Perfect imaging of a point charge in the quasistatic regime

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An exact calculation of the local electric potential field $\psi(\mathbf{r})$ in the quasistatic limit is described for the case of a point electric charge q in a two-constituent composite medium. In the case of an $\epsilon_2, \epsilon_1, \epsilon_2$ three-parallel-slab microstructure, where q is in the top ϵ_2 layer and both ϵ_2 layers are infinitely thick while the ϵ_1 layer has a finite thickness L_1, a perfect imaging of the point charge is expected if $\epsilon_1 = -\epsilon_2$ is real [J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000); R. J. Blaikie and D. O. S. Melville, J. Opt. A **7**, S176 (2005); U. Leonhardt, New J. Phys. **11**, 093040 (2009)]. Among our results we find that an infinite resolution image of the point charge q is only achievable if the actual charge is situated at a distance that is between $L_1/2$ and L_1 away from the ϵ_1 layer.

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When rays of light are emitted from a point source in a medium with refractive index n_2 and are then made to pass through a flat layer with refractive index $n_1 = -n_2$ before re-emerging in the n_2 medium, two foci are predicted to occur sometimes—see the left part of Fig. 1 and Ref. [1]. This situation is usually called a "Veselago lens." In a pioneering paper Pendry then argued that the size of those foci is not limited by the wavelength, thus raising the possibility that such a negative index lens would be able to achieve a perfect image of any object [2]. Since then, much work has been done on the effects of dissipation and other deviations from ideal materials, such as surface roughness, upon this intriguing possibility [3–5]. Experimental efforts have also been made to achieve the predicted enhancement of resolution [6]. It was also argued that if the system is in the quasistatic regime, where the magnetic permeability is unimportant, then it will suffice to have $\epsilon_1 = -\epsilon_2$ real in order to achieve a perfect image [2].

We would first like to point out that this idea presents two problems: (a) If the object is a point source (e.g., a point charge or some other point multipole charge) then the electromagnetic field has an appropriate mathematical singularity there. Such a singularity cannot be reproduced at any other point by Maxwell's equations. (b) When $\epsilon_1 = -\epsilon_2$ that is a highly singular point of those equations. That is, it is an accumulation point of the eigenvalues of Maxwell's equations in the quasistatic limit [7].

Here we tackle the perfect imaging problem by expanding the local static electric potential field $\psi(\mathbf{r})$ in a series of eigenfunctions of the quasistatic Maxwell equations. This leads to an exact expression for $\psi(\mathbf{r})$, when ϵ_1 and ϵ_2 have arbitrary values, as a one-dimensional integral. When $\epsilon_1/\epsilon_2 = -1$ that integral can be calculated in closed form. For other values of ϵ_1/ϵ_2, in particular when this value has a nonvanishing imaginary part, the integral can easily be evaluated numerically. Particularly interesting is its behavior when $\epsilon_1/\epsilon_2 = -1 + i\delta$ with $\delta \ll 1$. We will show that this has important implications for the possibility of achieving perfect imaging.

Consider a generic two-constituent composite structure which fills up the entire volume of a large parallel plate capacitor, like the one shown in Fig. 1. In the static limit Maxwell's equations reduce to Poisson's equation for $\psi(\mathbf{r})$:

$$-4\pi\rho(\mathbf{r}) = \nabla\cdot(\epsilon_1\theta_1 + \epsilon_2\theta_2)\nabla\psi = \epsilon_2\nabla\cdot(1 - u\theta_1)\nabla\psi, \tag{1}$$

$$\theta_1(\mathbf{r}) \equiv 1 - \theta_2(\mathbf{r}) = \begin{cases} 1 & \text{if} \quad \epsilon(\mathbf{r}) = \epsilon_1, \\ 0 & \text{if} \quad \epsilon(\mathbf{r}) = \epsilon_2, \end{cases} \tag{2}$$

$$u \equiv 1 - \frac{\epsilon_1}{\epsilon_2}, \tag{3}$$

where θ_1 and $\theta_2 \equiv 1 - \theta_1$ are step functions that characterize the microstructure of the composite medium. The function $\rho(\mathbf{r})$ which appears on the left-hand side of Eq. (1) represents a charge density distribution, including the possibility that $\rho(\mathbf{r}) = q\delta^3(\mathbf{r} - \mathbf{r}_0)$, i.e., a point charge at $\mathbf{r}_0$. The capacitor plates at $z = -L_2$ and $z = L'_2$ are included in order that appropriate boundary conditions on $\psi(\mathbf{r})$ may be imposed there so as to result in a unique solution for $\psi(\mathbf{r})$. When convenient, we shall take the limits $L_2 \to \infty$ and $L'_2 \to \infty$ at the end of the calculation.

Further progress is achieved by defining Green's function $G_0(\mathbf{r},\mathbf{r}')$ for this system by

$$\nabla^2 G_0(\mathbf{r},\mathbf{r}') = -\delta^3(\mathbf{r} - \mathbf{r}), \tag{4}$$

$$G_0(\mathbf{r},\mathbf{r}') = 0 \quad \text{for} \quad z = -L_2 \quad \text{and} \quad z = L'_2. \tag{5}$$

Using G_0, Eq. (1) is transformed into the following integro-differential equation for $\psi(\mathbf{r})$ [7]:

$$\psi(\mathbf{r}) = \psi_0(\mathbf{r}) + u\hat{\Gamma}\psi, \tag{6}$$

$$\hat{\Gamma}\psi \equiv \int dV'\theta_1(\mathbf{r}')\nabla' G_0(\mathbf{r},\mathbf{r}')\cdot\psi(\mathbf{r}'). \tag{7}$$

The integro-differential operator $\hat{\Gamma}$ is Hermitian (or self-adjoint) if the scalar product of scalar functions $\phi(\mathbf{r})$, $\psi(\mathbf{r})$ is defined as

$$\langle\psi(\mathbf{r})|\phi(\mathbf{r})\rangle \equiv \int dV\theta_1(\mathbf{r})\nabla\psi(\mathbf{r})^*\cdot\nabla\phi(\mathbf{r}). \tag{8}$$

The function $\psi_0(\mathbf{r})$ is a solution of Poisson's equation in a uniform host of ϵ_2 material,

$$\epsilon_2\nabla^2\psi_0(\mathbf{r}) = -4\pi\rho(\mathbf{r}), \tag{9}$$

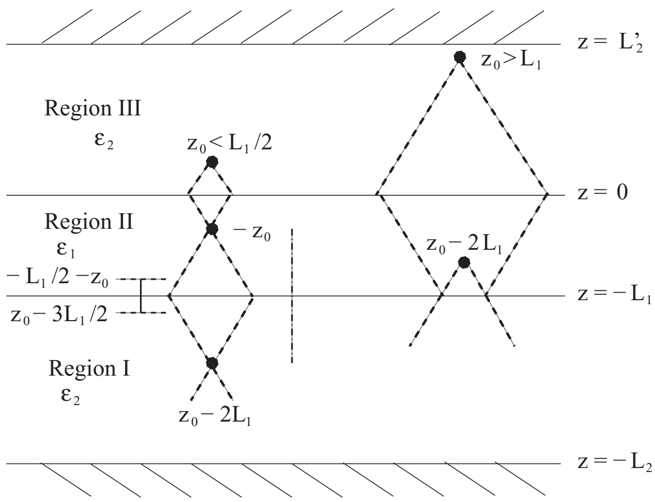

FIG. 1. A three-parallel-slab microstructure that fills the entire volume of a large parallel-plate capacitor. The upper layer (region III), where $\epsilon = \epsilon_2$, includes a point charge q located at $\mathbf{r}_0 = (0,0,z_0)$. In the left part $z_0 < L_1/2$ while in the right part $z_0 > L_1$, where L_1 is the thickness of the intermediate ϵ_1 layer (region II). Even when all the other linear sizes of this structure tend to ∞, this configuration is still unsolvable in any simple fashion. The diagonal dashed lines show how a geometric optics or light rays description would lead to a focusing of the original point charge in region III at new points in regions I and II when $\epsilon_2 = -\epsilon_1$. The vertical dot-dashed lines indicate the regions where $\psi(\mathbf{r})$ then diverges in the case shown on the left side, while the vertical solid lines show where the dissipation rate diverges.

which must satisfy the same boundary conditions that are imposed upon $\psi(\mathbf{r})$. That is because $G_0(\mathbf{r},\mathbf{r}')$ satisfies the homogeneous version of those conditions, i.e., it vanishes when $z = -L_2$ and $z = L_2'$.

The eigenstates of $\hat{\Gamma}$, which satisfy

$$s_n\phi_n(\mathbf{r}) = \hat{\Gamma}\phi_n, \tag{10}$$

are also eigenstates of the zero charge density and zero boundary conditions modification of Eq. (1). The eigenvalues s_n are clearly all real valued since $\hat{\Gamma}$ is self-adjoint. It is also easy to show that they all lie between 0 and 1 [7]. That is, a nonvanishing solution of

$$s\phi(\mathbf{r}) = \hat{\Gamma}\phi, \quad s \equiv \frac{1}{u} = \frac{\epsilon_2}{\epsilon_2 - \epsilon_1}$$

cannot exist when ϵ_1 and ϵ_2 are both real and positive. It is somewhat more difficult to show that $\hat{\Gamma} - 1/2$ is a continuous operator, from which it follows that the values of $(s_n - 1/2)^{-1}$ form a discrete unbounded sequence. Thus the eigenvalues s_n form a discrete sequence with one accumulation point at $s = 1/2$ [7].

The following formal solution of Eq. (6)

$$\phi = \phi_0 + \frac{\hat{\Gamma}}{s - \hat{\Gamma}}\phi_0 \tag{11}$$

can be expanded in a series of the eigenfunctions ϕ_n by using the expansion of the unity operator $\hat{I}$

$$\hat{I} = \sum_n |\phi_n\rangle\langle\phi_n| \tag{12}$$

$$\Longrightarrow \phi(\mathbf{r}) = \phi_0(\mathbf{r}) + \sum_n \frac{s_n}{s - s_n}\langle\phi_n|\phi_0\rangle\phi_n(\mathbf{r}). \tag{13}$$

We are interested in calculating $\psi(\mathbf{r})$ in a two-constituent composite medium for the case where, in the absence of the ϵ_1 constituent, the potential field $\psi_0(\mathbf{r})$ would be that of a point charge q at $\mathbf{r}_0$, namely,

$$\psi_0(\mathbf{r}) = \frac{q/\epsilon_2}{|\mathbf{r} - \mathbf{r}_0|}. \tag{14}$$

The field $\psi(\mathbf{r})$ is thus equal, up to a constant multiplicative factor, to Green's function in the composite medium:

$$\frac{\epsilon_2}{q}\psi(\mathbf{r}) = \frac{1}{|\mathbf{r} - \mathbf{r}_0|} + \sum_n \frac{s_n}{s - s_n}\left\langle\phi_n\left|\frac{1}{|\mathbf{r} - \mathbf{r}_0|}\right.\right\rangle\phi_n(\mathbf{r}). \tag{15}$$

Clearly, when s becomes equal to any eigenvalue s_n the appropriate term in this expression will diverge. Recalling that the sequence of eigenvalues s_n has a single accumulation point at 1/2, i.e., $s_n \to 1/2$ as $n \to \infty$, we can also conclude that when $s = 1/2$, i.e., when $\epsilon_1/\epsilon_2 = -1$, the coefficient $s_n/(1/2 - s_n)$ diverges as $n \to \infty$. Whether $\psi(\mathbf{r})$ also diverges then depends upon the behavior of the other terms in the sum. Clearly, the sum will diverge unless $\langle\phi_n|1/|\mathbf{r} - \mathbf{r}_0|\rangle \to 0$.

It is also worth recalling that when $\mathrm{Re}(\epsilon_1)$ or $\mathrm{Re}(\epsilon_2)$ is negative, a nonzero imaginary part must be present. If ϵ_2 represents a conventional dielectric (i.e., ϵ_2 is real and positive) while ϵ_1 represents a conventional metal in the optical frequency range (i.e., ϵ_1 has a negative real part and a nonzero imaginary part) then ϵ_1/ϵ_2 will also have a negative real part and a nonzero imaginary part with the same sign as $\mathrm{Im}(\epsilon_1)$. Consequently

$$s = \frac{1 - \mathrm{Re}\frac{\epsilon_1}{\epsilon_2} + i\,\mathrm{Im}\frac{\epsilon_1}{\epsilon_2}}{\left(1 - \mathrm{Re}\frac{\epsilon_1}{\epsilon_2}\right)^2 + \left(\mathrm{Im}\frac{\epsilon_1}{\epsilon_2}\right)^2}.$$

Due to the nonzero value of $\mathrm{Im}(s)$ all the terms in the sum of Eq. (15) will be finite for such a composite. Nevertheless, the behavior of that sum when s is precisely equal to 1/2, i.e., when $\epsilon_1/\epsilon_2 = -1$, is of interest because at that value the perfect imaging phenomenon is supposed to occur [2,8,9].

We now apply this approach to the case of three parallel slabs, as shown in Fig. 1. Here we will focus upon the local electric potential field produced by a point charge q located at $\mathbf{r}_0 = (0,0,z_0)$, which is inside region III of the ϵ_2 constituent. We will make the lateral dimensions of the capacitor L_x and L_y and the thicknesses of the ϵ_2 layers L_2' and $L_2 - L_1$ very large but leave the thickness L_1 of the ϵ_1 layer, as well as z_0, z, and $\rho \equiv \sqrt{x^2+y^2}$, finite.

The relevant eigenfunctions and eigenvalues are [10]

$$\phi_{\mathbf{k}}^{\pm}(\mathbf{r}) = e^{i\mathbf{k}\cdot\rho}\cdot\begin{cases} A_{\mathbf{k}}^{\pm}\sinh[k(z+L_2)], & z\in \mathrm{I},\\ B_{\mathbf{k}}^{\pm}\sinh(kz)+B_{\mathbf{k}}^{\prime\pm}\sinh[k(z+L_1)], & z\in \mathrm{II},\\ C_{\mathbf{k}}^{\pm}\sinh[k(z-L_2')], & z\in \mathrm{III},\end{cases} \tag{16}$$

$$s_{\mathbf{k}}^{\pm} = \frac{1\mp e^{-kL_1}}{2}, \tag{17}$$

where we assumed $kL_2 \gg 1$ and $kL_2' \gg 1$ to get Eq. (17), and will continue to assume that whenever it is convenient. The coefficients $A_{\mathbf{k}}^{\pm}$, $B_{\mathbf{k}}^{\pm}$, $B_{\mathbf{k}}^{\prime\pm}$, $C_{\mathbf{k}}^{\pm}$, as well as the eigenvalues $s_{\mathbf{k}}^{\pm}$, are determined by imposing the continuity conditions on $\phi_{\mathbf{k}}^{\pm}$ at the interfaces and the normalization condition $\langle\phi_{\mathbf{k}}^{\pm}|\phi_{\mathbf{k}}^{\pm}\rangle = 1$. Doing this we find

$$A_{\mathbf{k}}^{\pm} = -B_{\mathbf{k}}^{\pm}\frac{\sinh(kL_1)}{\sinh[k(L_2-L_1)]}, \quad B_{\mathbf{k}}^{\prime\pm} = \mp B_{\mathbf{k}}^{\pm},$$
$$C_{\mathbf{k}}^{\pm} = \pm B_{\mathbf{k}}^{\pm}\frac{\sinh(kL_1)}{\sinh(kL_2')}.$$

The eigenvalues clearly satisfy

$$0 < s_{\mathbf{k}}^{+} < \tfrac{1}{2} < s_{\mathbf{k}}^{-} < 1, \tag{18}$$

and they all tend to the common accumulation point at $s = 1/2$ when $k\to\infty$. The normalization condition $\langle\phi_{\mathbf{k}}^{\pm}|\phi_{\mathbf{k}}^{\pm}\rangle = 1$ leads to

$$1 = 2kL_xL_y|B_{\mathbf{k}}^{\pm}|^2\sinh(kL_1)[\cosh(kL_1)\mp 1]. \tag{19}$$

The scalar product of ψ_0 and $\phi_{\mathbf{k}}^{\pm}$ is given by

$$\langle\psi_0|\phi_{\mathbf{k}}^{\pm}\rangle = \frac{2\pi q}{\epsilon_2}B_{\mathbf{k}}^{\pm}e^{-kz_0}(1\pm e^{-kL_1})[1\mp\cosh(kL_1)].$$

Substituting these results in Eq. (15) we get the following *exact results* for $\psi(\mathbf{r})$ (J_0 is a regular Bessel function):

$$\psi = \frac{4s(1-s)q}{\epsilon_2}\int_0^\infty dk\, J_0(k\rho)\,\frac{e^{-k(z_0-z)}}{e^{-2kL_1}-(2s-1)^2}$$
$$= 4q\epsilon_1\int_0^\infty dk\, J_0(k\rho)\,\frac{e^{-k(z_0-z)}}{(\epsilon_2-\epsilon_1)^2e^{-2kL_1}-(\epsilon_2+\epsilon_1)^2} \quad \text{in} \quad \mathrm{I}, \tag{20}$$

$$\psi = \frac{2sq}{\epsilon_2}\int_0^\infty dk\, J_0(k\rho)\,e^{-k(z_0-z)}\frac{e^{-2k(z+L_1)}-2s+1}{e^{-2kL_1}-(2s-1)^2}$$
$$= 2q\int_0^\infty dk\, J_0(k\rho)\,e^{-k(z_0-z)}\frac{(\epsilon_2-\epsilon_1)e^{-2k(z+L_1)}-(\epsilon_2+\epsilon_1)}{(\epsilon_2-\epsilon_1)^2e^{-2kL_1}-(\epsilon_2+\epsilon_1)^2} \quad \text{in} \quad \mathrm{II}, \tag{21}$$

$$\psi = \psi_0 + \frac{q(2s-1)/\epsilon_2}{\sqrt{\rho^2+(z+z_0)^2}} - \frac{4s(1-s)(2s-1)q}{\epsilon_2}\int_0^\infty dk\, J_0(k\rho)\,\frac{e^{-k(z_0+z)}}{e^{-2kL_1}-(2s-1)^2} = \frac{q/\epsilon_2}{\sqrt{\rho^2+(z-z_0)^2}}$$
$$+\frac{\epsilon_2+\epsilon_1}{\epsilon_2-\epsilon_1}\frac{q/\epsilon_2}{\sqrt{\rho^2+(z+z_0)^2}} + 4q\epsilon_1\frac{\epsilon_2+\epsilon_1}{\epsilon_2-\epsilon_1}\int_0^\infty dk\, J_0(k\rho)\,\frac{e^{-k(z_0+z)}}{(\epsilon_2-\epsilon_1)^2e^{-2kL_1}-(\epsilon_2+\epsilon_1)^2} \quad \text{in} \quad \mathrm{III}. \tag{22}$$

The integrals in these expressions must be calculated with caution. When $\mathrm{Im}(s)\neq 0$, the denominators in the integrands never vanish and the integrations converge. By contrast, when $s\in(0,1)$ is real and differs from $1/2$ (i.e., $\epsilon_1/\epsilon_2 < 0$ but differs from -1) the denominators always vanish at some point and the integrations diverge. At the special point $s = 1/2$, when $\epsilon_1/\epsilon_2 = -1$, the integrations diverge when z lies in certain ranges. Thus, in region I $\psi(\mathbf{r})$ diverges when $z_0 - 2L_1 < z < -L_1$ while in region II it diverges when $-L_1 < z < -z_0$. When $\mathrm{Im}(s)$ is nonzero but small, $\psi(\mathbf{r})$ includes a contribution that is proportional to a power of $\mathrm{Im}(s)$ which depends upon z and is negative when there is a divergence.

An interesting matter is the behavior of the dissipation rate $W(\mathbf{r}) = \mathrm{Im}[\epsilon(\mathbf{r})]|\nabla\psi|^2/(8\pi)$ when $\psi(\mathbf{r})$ diverges. A detailed analysis leads to the following conclusions:

$$W(\mathbf{r}) \propto \begin{cases}\mathrm{Im}(\epsilon_2)[\mathrm{Im}(\epsilon_1+\epsilon_2)]^{\frac{2(z_0-z-2L_1)}{L_1}} & \text{for} \quad \mathbf{r}\in\mathrm{I},\\ \mathrm{Im}(\epsilon_1)[\mathrm{Im}(\epsilon_1+\epsilon_2)]^{\frac{2z+2z_0}{L_1}} & \text{for} \quad \mathbf{r}\in\mathrm{II}.\end{cases}$$

What this means is that even in the limit where $\mathrm{Im}(\epsilon_1)$ and $\mathrm{Im}(\epsilon_2)$ are comparable and tend to 0, the local dissipation rate will diverge unless

$$z < z_0 - 3L_1/2 \quad \text{for} \quad \mathbf{r}\in\mathrm{I},$$
$$z > -z_0 - L_1/2 \quad \text{for} \quad \mathbf{r}\in\mathrm{II}.$$

From this we can conclude that the total rate of dissipation remains finite only if $L_1/2 < z_0 < L_1$. In that case the lower focus $z_0 - 2L_1$ satisfies

$$-3L_1/2 < z_0 - 2L_1 < -L_1. \tag{23}$$

This means that it lies at a distance which is at most $L_1/2$ below the ϵ_1 layer. The same results, regarding the divergence of $\psi(\mathbf{r})$ at well as of $W(\mathbf{r})$, were already obtained in Ref. [5] from an approximate calculation of $\psi(\mathbf{r})$. By contrast, our results are based upon Eqs. (20)–(22) *which are exact*. Obviously, when $W(\mathbf{r})\to\infty$ we should not ignore the decrease in magnitude of $\psi(\mathbf{r})$ and $\nabla\psi(\mathbf{r})$ as $|\mathbf{r}-\mathbf{r}_0|$ increases. This would require repeating our discussion away from the quasistatic limit. Nevertheless, the apparent impending divergence of $W(\mathbf{r})$ shows that trying to achieve enhanced resolution with a Veselago lens must be done with great care.

It is worth noting that when $s = 1/2$, $\psi(\mathbf{r})$ has the following exact closed form expressions in those regions of z where it is nondiverging:

$$\psi(\mathbf{r}) = \begin{cases} \frac{q/\epsilon_2}{\sqrt{\rho^2+(z-z_0+2L_1)^2}}, & \mathbf{r} \in \mathrm{I}, \\ \frac{q/\epsilon_2}{\sqrt{\rho^2+(z+z_0)^2}}, & \mathbf{r} \in \mathrm{II}, \\ \frac{q/\epsilon_2}{\sqrt{\rho^2+(z-z_0)^2}}, & \mathbf{r} \in \mathrm{III}. \end{cases}$$

These are what would be expected if the point charge is perfectly imaged at the ray optics foci. What we have shown is that these perfect images are only valid if the object position z_0 satisfies the above inequalities.

It is also interesting to review what happens when the point charge q is situated above the ϵ_1 layer by more than its thickness L_1. In that case there is no real focal point even in ray optics. It is easy to see that the rays in region I appear to emanate from a virtual focal point $\mathbf{r}_2 = (0,0,z_0 - 2L_1)$ which lies in region II if $z_0 < 2L_1$ or in region III if $z_0 > 2L_1$—see the right part of Fig. 1. In that case there is no possibility of a perfect real image. However, because the virtual focus lies below z_0, the diverging beam of rays emanating from that focal point is narrower than the original beam emanating from the object point. This is the situation in the experiment described in Ref. [6]. The improved resolution observed there is therefore unrelated to the "perfect imaging" phenomenon.

In summary, we have calculated the quasistatic eigenstates of a three-parallel-slab ϵ_2, ϵ_1, ϵ_2 microstructure, where the thickness of the intermediate ϵ_1 layer is L_1, and used them to get an exact result for the electric potential of a point charge q. We found that the local potential and electric fields always diverge in certain parts of the system whenever $\epsilon_1 = -\epsilon_2$ if q is closer than L_1 to the intermediate layer. We have shown that a perfect resolution image is achievable in a certain limited sense if $\epsilon_1 = -\epsilon_2$ is real when the object lies at a distance between L_1 and $L_1/2$ above the ϵ_1 layer. At smaller distances the total dissipation diverges, even when $\mathrm{Im}(\epsilon_1)$ and $\mathrm{Im}(\epsilon_2)$ both tend to 0. At greater distances there is only a virtual image. We are currently trying to extend the approach described here so as to apply away from the quasistatic regime. Some remarks are now in order:

(i) The above considerations will obviously apply to other point sources of an electric field, e.g., a point electric dipole.

(ii) The eigenstates of (16) are reminiscent of the evanescent and amplifying (in the absence of an appropriate antonym for evanescent) waves invoked by Pendry in Ref. [2] to explain the unlimited resolution in perfect imaging. In particular, the $\sinh(kz)$ and $\cosh(kz)$ functions are clearly linear combinations of an increasing and a decreasing exponential function. Nevertheless the discussion of this situation that is based upon the quasistatic eigenstates leads to new insight. In particular, we now understand how the electric field depends upon $\mathbf{r}$ in the vicinity of the "perfect image" focal points.

(iii) In a previous discussion of the perfect imaging phenomenon it was assumed that even in the quasistatic limit, the object to be imaged is essentially a nonuniform plane wave incident from the top [2]. This differs from the situation discussed here wherein the incident wave issues from a point source, which results in an outgoing wave in all directions. The latter picture is perhaps more relevant to the possibility of achieving an image with unlimited resolution.

(iv) Our results for the field $\nabla\psi(\mathbf{r})$ could also have been obtained by applying the general results of Ref. [11] for electromagnetic fields of point sources in a flat multilayered medium to the problem discussed in this article.

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[10] There is actually one more eigenstate, namely,

$$\phi_0(\mathbf{r}) = \begin{cases} A_0(z + L_1), & -L_1 < z < 0, \\ B_0(z - L_2), & 0 < z < L_2, \end{cases}$$
$$s_0 = p_2,$$

where $p_i \equiv L_i/(L_1 + L_2)$ is the volume fraction of the ϵ_i constituent and the special values of the two ϵ_i satisfy $\epsilon_1/\epsilon_2 = -p_1/p_2$. This eigenstate does not contribute in the calculation of $\psi(\mathbf{r})$ produced by a point charge.
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