classical wireless communication cost and quantum communication delay.

A. Classical wireless communication cost

We define classical wireless communication cost as the number of the data transmission required in the scheme. It includes all the classical information transmitted by the nodes. Therefore, the cost of one transmission is the product of the number of data and the hop number that the information need to be transmitted. The total communication cost is the sum of all the transmissions in the process. Let H_d^i denote the classical wireless communication hop count from the ith node to the destination node on quantum paths, where $i = 1,\ldots,N-1$, with $i = 1$ for the source node and N represents the number of nodes on quantum paths. Therefore, the classical wireless communication cost of scheme A is

$$
C_A^{\text{c-bit}} = 4 \times \sum_{i=1}^{N-2} H_d^i + 2H_d^{N-1}. \tag{37}
$$

Note that the destination node knows the Bell pair type they share and thus the last hop node only needs to transmit its two c-bits measurement outcomes to the destination node. In the example shown in Fig. 7, the wireless communication paths to the destination node are the same with the quantum paths. Therefore, in this case $H_d^i = N - i$, and the cost is then $\widetilde{C}_A^{\text{c-bit}} = 4 \times \sum_{i=1}^{N-2}(N-i) + 2 \times 1 = 2(N^2 - N - 1)$. For $N = 5$, the classical wireless communication cost is 38 c-bits.

Although the measurement outcomes discussed above are in the unit of "c-bit," in realistic communication they are all transmitted in the form of packet. In classical wireless communications protocol, such as Wimax, WiFi, and LTE, a packet usually requires at least 30 Bytes [27,28]. It seems that the communication cost is more reasonable to be expressed in the unit of "packet," which contains the useful information and communication overhead. Consequently, the classical wireless communication cost of scheme A is written as

$$
C_A^{\text{packet}} = \sum_{i=1}^{N-1} H_d^i, \tag{38}
$$

and 10 packets are required in the example shown in Fig. 7.

In scheme B, the classical information is exchanged between intermediate nodes. The classical wireless communication cost is then

$$
C_B^{\text{c-bit}} = 2 \times \sum_{i=2}^{N-1} H_u^i + 2H_d^1, \tag{39}
$$

where H_u^i denotes the hop count from the ith node on quantum paths to its upstream $(i+1)$th node. Explicitly, $2 \times \sum_{i=2}^{N-1} H_u^i$ is the transmission among intermediate nodes, and $2H_d^1$ represents the measurement outcomes of teleportation transmitted from the source node. In the example shown in Fig. 7, the classical wireless communication cost is then $\widetilde{C}_B^{\text{c-bit}} = 2 \times \sum_{i=2}^{N-1} 1 + 2(N-1) = 4N - 6$. Therefore, for $N = 5$, scheme B requires 14 c-bits. Similarly, in the unit

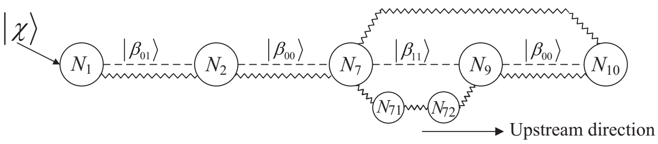

FIG. 8. Multihop quantum communication without direct wireless paths between adjacent nodes.

of packet, we have

$$C_B^{\text{packet}} = \sum_{i=2}^{N-1} H_u^i + H_d^1, \tag{40}$$

and scheme B needs seven packets in the example shown in Fig. 7.

We can see that the communication cost of scheme A is more than scheme B in the example case. However, this is not always true in complicated network environment. When the path and topology change, the cost difference could also change. For example, when there is no direct classical wireless communication path between neighbor nodes in quantum paths, as shown in Fig. 8, the wave and dashed lines denote wireless classical and quantum communication channels, respectively. The node N_7 needs to hop through N_{71} and N_{72} to communicate with its neighbor node N_9. However, N_7 can communicate with the destination node N_{10} directly. In this case, our scheme needs less packets. Generally, comparing Eqs. (38) and (40), we have $C_A^{\text{packet}} \leqslant C_B^{\text{packet}}$, when $\sum_{i=2}^{N-1} H_d^i \leqslant \sum_{i=2}^{N-1} H_u^i$. It indicates that when the sum of hop count from each intermediate node to destination is less than or equal to the sum of hop count between each intermediate node and its upstream node, scheme A requires less or equal packets than scheme B. We can also find the condition for scheme A requiring less or equal communication cost in unit of "c-bit" than scheme B, which is $2\sum_{i=2}^{N-2} H_d^i + H_d^1 \leqslant \sum_{i=2}^{N-2} H_u^i$.

B. Quantum communication delay

The measurements and wireless communication introduce extra delay in quantum communication, which cannot be ignored, explicitly including node processing delay, wireless medium access delay, the transmission, and propagation delay. The short delay is pursued due to the limited decoherence time in quantum memory and QOS (quality of service) need. Assume each measurement takes D_m seconds and each wireless communication takes D_w seconds.

In scheme A, the measurements are performed without dependence on any measurement sequence. The wireless communication delay is equal to the time needed for one packet transmitted from the farthest node to the destination node, because when the farthest node sends its classical information, the other intermediate nodes can send theirs simultaneously without any mutual interference. The farthest node is defined as the node which has the biggest hop count to destination, i.e., $H_f = \max\{H_d^i\}$. Therefore, the total quantum communication delay of scheme A is written as

$$D_A = D_m + D_w \times H_f = D_m + D_w \times \max\left\{H_d^i\right\}. \tag{41}$$

In scheme B, the measurements and wireless communication are performed in sequence. The intermediate node depends on the two c-bits information from its downstream node and only performs measurements for entanglement swapping after receiving this information. Therefore, the total communication delay of scheme B is

$$\begin{aligned} D_B &= \sum_{i=2}^{N-1} \left(D_m + D_w H_u^i\right) + \left(D_m + D_w H_d^1\right) \\ &= (N-1)D_m + D_w \left(H_d^1 + \sum_{i=2}^{N-1} H_u^i\right). \end{aligned} \tag{42}$$

Since $\max\{H_d^i\} < H_d^1 + \sum_{i=2}^{N-1} H_u^i$, from Eqs. (41) and (42), we obtain $D_A < D_B$. Therefore, scheme A requires less time for quantum communication, and it has shorter delay in general.

To the end, we should state that these two schemes can be used not limited in wireless networks, but in wider applicable scenarios. All of the results of this paper could be applied to both wireless and wired multihop networks.

VI. CONCLUSIONS

In summary, we studied quantum communication in a wireless multihop network, by proposing a scheme for quantum teleportation between two nodes in the network, which have no direct Bell pair shared. We made detailed calculations on one-hop and two-hop cases and generalized all the results to the multihop case. Our scheme employs a simultaneous and independent entanglement swapping method. All the intermediate nodes make Bell measurements independently on any measurement results of other nodes, and only need to send measurement outcomes and Bell pair types to the destination node. After gathering all the information, the destination node performs local operations to recover the initial qubit state. We gave two efficient methods to determine the operations.

Our scheme has several merits. First, our scheme does not require the Bell pairs shared by the intermediate nodes in the network to be the same type. This merit is of practical importance since in realistic networks, the nodes may have different accordance on Bell pair types. Second, we made detailed comparisons on our scheme and the sequential entanglement swapping scheme usually used in quantum repeaters. We find that our scheme can reduce communication delay, although in some networks our scheme may cost more classical information. Third, our scheme does not require communication channels between all the neighbor nodes but only needs one path from each intermediate node to the destination node. This merit may facilitate its application in some networks. Finally, our scheme is scalable and can be also used to wired or hybrid quantum networks.

Quantum communication schemes are crucial for designing efficient quantum networks. We hope our approach may stimulate more investigations on communication proposals for quantum networks.

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