Dissipative creation of three-dimensional entangled state in optical cavity via spontaneous emission

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We present a dissipative protocol to engineer two ^{87}Rb atoms into a form of three-dimensional entangled state via spontaneous emission. The combination of coupling between ground states via microwave fields and dissipation induced by spontaneous emission makes the current scheme deterministic and a stationary entangled state is always achievable without state initialization. Moreover, this scheme can be straightforwardly generalized to preparation of an N-dimensional entangled state in principle.

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I. INTRODUCTION

For an open quantum system, the dissipation process must be accompanied by entanglement generation, i.e., the populations of quantum states are altered due to entanglement with an external environment. Thus researchers are dedicating themselves to finding efficient ways of avoiding decoherence during the quantum information process. Currently, the feasible methods include an active error-correction approach based on the assumption that the most probable errors occur independently to a few qubits, which can be corrected via subsequent quantum operation [1–5], and an alternative passive error-prevention scheme, where the logical qubits are encoded into subspaces which do not decohere because of symmetry [6–10]. Recently, the function of dissipation was reexamined in Refs. [11–20], where the environment alone can be used as a resource to prepare entanglement and implement universal quantum computing. In particular, Kastoryano *et al.* consider a dissipative scheme for preparing a maximally entangled state of two Λ atoms in a high finesse optical cavity [14], in which a pure steady singlet state is achieved with no need of state initialization.

Compared with other kinds of entanglement, high-dimensional entangled states have attracted much interest, since they can enhance the violations of local realism and the security of quantum cryptography. In the fields of linear optics, two experiments utilize the spatial modes of the electromagnetic field carrying orbital angular momentum to create high-dimensional entanglement [21,22]. In the context of cavity quantum electrodynamics (QED), three-dimensional entanglement has also been realized in the unitary evolutionary dynamics based on resonant, and dispersive atom-cavity interactions [23–26]. In this paper, we put forward a dissipative method for preparing a stationary three-dimensional entangled state. The motivation of our proposal is mainly based on the following truth: The typical decoherence factors in a cavity QED system consist of atomic spontaneous emission and cavity decay, which have detrimental effects on schemes based on unitary dynamics. However, the loss of cavity can be utilized to stabilize a maximal pure entangled state when a suitable feedback control is applied [27–30]. Thus the spontaneous emission of atom becomes the only one detrimental factor. The result of our work shows that atomic spontaneous emission is capable of being a useful resource with respect to entanglement preparation; in particular, the fidelity of target state can even be better than the unitary evolution based schemes.

The structure of the paper is as follows. We derive the effective Hamiltonian for open systems in Sec. II, and analyze the role of spontaneous decay rates for the preparation of a three-dimensional entangled state in Sec. III. We then generalize the scheme to realize an N-dimensional entangled state via introducing multilevel atoms and multimode cavity, and discuss the effect of cavity decay on the fidelity in Sec. IV. This paper ends with a conclusion in Sec. V.

II. EFFECTIVE HAMILTONIAN FOR OPEN SYSTEMS

We take into account a system composed of two ^{87}Rb atoms trapped in a bimode optical field, as shown in Fig. 1. The quantum states $|g_L\rangle$, $|g_0\rangle$, $|g_R\rangle$, and $|g_a\rangle$ correspond to atomic levels $|F=1,m_f=-1\rangle$, $|F=1,m_f=0\rangle$, $|F=1,m_f=1\rangle$, and $|F=2,m_f=0\rangle$ of $5S_{1/2}$, and $|e_L\rangle$, $|e_0\rangle$, $|e_R\rangle$ correspond to $|F=1,m_f=-1\rangle$, $|F=1,m_f=0\rangle$, and $|F=2,m_f=1\rangle$ of $5P_{3/2}$. Without loss of generality, we apply two off-resonance π-polarized optical lasers, with Rabi frequencies $\Omega_{1(2)}$, detuning Δ to drive the transitions $|e_0\rangle \leftrightarrow |g_a\rangle$ for the first atom and $|e_L\rangle \leftrightarrow |g_L\rangle$ and $|e_R\rangle \leftrightarrow |g_R\rangle$ for the second atom. The transition $|e_{0(R)}\rangle \leftrightarrow |g_{L(0)}\rangle$ and $|e_{0(L)}\rangle \leftrightarrow |g_{R(0)}\rangle$ are coupled to the cavity modes a_L and a_R with coupling strength g_L and g_R, detuning $\Delta-\delta$, respectively. In addition, two microwave fields with Rabi frequencies ω_1 and ω_2 are introduced to resonantly couple ground states as to acquire a steady-state entanglement during the dissipative process.

In a rotating frame, the master equation describing the interaction between quantum systems and external environment is in the Lindblad form

$$\dot{\hat{\rho}} = i[\hat{\rho},\hat{H}] + \sum_j \hat{L}_j\hat{\rho}\hat{L}_j^\dagger - \frac{1}{2}(\hat{L}_j^\dagger\hat{L}_j\hat{\rho} + \hat{\rho}\hat{L}_j^\dagger\hat{L}_j). \quad (1)$$

In connection with cavity QED, the Lindblad operator L_j is closely related to two typical decoherence factors, i.e., the spontaneous emission rate γ from the excited state of the ^{87}Rb atom and the leaky rate κ of photon from the optical cavity. Thus the nine dissipation channels are denoted by

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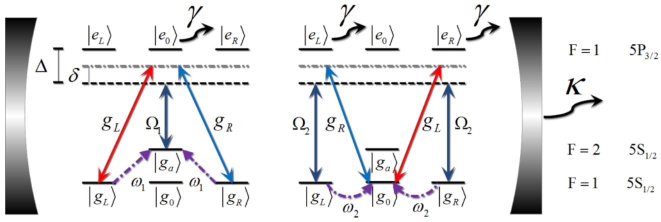

FIG. 1. (Color online) Schematic view of the configuration of atoms. The system consists of two ^{87}Rb atoms simultaneously driven by optical lasers and microwave fields, coupled to a bimode cavity. The excited state of the first atom $|e_0\rangle$ can spontaneously decay into ground states $|g_L\rangle$, $|g_a\rangle$, and $|g_R\rangle$ with branching rate $\gamma_1/3$, while the upper levels $|e_{L(R)}\rangle$ for the second atom are translated into $|g_{L(R)}\rangle$ and $|g_0\rangle$ with rate $\gamma_2/2$, and we assume $\gamma_1 = \gamma_2 = \gamma$ throughout this paper. Also, the decay rates for the cavity modes are set to be the same κ.

$L^{\gamma_1,g_{L(a,R)}} = \sqrt{\gamma/3}|g_{L,(a,R)}\rangle\langle e_0|$, $L^{\gamma_2,g_{L(0)}} = \sqrt{\gamma/2}|g_{L,(0)}\rangle\langle e_L|$, $L^{\gamma_2,g_{R(0)}} = \sqrt{\gamma/2}|g_{R,(0)}\rangle\langle e_R|$, $L^{a_L} = \sqrt{\kappa}a_L$, and $L^{a_R} = \sqrt{\kappa}a_R$, respectively. For the sake of convenience, we have assumed a uniform dissipation rate for atoms and cavity modes. The Hamiltonian of the total system reads

$$\hat{H} = \hat{H}_0 + \hat{H}_g + \hat{V}_+ + \hat{V}_-, \tag{2}$$

$$\begin{aligned}\hat{H}_0 = {} & \delta\hat{a}_L^\dagger\hat{a}_L + [g_L(|g_L\rangle_{11}\langle e_0| + |g_0\rangle_{22}\langle e_R|)\hat{a}_L^\dagger + \text{H.c.}] \\ & + \delta\hat{a}_R^\dagger\hat{a}_R + [g_R(|g_R\rangle_{11}\langle e_0| + |g_0\rangle_{22}\langle e_L|)\hat{a}_R^\dagger + \text{H.c.}] \\ & + \Delta(|e_0\rangle_{11}\langle e_0| + |e_L\rangle_{22}\langle e_L| + |e_R\rangle_{22}\langle e_R|),\end{aligned} \tag{3}$$

$$\begin{aligned}\hat{H}_g = {} & \omega_1(|g_L\rangle_{11}\langle g_a| + |g_R\rangle_{11}\langle g_a|) \\ & + \omega_2(|g_L\rangle_{22}\langle g_0| + |g_R\rangle_{22}\langle g_0|) + \text{H.c.},\end{aligned} \tag{4}$$

$$\begin{aligned}\hat{V}_+ = \hat{V}_-^\dagger = {} & \Omega_1(|e_0\rangle_{11}\langle g_a|) \\ & + \Omega_2(|e_L\rangle_{22}\langle g_L| + |e_R\rangle_{22}\langle g_R|),\end{aligned} \tag{5}$$

where $\hat{H}_0$ characterizes the strong interaction between atoms and quantized cavity fields, and $\hat{H}_g$ and $\hat{V}_\pm$ correspond to the weakly driven fields of microwave and optical lasers, respectively. For simplicity, we set $g_L = g_R = g$, $\Omega_1 = \Omega_2 = \Omega$, and $\omega_1 = -\omega_2 = \omega$ in the following. To gain better insight into the effect of spontaneous emission on the preparation of an entanglement state, we first consider a perfect cavity without decay. According to the effective operator method [31], the excited states of the atoms and the cavity field modes can be adiabatically eliminated, provided that the Rabi frequency Ω of the optical pumping laser is sufficiently weak enough compared with g, δ, and Δ, and the excited states are not initially populated. Then we obtain the effective master equation as

$$\begin{aligned}\dot{\hat{\rho}} = {} & i[\hat{\rho},\hat{H}_{\text{eff}}] + \sum_j \hat{L}_{\text{eff},j}\hat{\rho}\hat{L}_{\text{eff},j}^\dagger \\ & - \frac{1}{2}(\hat{L}_{\text{eff},j}^\dagger\hat{L}_{\text{eff},j}\hat{\rho} + \hat{\rho}\hat{L}_{\text{eff},j}^\dagger\hat{L}_{\text{eff},j}),\end{aligned} \tag{6}$$

where

$$\begin{aligned}\hat{H}_{\text{eff}} &= -\tfrac{1}{2}\big[\hat{V}_-\hat{H}_{NH}^{-1}\hat{V}_+ + \hat{V}_-\big(\hat{H}_{NH}^{-1}\big)^\dagger\hat{V}_+\big] + \hat{H}_g, \\ \hat{L}_{\text{eff},j} &= \hat{L}_j\hat{H}_{NH}^{-1}\hat{V}_+.\end{aligned} \tag{7}$$

In the above expression, $\hat{H}_{NH} = \hat{H}_0 - \frac{i}{2}\sum_j\hat{L}_j^\dagger\hat{L}_j$ is a non-Hermitian Hamiltonian, and its inverted matrix can be written as $\hat{H}_{NH}^{-1} = \hat{H}_{NH_1}^{-1} + \hat{H}_{NH_2}^{-1} + \hat{H}_{NH_3}^{-1}$, explicitly

$$\begin{aligned}\hat{H}_{NH_1}^{-1} = {} & \frac{g^2 - 3\delta\Delta'}{9g^2\Delta' - 3\delta\Delta'^2}|X_1\rangle\langle X_1| + \frac{1}{9}\left[\frac{8}{\Delta'} - \frac{\delta}{3g^2 - \delta\Delta'}\right]|X_2\rangle\langle X_2| - \frac{1}{g^2 - \delta\Delta'}(\delta|X_3\rangle\langle X_3| + \Delta'|-\rangle\langle -|) - \frac{\Delta'}{3g^2 - \delta\Delta'}|+\rangle\langle +| \\ & + \left[\frac{2\sqrt{2}g^2}{9g^2\Delta' - 3\delta\Delta'^2}|X_2\rangle\langle X_1| + \frac{2\sqrt{2}g}{\sqrt{3}(3g^2 - \delta\Delta')}|+\rangle\langle X_1| - \frac{g}{\sqrt{3}(3g^2 - \delta\Delta')}|+\rangle\langle X_2| + \frac{g}{g^2 - \delta\Delta'}|-\rangle\langle X_3| + \text{H.c.}\right],\end{aligned} \tag{8}$$

$$\begin{aligned}\hat{H}_{NH_2}^{-1} = {} & -\frac{\delta}{2g^2 - \delta\Delta'}(|e_0g_L\rangle\langle e_0g_L| + |e_0g_R\rangle\langle e_0g_R|) + \left[\frac{1}{\delta} - \frac{g^2}{2g^2\delta - \delta^2\Delta'}\right](|g_Lg_L\rangle|1_L\rangle\langle g_Lg_L|\langle 1_L| + |g_Rg_L\rangle|1_R\rangle\langle g_Rg_L|\langle 1_R| \\ & + |g_Lg_R\rangle|1_L\rangle\langle g_Lg_R|\langle 1_L| + |g_Rg_R\rangle|1_R\rangle\langle g_Rg_R|\langle 1_R|) + \bigg\{\frac{g}{2g^2 - \delta\Delta'}[(|g_Lg_L\rangle|1_L\rangle + |g_Rg_L\rangle|1_R\rangle)\langle e_0g_L| + (|g_Lg_R\rangle|1_L\rangle \\ & + |g_Rg_R\rangle|1_R\rangle)\langle e_0g_R|] - \frac{g^2}{2g^2\delta - \delta^2\Delta'}(|g_Rg_L\rangle|1_R\rangle\langle g_Lg_L|\langle 1_L| + |g_Rg_R\rangle|1_R\rangle\langle g_Lg_R|\langle 1_L|) + \text{H.c.}\bigg\},\end{aligned} \tag{9}$$

$$\begin{aligned}\hat{H}_{NH_3}^{-1} = {} & -\frac{\delta}{g^2 - \delta\Delta'}(|g_ae_L\rangle\langle g_ae_L| + |g_ae_R\rangle\langle g_ae_R| + |g_Le_L\rangle\langle g_Le_L| + |g_Re_R\rangle\langle g_Re_R|) - \frac{\Delta'}{g^2 - \delta\Delta'}(|g_ag_0\rangle|1_R\rangle\langle g_ag_0|\langle 1_R| \\ & + |g_ag_0\rangle|1_L\rangle\langle g_ag_0|\langle 1_L| + |g_Lg_0\rangle|1_R\rangle\langle g_Lg_0|\langle 1_R| + |g_Rg_0\rangle|1_L\rangle\langle g_Rg_0|\langle 1_L|) + \frac{g}{g^2 - \delta\Delta'}(|g_ag_0\rangle|1_R\rangle\langle g_ae_L| \\ & + |g_ag_0\rangle|1_L\rangle\langle g_ae_R| + |g_Lg_0\rangle|1_R\rangle\langle g_Le_L| + |g_Rg_0\rangle|1_L\rangle\langle g_Re_R| + \text{H.c.}),\end{aligned} \tag{10}$$

where $\Delta' = \Delta - \frac{i\gamma}{2}$, the vacuum states of cavity modes are discarded, and we have adopted the notation $|X_1\rangle = \frac{1}{\sqrt{3}}(|g_Le_R\rangle + |g_Re_L\rangle + |e_0g_a\rangle)$, $|X_2\rangle = \frac{1}{\sqrt{6}}(|g_Le_R\rangle + |g_Re_L\rangle - 2|e_0g_a\rangle)$, $|X_3\rangle = \frac{1}{\sqrt{2}}(|g_Le_R\rangle - |g_Re_L\rangle)$, and $|+\rangle =$

$\frac{1}{\sqrt{2}}(|g_Lg_0\rangle|1_L\rangle + |g_Rg_0\rangle|1_R\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|g_Lg_0\rangle|1_L\rangle - |g_Rg_0\rangle|1_R\rangle)$. On the basis of Eq. (7), we have the effective Hamiltonian as

$$\begin{aligned}\hat{H}_{\text{eff}} = {}& \Omega^2\text{Re}\left[\frac{\delta}{g^2-\delta'\Delta'} + \frac{\delta}{2g^2-\delta'\Delta'}\right](|g_ag_L\rangle\langle g_ag_L| + |g_ag_R\rangle\langle g_ag_R|) + \Omega^2\text{Re}\left[\frac{\delta}{g^2-\delta\Delta'}\right](|g_Lg_L\rangle\langle g_Lg_L| + |g_Rg_R\rangle\langle g_Rg_R|\\ & + |T_3\rangle\langle T_3|) - \Omega^2\text{Re}\left[\frac{g^2-3\delta\Delta'}{9g^2\Delta'-3\delta\Delta'2}\right]|T_1\rangle\langle T_1| - \Omega^2\text{Re}\left[\frac{2\sqrt{2}g^2}{9g^2\Delta'-3\delta\Delta'2}\right](|T_1\rangle\langle T_2| + \text{H.c.})\\ & - \Omega^2\text{Re}\left\{\frac{1}{9}\left[\frac{8}{\Delta'} - \frac{\delta}{3g^2-\delta\Delta'}\right]\right\}|T_2\rangle\langle T_2| + \hat{H}_g,\end{aligned} \tag{11}$$

where

$$|T_1\rangle = \frac{1}{\sqrt{3}}(|g_Lg_R\rangle + |g_Rg_L\rangle + |g_ag_0\rangle) \tag{12}$$

is the desired three-dimensional entangled state and $|T_2\rangle = \frac{1}{\sqrt{6}}(|g_Lg_R\rangle + |g_Rg_L\rangle - 2|g_ag_0\rangle)$, $|T_3\rangle = \frac{1}{\sqrt{2}}(|g_Lg_R\rangle - |g_Rg_L\rangle)$.

III. THE ROLE OF SPONTANEOUS DECAY RATES

Referring to Eq. (7), the effective Lindblad operators induced by the spontaneous emission take the form of

$$\begin{aligned}\hat{L}_{\text{eff}}^{\gamma_1,g_{L(a,R)}} = {}& \frac{\Omega\sqrt{\gamma}}{\sqrt{3}}\left\{\frac{1}{\sqrt{3}}|g_{L(a,R)}g_0\rangle\left[\left(\frac{g^2-3\delta\Delta'}{9g^2\Delta'-3\delta\Delta'^2} - \frac{4g^2}{9g^2\Delta'-3\delta\Delta'^2}\right)\langle T_1|\right.\right.\\ & \left.\left.+\frac{\sqrt{2}}{\sqrt{3}}\left(\frac{2g^2}{9g^2\Delta'-3\delta\Delta'^2} - \frac{8}{9\Delta'} + \frac{\delta}{3g^2-\delta\Delta'}\right)\langle T_2|\right] - \frac{\delta}{2g^2-\delta\Delta'}(|g_{L(a,R)}g_L\rangle\langle g_ag_L| + |g_{L(a,R)}g_R\rangle\langle g_ag_R|)\right\},\end{aligned} \tag{13}$$

$$\begin{aligned}\hat{L}_{\text{eff}}^{\gamma_2,g_{L(R)}} = {}& \frac{\Omega\sqrt{\gamma}}{\sqrt{2}}\left\{\frac{1}{\sqrt{3}}|g_{R(L)}g_{L(R)}\rangle\left[\left(\frac{g^2-3\delta\Delta'}{9g^2\Delta'-3\delta\Delta'^2} + \frac{2g^2}{9g^2\Delta'-3\delta\Delta'^2}\right)\langle T_1|\right.\right.\\ & \left.+\frac{\sqrt{2}}{\sqrt{3}}\left(\frac{2g^2}{9g^2\Delta'-3\delta\Delta'^2} + \frac{8}{9\Delta'} - \frac{\delta}{3g^2-\delta\Delta'}\right)\langle T_2|\right] - \frac{\delta}{g^2-\delta\Delta'}(|g_ag_{L(R)}\rangle\langle g_ag_{L(R)}| + |g_{L(R)}g_{L(R)}\rangle\langle g_{L(R)}g_{L(R)}|\\ & \left.\mp\frac{1}{\sqrt{2}}|g_{R(L)}g_{L(R)}\rangle\langle T_3|)\right\},\end{aligned} \tag{14}$$

$$\begin{aligned}\hat{L}_{\text{eff}}^{\gamma_2,g_0} = {}& \frac{\Omega\sqrt{\gamma}}{\sqrt{2}}\left\{\frac{1}{\sqrt{3}}(|g_Rg_0\rangle + |g_Lg_0\rangle)\left[\left(\frac{g^2-3\delta\Delta'}{9g^2\Delta'-3\delta\Delta'^2} + \frac{2g^2}{9g^2\Delta'-3\delta\Delta'^2}\right)\langle T_1|\right.\right.\\ & \left.+\frac{\sqrt{2}}{\sqrt{3}}\left(\frac{2g^2}{9g^2\Delta'-3\delta\Delta'^2} + \frac{8}{9\Delta'} - \frac{\delta}{3g^2-\delta\Delta'}\right)\langle T_2|\right] - \frac{\delta}{g^2-\delta\Delta'}\big[|g_ag_0\rangle\\ & \left.\times(\langle g_ag_L| + \langle g_ag_R|) + |g_Lg_0\rangle\langle g_Lg_L| + |g_Rg_0\rangle\langle g_Rg_R| - \frac{1}{\sqrt{2}}(|g_Rg_0\rangle - |g_Lg_0\rangle)\langle T_3|\big]\right\}.\end{aligned} \tag{15}$$

In order to have a compact form for the above expression, we have employed $|g_Lg_R\rangle$, $|g_Rg_L\rangle$, and $|g_ag_0\rangle$ to represent the *kets* instead of $|T_{1(2,3)}\rangle$. It is worth noting that if the cavity detuning δ from two photon resonance meets the requirements $\delta = g^2/\Delta$, $\Delta \gg \gamma$, other decay rates approximately equal to zero except the following dominant parts:

$$\hat{L}_{\text{eff}}^{\gamma_2,g_{L(R)}} = \frac{\sqrt{\gamma}}{\sqrt{2}}\frac{g_{\text{eff}}}{\delta\gamma/(2\Delta)}\left[\left(\frac{1}{2}|T_3\rangle \mp \frac{1}{\sqrt{6}}|T_1\rangle \mp \frac{1}{2\sqrt{3}}|T_2\rangle\right)\langle T_3| + |g_{L(R)}g_{L(R)}\rangle\langle g_{L(R)}g_{L(R)}| + |g_ag_{L(R)}\rangle\langle g_ag_{L(R)}|\right], \tag{16}$$

$$\begin{aligned}\hat{L}_{\text{eff}}^{\gamma_2,g_0} = {}& \frac{\sqrt{\gamma}}{\sqrt{2}}\frac{g_{\text{eff}}}{\delta\gamma/(2\Delta)}\left[\left(\frac{1}{\sqrt{3}}|T_1\rangle - \frac{2}{\sqrt{6}}|T_2\rangle\right)(\langle g_ag_L| + \langle g_ag_R|) + |g_Lg_0\rangle\langle g_Lg_L| + |g_Rg_0\rangle\langle g_Rg_R|\right.\\ & \left.-\frac{1}{\sqrt{2}}(|g_Rg_0\rangle - |g_Lg_0\rangle)\langle T_3|\right],\end{aligned} \tag{17}$$

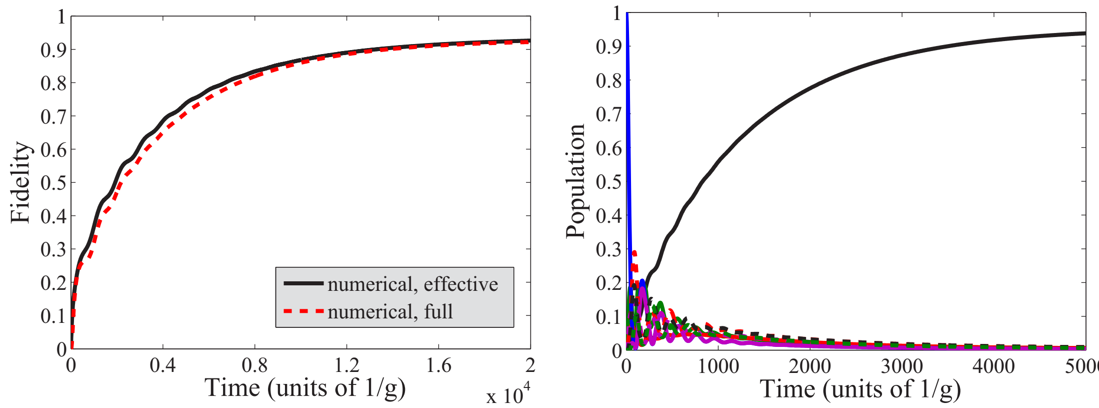

FIG. 2. (Color online) Left panel: The comparison of fidelities for preparation of the three-dimensional entangled state $|T_1\rangle$ from an initial state $|g_a g_L\rangle$ with the full master equation (red dashed curve) and the effective one (black curve) under the given parameters $\Omega = 0.02g$, $\omega = 0.1\Omega$, $\Delta = g$ and $\kappa = 0$, $\gamma = 0.1g$. Right panel: The populations of quantum states with optimized parameters $\Omega = 0.03g$, $\omega = 0.4\Omega$, and $\Delta = g$ to achieve a stationary state within a short time corresponding to the same dissipation rate $\kappa = 0$, $\gamma = 0.1g$.

where $g_{\rm eff} = g\Omega/\Delta$. The dissipative dynamics alone results in populations of $|T_1\rangle$, $|T_2\rangle$, $|g_L g_0\rangle$, and $|g_R g_0\rangle$.On the other hand, the coherent dynamics offered by microwave fields between ground states guarantees $|T_1\rangle$ remains invariant while the other three ground states are driven out of the steady subspace of Eqs. (16) and (17). Thus the three-dimensional entangled state $|T_1\rangle$ is able to be achieved from an arbitrary initial state via the effective dissipation induced by spontaneous emission. In the left panel of Fig. 2, we plot the fidelities $F(|T_1\rangle,\hat{\rho}) = \langle T_1|\hat{\rho}|T_1\rangle$ for creation of $|T_1\rangle$ with the full and the effective master equations, from which we see that under the given parameters the full and the effective dynamics of the system are in excellent agreement. In the right panel, we further optimize the parameters to make the entangled state reach stability in a shorter time.

IV. GENERALIZATION TO HIGH-DIMENSIONAL ENTANGLED STATE

The successful use of dissipation to deterministic creation of a three-dimensional entangled state mainly relies on the effective level structure of atoms, i.e., we require transitions from a common excited (ground) state of first (second) atom to two ground (excited) states coupled by two orthogonal cavity modes, while other transitions are driven by off-resonance optical lasers. Thus it is possible to generalize our model to prepare a high-dimensional entangled state if we design the atomic energy-level diagram following similar rules. In Fig. 3, we suppose two potential multilevel atoms strongly interact with a multimode optical cavity, which is a direct extension of Fig. 1. By introducing microwave fields that drive the transitions $|g_0\rangle \leftrightarrow |g_i\rangle$ where $i = 1,\ldots,N-1$, an N-dimensional entangled state $1/\sqrt{N}(|g_a g_a\rangle + |g_1 g_1\rangle + |g_2 g_2\rangle + \cdots + |g_{N-1} g_{N-1}\rangle)$ will be carried out via spontaneous emission. In confirmation of our assumption, we numerically simulate the fidelity for generating the four-dimensional entangled state with the full master equation in the left panel of Fig. 4. Compared with the case of the three-dimensional entangled state, a longer time is required to stabilize the target state above the fidelity 90%. Hence it is not difficult to conclude that the increase of dimension is at the cost of convergence time.

Now we briefly discuss the effect of cavity decay on the performance for entanglement preparation. In the right panel of Fig. 4, we plot the fidelity by numerically solving the full master equation of Eq. (1) incorporating κ; the three curves correspond to different parameters of dissipation, i.e., $\kappa = \gamma = 0.05g$, $\kappa = \gamma = 0.1g$, and $\kappa = \gamma/2 = 0.1g$. The decrease of population for $|T_1\rangle$ undoubtedly accompanies the increase of populations for other states. As the system approaches equilibrium, we will obtain a steady-mixed entanglement state. For certain cavity setup, the coupling strength between atom and cavity g, the cavity leakage rate κ, and the spontaneous emission rate γ are fixed, thus we are allowed to modulate other parameters to achieve a three-dimensional entangled state with a relatively high fidelity. Figure 5 illustrates the evolution of fidelity versus time in units of g^{-1} with cavity parameters extracted from a recent experiment $(g,\kappa,\gamma) \sim 2\pi \times (750,2.62,3.5)$ MHz [32]. A selection of $\Omega = 0.02g$, $\omega = 0.4\Omega$, $\Delta = g$ will lead to a fidelity about 98%, which overwhelms with the values based on the unitary dynamics [24–26].

V. CONCLUSION

In conclusion, we have achieved a stationary three-dimensional entangled state via using the dissipation caused

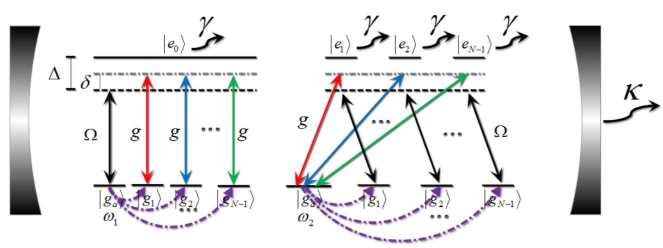

FIG. 3. (Color online) A potential atomic energy-level diagram to be used for generating an N-dimensional entangled state $1/\sqrt{N}(|g_a g_a\rangle + |g_1 g_1\rangle + |g_2 g_2\rangle + \cdots + |g_{N-1} g_{N-1}\rangle)$.

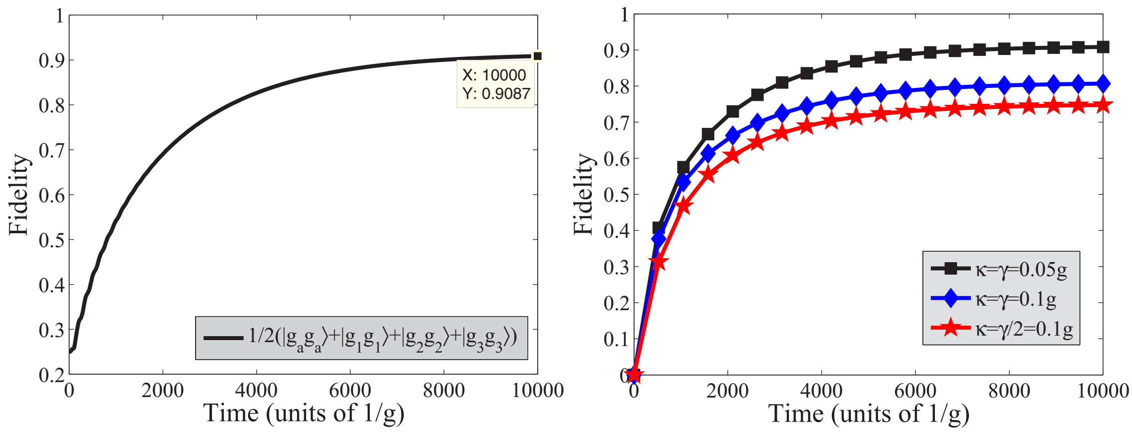

FIG. 4. (Color online) Left panel: The fidelity for preparation of four-dimensional entangled state with the same parameters shown in the right panel of Fig. 2, the initial state is randomly chosen as $|g_a g_a\rangle$. Right panel: The effect of cavity loss on the preparation of three-dimensional entangled state.

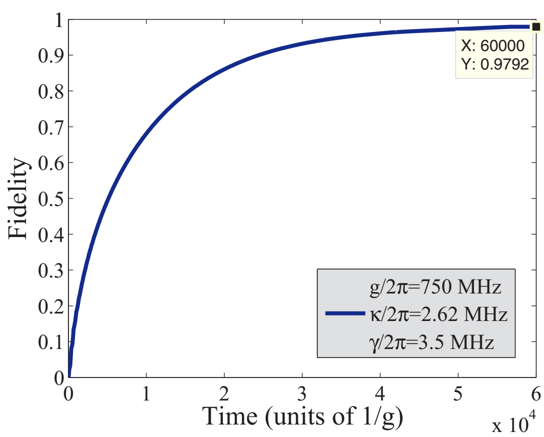

FIG. 5. (Color online) Fidelity for generation of three-dimensional entangled state using an experimental cavity parameters.

by spontaneous emission of atoms. The numerical simulation reveals the theory for effective operator and agrees well with the full master equation under given parameters. This proposal is then extended to realize the N-dimensional entangled state in theory by considering two multilevel atoms interacting with a multimode cavity, which is confirmed by the simulation of implementing a four-dimensional entangled state. The cavity decay plays a negative role on the state preparation, thus corresponding to different experimental situations, we need to regulate the Rabi frequencies of both optical and microwave fields accurately so as to obtain a relatively high fidelity. We believe that our work will be useful for the experimental realization of quantum information in the near future.

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