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Mass shell of strong-field quantum electrodynamics

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It has long been known that a free electron in an intense plane-wave field has a mass shell that differs from the usual free-electron mass shell, with a form that implies that an intensity-dependent increase in mass occurs. It has been an attractive but elusive goal to observe this mass shift. Many schemes have been proposed by which a definitive measurement might be made, and some claims of success exist, but these tests are not conclusive. It is shown here that the intense-field mass shell is not the result of a change in mass. Rather, it is a consequence of the potential energy that a charged particle must possess in the presence of a plane-wave field. When the effects of this potential are incorporated in a properly covariant form, the mass shift no longer appears and kinematic relations are conventional. If the plane-wave pulse is sufficiently long to allow the electron to exit the field adiabatically, then there is no alteration at all of the mass shell expression. Other aspects of the role played by a ponderomotive 4-potential are examined. It is also shown that the putative "relativistic mass" of the electron is illusory when confronted with covariance requirements. Both "mass increases" of the free electron are thereby nullified by fundamental principles.

I. INTRODUCTION

It has been known since the earliest nonperturbative treatments of Compton scattering [1] and of pair production from the vacuum [2,3] that a charged particle (hereafter referred to generically as an "electron") immersed in an intense plane-wave field exhibits an intensity-dependent alteration of its mass shell condition. The field-free mass shell of ordinary quantum electrodynamics (QED),

$$p^\mu p_\mu = (mc)^2, \tag{1}$$

is replaced by the mass shell of strong-field QED (SFQED),

$$p^\mu p_\mu = (mc)^2(1 + z_f), \tag{2}$$

where

$$z_f := 2U_p/mc^2. \tag{3}$$

The quantity U_p is the ponderomotive potential of the electron in the plane-wave field. The terminology of Ref. [3] is used, since the conclusion of Ref. [3] that z_f is the coupling constant of the electron to the plane-wave field is relevant here. (In [2,3], the z_f parameter was designated as z without a subscript.) Equation (2) appears to indicate that the mass of the electron has increased as a result of interaction with the strong field.

A problem of interpretation arises because Eq. (2) can be placed in the form

$$p^\mu p_\mu = (m^2 + \Delta m^2)c^2, \tag{4}$$

whereas a simple shift in mass Δm would lead to

$$p^\mu p_\mu = (m + \Delta m)^2 c^2. \tag{5}$$

A considerable literature has arisen with respect to this revision of the mass shell condition. An alteration of the mass of the electron that is dependent on the intensity of the field in which it is immersed has implications for the foundations of QED. For example, Ref. [4] examines the effects on basic symmetries suggested by an intense-field mass shift.

Within the growing body of literature on the subject of the intensity-dependent mass, differences of opinion inevitably arise [5,6]. From very early on, ways to observe the mass shift by laboratory measurements have been suggested [7,8]. These suggestions continue to the present day [9]. Most of these proposals amount to a quest for an alteration in kinematics following from the presence of the ponderomotive potential U_p. A study of the implications of the ponderomotive potential is a focus of this article.

Possibly the earliest attempt to measure the mass shift was in 1971 [10]. Claims have been made [11–13] that the mass shift was observed, although these include the caution that the evidence may not be decisive [12].

A direct approach is taken here to understanding the origin of the free-electron mass shell condition of SFQED. It is found that it is a straightforward matter to trace the origins of this alteration, with the result that one can assign a simple meaning to it. Rather than focusing on mass *per se*, it is more fruitful to consider energy and momentum conditions. The result is that the ponderomotive potential provides the essential key to the explanation of the electron's modified kinematic properties. The ponderomotive potential has several aspects to its physical significance that will be explicated after revealing its mass shell implications.

Section II analyzes the mass shell condition in terms of the requirement for a potential energy due to the interaction of a charged particle with the plane-wave field in which it is immersed. When this potential energy is explored in covariant terms, it is found to provide a complete explanation for the modified mass shell condition (2) or (4).

An analog to the ponderomotive potential of transverse (i.e., plane-wave) fields is the so-called "quiver" energy of an electron in a longitudinal field. Despite an apparent equivalence, the two concepts are fundamentally different, as is explained in Sec. II.

The quantity z_f may be regarded as a dimensionless expression of U_p. All investigators of free electrons in strong fields have encountered the equivalent of the z_f parameter, albeit expressed with a multitude of different notations. Some authors prefer a parameter that is proportional to the strength of the electric field, apparently seeking an equivalence with

longitudinal field quantities where only the electric field is of significance. The parameter thus defined is proportional to $z_f^{1/2}$. As explained in Sec. III, this reasoning is regarded as misleading. The quantity z_f is the coupling constant of SFQED, and its relation to the perturbative coupling given by the fine-structure constant α is very instructive. The ponderomotive potential U_p and its covariant extension to a ponderomotive 4-potential lie at the heart of SFQED.

Section IV examines the concept of a so-called "relativistic mass" that has previously been discredited [14]. However, the notion continues in general use, and a simple covariance argument is used here to show how unphysical that concept actually is. The conclusion is that neither the intense-field mass shift nor the relativistic mass shift actually exists.

Section V gives a brief summary.

II. INTENSE-FIELD MASS SHELL

A. Ponderomotive 4-momentum

The structure of the mass shell expressed by Eq. (2) is open to an interpretation quite apart from any change of mass. With the left-hand side expanded, and Eq. (3) inserted on the right-hand side, the expression becomes

$$\left(\frac{E}{c}\right)^2 - \mathbf{p}^2 = (mc)^2 + 2U_p m. \tag{6}$$

A simple rearrangement gives

$$E^2 = (mc^2)^2 + 2U_p mc^2 + \mathbf{p}^2 c^2. \tag{7}$$

The mass shell condition follows from the expression for the minimum value that E can have. The ponderomotive potential U_p is a true potential energy. If the electron were to emerge adiabatically from the plane-wave field, only then would that energy become a kinetic energy. The electron cannot exist as a physical particle within the plane-wave field unless it possesses the ponderomotive potential energy U_p. The electron must acquire the energy U_p from the electromagnetic field. In a uniform-intensity monochromatic field, photons of that field are unidirectional, and an amount of field energy U_p is associated with a field momentum U_p/c in the direction of propagation. Therefore, the minimum energy of the electron is, from (7) with the minimal 3-momentum $|\mathbf{p}| = U_p/c$ inserted,

$$E_{\min}^2 = (mc^2)^2 + 2U_p mc^2 + U_p^2. \tag{8}$$

The minimum energy condition is thus

$$E_{\min} = mc^2 + U_p. \tag{9}$$

This is an expression of the requirement that the electron in the field must have, at minimum, the rest energy mc^2 plus the ponderomotive potential U_p. An important additional concept is that the energy U_p acquired from the field comes with the photon momentum U_p/c associated with that amount of field energy. The analysis [15,16] of recent experiments [17] that observed radiation pressure confirms the U_p energy and U_p/c momentum assignments.

An alternative approach starts with the knowledge that an electron in a transverse field must possess an energy U_p. Covariance requires that an energy must be the time part of a 4-momentum,

$$U^\mu : (U_p, U_p \widehat{\mathbf{k}}), \tag{10}$$

where $\widehat{\mathbf{k}}$ is a unit 3-vector in the direction of field propagation. The direction and relative amplitude of the space part of Eq. (10) come from the fact that the *4-potential* U^μ arises from interaction with the plane-wave field. That is, the 4-potential U^μ is a lightlike 4-vector that is parallel to the propagation 4-vector:

$$k^\mu : \left(\frac{\omega}{c}, \mathbf{k}\right), \tag{11}$$

with the equivalence

$$U^\mu = \frac{U_p}{\omega/c} k^\mu. \tag{12}$$

The mass shell is then found from the scalar product

$$\left(p^\mu + \frac{1}{c}U^\mu\right)\left(p_\mu + \frac{1}{c}U_\mu\right), \tag{13}$$

where p^μ is the free-particle 4-momentum that satisfies Eq. (1). Carrying out the multiplication indicated in Eq. (13) gives

$$p^\mu p_\mu + \frac{2}{c}p^\mu U_\mu + \frac{1}{c^2}U^\mu U_\mu = (mc)^2 + \frac{2U_p}{\omega}p^\mu k_\mu \tag{14}$$

using (12) and the fact that U^μ is on the light cone. The expressions (13) and (14) are Lorentz-invariant, so they must be true in the frame where $p^\mu \to p_0^\mu : (mc, \mathbf{0})$, so the product (13) is

$$\left(p^\mu + \frac{1}{c}U^\mu\right)\left(p_\mu + \frac{1}{c}U_\mu\right) = (mc)^2 + 2mU_p. \tag{15}$$

This result corresponds exactly to Eqs. (2) and (3).

The conclusion is that the free-electron canonical 4-momentum must be supplemented by the ponderomotive 4-potential because the electron exists in the presence of a plane-wave field. The intense-field mass shift has now vanished, since the modified mass shell condition arises entirely from the known presence of the ponderomotive potential when the electron is in interaction with the field. The full mass-shell condition (15) contains no added-mass considerations.

The mass shell condition (2) is straightforward. The kinetic energy T is found from

$$T = E - E_{\min} = \sqrt{(mc^2)^2 + 2U_p mc^2 + \mathbf{p}^2 c^2} - (mc^2 + U_p) \tag{16}$$

$$= mc^2 \sqrt{1 + \frac{2U_p}{mc^2} + \frac{\mathbf{p}^2}{m^2 c^2}} - (mc^2 + U_p) \tag{17}$$

$$\approx \frac{\mathbf{p}^2}{2m}, \tag{18}$$

where the approximation in the last step (18) corresponds to the nonrelativistic limit

$$mc^2 \gg U_p, \quad m^2 c^2 \gg \mathbf{p}^2. \tag{19}$$

If that nonrelativistic assumption is not justified, then the full expression (16) or (17) must be used.

The conclusion is that there is nothing out of the ordinary about the kinematics. There need not be any allowance for

a shifted mass. The appearance of U_p in Eqs. (16) and (17) is due simply to the well-known ponderomotive potential of a charged particle in a plane-wave field. The nonappearance of U_p in Eq. (18) means that the ponderomotive potential of a free-particle interaction is very difficult to observe in a nonrelativistic situation.

There is, however, the caveat that return of the ponderomotive energy to the emergent particle is possible if the laser pulse is sufficiently long that the electron can exit adiabatically from the field. If U_p is returned from the field to the electron, then so is the associated momentum of magnitude U_p/c in the direction of laser propagation, and the simple mass shell condition (1) would then be recovered. (This long-pulse behavior might be the cause of the null result of the experiment reported in Ref. [10], designed to detect the presence of U_p.)

The conclusion just reached about the ponderomotive potential being the real source of the SFQED mass shell has not required any alteration of that expression. Rather, it is a statement that provision for U_p *must* be a part of kinematic considerations. From this point of view, the putative mass shift is not a supportable concept.

The qualitative puzzle about why the mass shell of SFQED has the form of Eq. (2) or (4) and not the simple mass-shift form of Eq. (5) can now be answered. The connection between the ponderomotive energy U_p supplied by the field, and the momentum U_p/c acquired in that transfer, is related to the zero-mass property of the photon rather than the nonzero-mass energy-momentum relationship of the electron. This precludes the form (5).

B. Lorentz and gauge invariance

These conclusions are both Lorentz-invariant and gauge-invariant because the ponderomotive energy U_p is both Lorentz-invariant and gauge-invariant [18]. Lorentz invariance follows immediately from the fact that U_p arises from the product $A^\mu A_\mu$ of the 4-vector potential of the field with itself, as is evident from the defining relation

$$U_p = \frac{e^2}{2mc^2}\langle|A^\mu A_\mu|\rangle,\tag{20}$$

where the angle brackets denote a cycle average, and the absolute value brackets are necessary because A^μ is a spacelike 4-vector. Gauge invariance of $A^\mu A_\mu$ is less obvious, but it can be shown [18] to follow from the requirement for a plane-wave field that dependence on the spacetime 4-vector x^μ can only be in the form of the covariant phase $k^\mu x_\mu$, where k^μ is the propagation 4-vector of the plane-wave field. This requirement is imposed as an ansatz by Schwinger [19] and by Sarachik and Schappert [20], and is shown to be a necessity in Ref. [18].

Gauge invariance is so easily demonstrated and so basic that the proof is replicated here from Ref. [18]. Since A^μ can depend on x^μ only in the form of the covariant phase $k^\mu x_\mu$, then the generating function Λ of the gauge transformation must also have that property, giving

$$A^\mu \to \widetilde{A}^\mu = A^\mu + \partial^\mu \Lambda = A^\mu + (\partial^\mu \varphi)\frac{d}{d\varphi}\Lambda$$
$$= A^\mu + k^\mu \Lambda',\tag{21}$$

where

$$\varphi := k^\mu x_\mu,\tag{22}$$

and Λ' is the total derivative of Λ with respect to φ. The inner product of $\widetilde{A}^\mu$ with itself is thus

$$\widetilde{A}^\mu \widetilde{A}_\mu = (A^\mu + k^\mu \Lambda')(A_\mu + k_\mu \Lambda') = A^\mu A_\mu \tag{23}$$

since $k^\mu k_\mu = 0$, and $k^\mu A_\mu = 0$ for a transverse field. Equation (23) is an expression of gauge invariance of U_p in view of (20).

C. Ponderomotive potential versus quiver energy

Some further remarks are important for the necessary identification of U_p as a true potential energy. In nonrelativistic laser physics, U_p is often referred to as a "quiver energy" associated with an oscillatory motion of an electron in an oscillating field. That identification would mean that U_p is a kinetic energy and not a potential energy. The reason for this is the fact that most nonrelativistic theory and interpretation is done in terms of the Göppert-Mayer (GM) gauge (also called the *length gauge*), which treats a transverse laser field as if it were a longitudinal field. In a longitudinal field, the potential energy of a particle of charge q in the field is given by $-q\mathbf{r}\cdot\mathbf{E}$ (where $\mathbf{E}$ is the electric field vector), and not by U_p. The ponderomotive potential U_p does not exist in the GM gauge as a true potential. It does occur, but in the guise of a kinetic energy. Its existence as a kinetic quiver energy arises from the presence of apparent charge and current sources in the GM gauge that do not actually exist in the laboratory [21]. The equations employed [22] for the motion of an electron within the GM gauge predict the quiver energy, but these equations of motion arise from the virtual sources that are a necessary adjunct of the GM gauge [21]. This is a clear contrast with plane-wave behavior. A hallmark of plane waves is that, once formed, they propagate without input from external sources.

Direct laboratory evidence exists that U_p is a potential energy and not a kinetic energy. In typical short-pulse laser ionization experiments, there is not sufficient time for the ponderomotive potential to be returned to the photoelectron before the end of the pulse. (See, for example, the discussion in Ref. [23].) Linear polarization spectra would have a minimum at U_p were that energy the kinetic quiver energy, and that fact would be very noticeable in practical strong-field experiments where U_p can be of the order of the binding energy or greater. That is not what is observed; linear polarization spectra typically peak near zero energy. This is consistent with the identity of U_p as a potential energy.

III. z_f AS COUPLING CONSTANT

Other properties of the ponderomotive potential are relevant; in particular, it acts as an effective coupling constant of charged particles with very strong electromagnetic fields. The purpose of Ref. [3] was to explore whether the radius of convergence of SFQED differs from that of QED. This is important because Dyson demonstrated [24] that QED has an essential singularity at the origin in a complex coupling constant plane, meaning that all the remarkable successes of QED actually follow from a theory that is only asymptotic.

The findings of Refs. [2] and [3] are that the fine-structure constant α, the basic coupling constant of QED, is replaced in SFQED by the intensity-dependent parameter z_f, and that the essential singularity at the origin in QED does not appear in SFQED. However, there are other perturbation-limiting singularities away from the origin that occur in SFQED in intensity-dependent locations.

A qualitative understanding about the intensity-dependent failure of perturbation theory comes from the observation that, when U_p increases to the point that the minimum number of photons required to achieve the energy threshold of the process being studied must index upward to the next larger integer, this marks an essential singularity in a complex coupling constant plane. This phenomenon occurs in both free-particle [3] and bound-particle (see Section IX of [25]) processes.

The defining expression for z_f in Eq. (3) has an alternative expression as

$$z_f = \alpha \rho \left(2\lambda \lambdabar_C^2 \right), \tag{24}$$

where ρ is the number of photons per unit volume, λ is the wavelength of the field, and $\lambdabar_C$ is the electron Compton wavelength. The multiplier α in (24) is the fine-structure constant, the coupling parameter of QED. The effective volume within the parentheses in Eq. (24) is approximately the volume of a right circular cylinder of radius $\lambdabar_C$ and length λ. In other words, the fine-structure constant α of QED is enhanced in SFQED by the number of photons contained within the volume of a cylinder with a radius of an electron Compton wavelength, and extended over a wavelength of the plane-wave field.

This conclusion is significant because the coupling constant z_f is directly proportional to U_p, meaning that U_p takes on the additional meaning of measuring the coupling of the electron to the field, as well as specifying the potential energy of an electron in a transverse field. The quantity z_f can be regarded as the dimensionless form of U_p.

A final remark concerns the "multiple-pole" structure of Volkov Green's functions in monochromatic beams. The mass shell condition found there takes the form [26–28]

$$(p^\mu - nk^\mu)(p_\mu - nk_\mu) = (mc)^2(1 + z_f) \tag{25}$$

for any integer n. This is a significant generalization of Eq. (2). The analysis given above applies only to $n = 0$, as well as requiring that p^μ be replaced by $p^\mu + U^\mu$. However, this is sufficient: only for $n = 0$ is the mass shell condition strictly applicable [29]. This follows from the fact that generalizing a monochromatic field to a wave packet of plane waves moves all poles except for $n = 0$ off the real axis in a complex representation of the Green's function in momentum space.

IV. RELATIVISTIC MASS

The primary purpose of this work has been to explore the "intense-field mass shift." There is a quite different concept of putative mass change known as "relativistic mass." This can be treated with brevity, so that it is possible to dismiss in a single paper both long-standing notions of mass alteration of an electron.

The relativistic mass concept holds that an electron has a rest-frame mass given by m_0, and that this is altered to

$$m = m_0 \gamma, \tag{26}$$

$$\gamma = 1/(1 - v^2/c^2)^{1/2} \tag{27}$$

when viewed in a frame moving at velocity v with respect to the rest frame. This point of view was rejected by Okun [14], who protested against a concept that, among other problems, would require different "transverse" and "longitudinal" masses. Nevertheless, the relativistic mass concept has been stoutly defended [30], and it continues to be part of a standard physics curriculum.

A Lorentz vector is defined as any quantity that transforms under a Lorentz transformation according to the same rule as the basic Lorentz spacetime vector x^μ. A relativistic velocity that is a simple vector follows from a derivative with respect to proper time τ:

$$u^\mu = \frac{d}{d\tau} x^u. \tag{28}$$

The relativistic momentum is then just the product of u^μ with the Lorentz scalar mass m:

$$p^\mu = m u^\mu. \tag{29}$$

No subscript is required for m since it is a Lorentz scalar that represents a unique property of the electron. By construction, as shown in Eqs. (28) and (29), u^μ and p^μ are obviously Lorentz vectors.

Confusion becomes possible when the relativistic velocity u^μ is written in terms of a nonrelativistic velocity. Since time undergoes an apparent dilation in any frame other than the rest frame, one has the connection

$$t = \gamma \tau, \tag{30}$$

so that Eq. (28) can be expressed as

$$u^\mu = \gamma \frac{d}{dt} x^\mu, \tag{31}$$

which makes it possible to write the momentum 4-vector as

$$p^\mu = (m)\left(\gamma \frac{d}{dt} x^\mu \right), \tag{32}$$

where each set of parentheses encloses a quantity with a clear Lorentz identity. If the factors in Eq. (32) are grouped instead as

$$p^\mu = (m\gamma)\left(\frac{d}{dt} x^\mu \right), \tag{33}$$

this makes it possible to introduce the confusing notion of a variable mass $m\gamma$. When the factors are grouped as in Eq. (33), the loss of covariance is obvious in that neither factor has any Lorentz identity. The form given in Eq. (33) loses covariance completely by multiplying the noncovariant factor $(m\gamma)$ by another noncovariant factor (dx^μ/dt). This is needless and obfuscating. The concept of a variable relativistic mass destroys the relativistic clarity of free-particle dynamics.

V. SUMMARY

The essential conclusions of this paper can be summarized as follows: The requirement that an electron in a plane-wave field must possess the ponderomotive potential U_p due to that field, coupled with the fact that the acquisition of U_p from the field also gives the electron a minimum momentum, has been shown to provide a complete explanation for the mass shell expression of strong-field QED. No change in the mass of the electron occurs. It is further remarked that the explanation for the existence of the apparent mass shift in terms of the ponderomotive potential precludes the mass-shift interpretation. The question of whether Eq. (2) might follow from a shift in mass or from the presence of the ponderomotive potential of an electron in a plane-wave field is resolved in favor of the latter explanation.

The potential U_p itself is Lorentz-invariant, gauge-invariant, and determines the strength of coupling between a plane-wave field and a charged particle. Its presence and its effects are fundamental. With that acknowledgment, the intensity-dependent mass shift hypothesis must be discarded.

The putative variable "relativistic mass" of an electron destroys otherwise straightforward covariance requirements. It has no redeeming features, and should not be used. The net conclusion is that both forms of variable electron mass are unnecessary, and serve only to muddle the underlying physics.

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