

Size of quantum superpositions as measured with classical detectors

Pavel Sekatski,^{*} Nicolas Sangouard, and Nicolas Gisin

Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland

(Received 13 June 2013; published 17 January 2014)

We propose a criterion which defines whether a superposition of two photonic components is macroscopic. It is based on the ability to discriminate these components with a particular class of “classical” detectors, namely, a photon number measurement with a resolution coarse-grained by noise. We show how our criterion can be extended to a measure of the size of macroscopic superpositions by quantifying the amount of noise that can be tolerated and taking the distinctness of two Fock states differing by N photons as a reference. After applying our measure to several well-known examples, we demonstrate that the superpositions which meet our criterion are very sensitive to phase fluctuations. This suggests that quantifying the macroscopicity of a superposition state through the distinguishability of its components with “classical” detectors not only is a natural measure but also explains why it is difficult to observe superpositions at the macroscopic scale.

DOI: [10.1103/PhysRevA.89.012116](https://doi.org/10.1103/PhysRevA.89.012116)

PACS number(s): 03.65.Ta, 03.67.Mn

I. INTRODUCTION

Quantum physics is sometimes presented as a theory of microscopic phenomena only, suggesting that there could be a boundary beyond which quantum laws do not apply. However, there is nothing in quantum physics itself that predicts the existence of such a boundary. So either quantum theory is incomplete or quantum effects apply at any scale but demand a particular effort to be maintained and revealed. This concern provided strong motivation over the last decades to prove through experiments that macroscopic systems can exhibit quantum effects. The question at issue is how to judge whether a quantum system is macroscopic.

Let us set the problem. Take an entangled bipartite state,

$$|\uparrow\rangle_A |A\rangle_B + |\downarrow\rangle_A |D\rangle_B, \quad (1)$$

where party A is a qubit and B involves two photonic components. Note that even though the terminology of macroscopic superposition is sometimes used, qubit A is necessary to fix components $|A\rangle$ and $|D\rangle$ (up to rotations). Assume that one knows how to reveal the entanglement in (1). To call this entanglement macroscopic, one wants states $|A\rangle$ and $|D\rangle$ to be macroscopically distinct [1]. But how does one tell whether this is the case? So far there is no consensus on what this criterion should be and there could be a variety of different, though related, concepts. However, we know what macroscopicity cannot be.

The notion of macroscopicity cannot be invariant under local unitaries, in strong contrast to entanglement. This is already clear in Schrödinger’s *gedanken* experiment, where a microscopic state of a photonic mode ($|0\rangle$ or $|1\rangle$) is mapped with a unitary transformation onto the macroscopic state of a cat ($|\text{Alive}\rangle$ or $|\text{Dead}\rangle$). Another example is a series of C-NOT gates that allows one to map a microscopic superposition of qubit states $|\uparrow\rangle$ and $|\downarrow\rangle$ onto a large GHZ-type superposition of $|\uparrow\rangle^{\otimes N}$ and $|\downarrow\rangle^{\otimes N}$. Therefore, in our quest for a macroscopicity criterion, the local unitary invariance has to be abandoned. Furthermore, finding a physically motivated way to break this invariance is the solution to the problem we are aiming at.

Several criteria have been proposed recently to define the notion of macroscopicity [2–8]. Specifically, Korsbakken and coauthors [4] linked the macroscopicity of a superposition state carried by an ensemble of qubits with the ease of distinguishing its components when only a few qubits are analyzed. This approach (as the majority of available criteria) relies on the partition of the total Hilbert space into individual particles, and there is no such partition for bosonic system. The criterion that we introduce recognizes that the entangled state, (1), is macroscopic if its components, $|A\rangle$ and $|D\rangle$, are well distinguishable. It demands that these components can be distinguished in a single shot with classical detectors; i.e., the components lead to very different results when measured with detectors whose limited resolution forbids the resolution of microscopic states. Indeed common sense tells us that a property (the distinctness here) is macroscopic if it is available first-hand for us to observe. This follows the intuition that there is no need for a microscopic resolution to distinguish the dead and alive components of the Schrödinger cat. More precisely, we focus on photon number measurements coarse-grained by noise, a measurement resolving large photon number differences only. It can distinguish a vacuum from an N -photon Fock state $|N\rangle$ (as long as N is larger than the detector’s uncertainty) but it is unable to discriminate a vacuum from a single photon. This supports the natural claim that state (1) with Fock states $|A\rangle = |M\rangle$ and $|D\rangle = |M+1\rangle$ is a micro-micro entangled state, whereas it corresponds to micro-macro entanglement for $|A\rangle = |M\rangle$ and $|D\rangle = |M+N\rangle$ (when $N \gg 1$). The choice of the photon number measurement for our criterion is arbitrary to some extent. The whole development could just as well be deployed starting with another observable, leading to another hierarchy of macroscopic states. However, the energy (photon number) is a very particular quantity. It is what the human eye measures, and more importantly, it is the only one that can be measured with a passive device and without phase reference (in the nondegenerate case) (see Appendix).

II. CRITERION FOR MACROSCOPICITY

A noisy photon number measurement is given by the textbook model depicted in Fig. 1. A classical pointer on a scale x interacts with state $|S\rangle$ of mode B and its position

^{*}pavel.sekatski@unige.ch

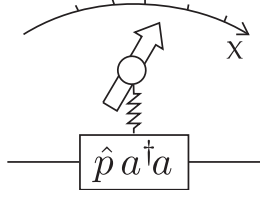


FIG. 1. A simple model of the photon number measurement.

is shifted by a value corresponding to the photon number in B.¹ The number of photons in $|S\rangle$ is inferred by reading out the final position of the pointer, which spans position x with probability $p_S(x) = \text{tr}_B p_i(x + a^\dagger a) |S\rangle \langle S|$. If the initial position of the pointer $p_i^0(x)$ is δ -peaked around 0, the final probability $p_S^0(x)$ exactly reproduces the statistics of $a^\dagger a$ and corresponds to a projective measurement. On the other hand, when the initial position of the pointer has a nonzero Gaussian spread, $p_i^\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{x^2}{2\sigma^2})$, the probability distribution $p_S^\sigma(x)$ available to the experimenter does not contain full information on the photon number statistics. Increasing σ lowers the resolution of the detector, making it more and more “classical.” A normally distributed position of the pointer is something one would expect from a classical object, where statistical fluctuations come from a lot of uncorrelated factors.²

According to our definition, the macroscopic state (1) involves components $|A\rangle$ and $|D\rangle$, which can be distinguished with such a detector. Consider a game where B receives one of these two components (for example, prepared by party A) and has to guess which one has been sent. The probability of making a correct guess in a single shot,

$$P^\sigma[|A\rangle, |D\rangle] = \frac{1}{2} \{1 + D[p_A^\sigma(x), p_D^\sigma(x)]\}, \quad (2)$$

is related to the trace distance $D[p_A^\sigma(x), p_D^\sigma(x)] = \frac{1}{2} \int dx |p_A^\sigma(x) - p_D^\sigma(x)|$ between the outcome distributions $p_A^\sigma(x)$ and $p_D^\sigma(x)$. The size of the superposition should be related to the amount of noise σ that can be tolerated.

III. QUANTIFYING THE SIZE

To define a measure of macroscopicity we have to make comparisons to a reference case for which there is a natural definition of the size of a superposition. Fock states provide a perfect chance for such a calibration, the probability of guessing between two Fock states $|M\rangle$ and $|M+N\rangle$ with a detector coarse-grained by Gaussian noise is independent of M and reads

$$P_{\text{Fock}}^\sigma[N] = \frac{1}{2} \left(1 + \text{erf} \left(\frac{1}{2\sqrt{2}} \frac{N}{\sigma} \right) \right). \quad (3)$$

The size of state (1) is given by N , for which $P_{\text{Fock}}^\sigma[N]$ coincides with $P^\sigma[|A\rangle, |D\rangle]$. However, that leaves a free

¹Physically this may, for example, correspond to a shift of the kinetic moment of a material pointer given by the radiation pressure Hamiltonian $H = \hat{x} a^\dagger a$.

²Note that if the initial state of the pointer was set to be pure instead, then the detector would perform a weak measurement of the photon number saturating the information-disturbance relation.

parameter, σ , and to deal with it, we fix the required minimal probability to correctly guess between the two components P_g . The maximal tolerable noise $\sigma^{P_g}[|A\rangle, |D\rangle]$ is then the solution of the equation $P^\sigma[|A\rangle, |D\rangle] = P_g$. Given $\sigma^{P_g}[|A\rangle, |D\rangle]$, the size of the superposition, (1), is obtained by inverting $P_{\text{Fock}}^{\sigma[|A\rangle, |D\rangle]}[N] = P_g$ and corresponds to the N for which the Fock states achieve the same probability P_g . If state (1) does not reach the required P_g , we set its size to be 0. In general, the parameter P_g can be suggested by a particular task that one has in mind. For the numerical application, we use $P_g = 2/3$, as it is common in the literature on probabilistic algorithms [9]; in this case,³

$$\text{Size}_{P_g=2/3}[|A\rangle, |D\rangle] \approx 0.86 \sigma[|A\rangle, |D\rangle]. \quad (4)$$

Remember that by changing the basis on side A, we are free to choose the components $|A'\rangle = |c_\theta A + s_\theta e^{i\varphi} D\rangle$ and $|D'\rangle = |c_\theta D - s_\theta e^{i\varphi} A\rangle$ that maximize the size.

A. Example 1. Optical cat states $|\beta/2\rangle$ and $|- \beta/2\rangle$

Our first example involves two coherent states with opposite phases, $|A\rangle = |-\beta/2\rangle$ and $|D\rangle = |\beta/2\rangle$. Such states have been at the core of several experiments [10–12]. Obviously $|A\rangle = |-\beta/2\rangle$ and $|D\rangle = |\beta/2\rangle$ have the same energy spectra with the sign information encoded in the phase relation between neighboring Fock components, and hence they are completely confused by our detector. However, it is easy to modify these states to circumvent this problem: Displacing mode B by $-\beta/2$ brings the components to $|A\rangle = |0\rangle$ and $|D\rangle = |\beta\rangle$ (example 1a), with the corresponding photon number distributions separated by $|\beta|^2$. For fixed $P_g \neq 1$ and large enough β , the size of this state,

$$\text{Size}_{P_g}^{(1)} = |\beta|^2 - 2(\text{erf}^{-1}(2P_g - 1))^2, \quad (5)$$

increases linearly with respect to the number of photons, as expected. Remember that the size of the superposition can be increased by further displacing the components to $|A\rangle = |\alpha\rangle$ and $|D\rangle = |\alpha + \beta\rangle$ (example 1b). In the limit $\alpha \gg \beta$ the size of this superposition increases linearly with α , and for $|\beta|^2 \gg \text{erf}^{-1}(2P_g - 1)$ the size is proportional to the product $\alpha\beta$. The maximal achievable guessing probability is $\lim_{\alpha \rightarrow \infty} P^\sigma = \frac{1}{2}(1 + \text{erf}(\frac{\beta}{\sqrt{2}}))$.

This example clearly shows that our measure is not invariant under displacement, since the latter is a nontrivial transformation of the energy spectrum. This is well known in the context of homodyne measurements, where the detector noise σ can be circumvented by displacing the measured mode. The next example also exploits the noninvariance of the size with respect to displacement.

B. Example 2. Coherent state and displaced single photon

We recently proposed [13] to investigate the quantum features of macro states through a displaced single-photon

³In general,

$$\text{Size}_{P_g}[|A\rangle, |D\rangle] = 2\sqrt{2}\text{erf}^{-1}(2P_g - 1)\sigma^{P_g}[|A\rangle, |D\rangle].$$

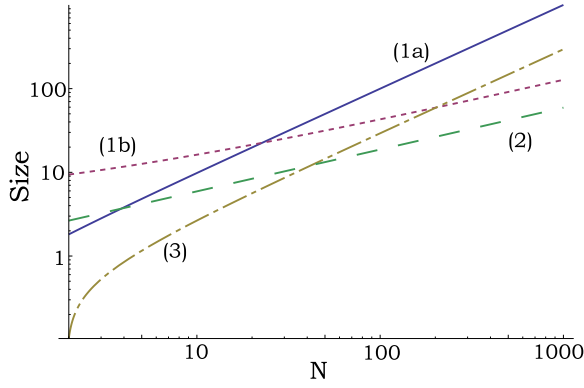


FIG. 2. (Color online) Size of several states for $P_g = 2/3$ and increasing “particle number” N . (1a) $|0\rangle$ and $|\beta\rangle$ with $N = |\beta|^2$. (1b) $|\alpha\rangle$ and $\mathcal{D}(\alpha)|\beta\rangle$ with $N = |\alpha|^2$ and $|\beta|^2 = 4$. (2) $\mathcal{D}(\alpha)|+\rangle$ and $\mathcal{D}(\alpha)|-\rangle$ with $N = |\alpha|^2$. (3) $|\phi_1\rangle^{\otimes N}$ and $|\phi_2\rangle^{\otimes N}$ with $\delta = 0.3$.

entangled state $\mathcal{D}(\alpha)_B(|1_A, 0_B\rangle - |0_A, 1_B\rangle)$, which can be rewritten as $\mathcal{D}(\alpha)_B(|+\rangle_A, -|-\rangle_B) - |-\rangle_A, +|+\rangle_B)$, with $|+\rangle = |0+1\rangle$ and $|-\rangle = |0-1\rangle$ (see [14] and [15] for the corresponding experiments). The photon number distributions for $|A\rangle = \mathcal{D}(\alpha)|+\rangle$ and $|D\rangle = \mathcal{D}(\alpha)|-\rangle$ are both of width α and have their means separated by 2α . For large enough $|\alpha|^2$ (>50) the statistic of a coherent state follows a normal distribution, and the guessing probability is a monotonous function of the ratio $\frac{\sigma}{\alpha}$ (contrary to $\frac{\sigma}{N}$ for Fock states), with $\lim_{\alpha \rightarrow \infty} P^\sigma \approx 0.899$. Consequently, the size of this state scales as the square root of the photon number and is precisely given by

$$\text{Size}_{P_g}^{(2)} = 2\alpha \text{erf}^{-1}(2P_g - 1) \sqrt{\frac{1}{\pi(2P_g - 1)^2} - 2}. \quad (6)$$

C. Example 3. GHZ-like state with overlapping components

Let us now focus on the state studied in [2], where the components $|A\rangle = |\phi_1\rangle^{\otimes N}$ and $|D\rangle = |\phi_2\rangle^{\otimes N}$ contain N copies of two nonorthogonal states, $|\langle\phi_1|\phi_2\rangle|^2 = 1 - \epsilon^2$, each copy corresponding to a two-level system. Although these states are not photonic but describe spin ensembles, it is easy to generalize our criterion to this case. To do so, replace the number of photons with the population in the excited states (number of $|e\rangle$'s) in the definition of the classical detector. It is then clear that the size of the superposition depends not only on the relative angle ϵ between $|\phi_1\rangle$ and $|\phi_2\rangle$ but also on their azimuthal angle. For $|\phi_j\rangle = \cos(\theta_j)|g\rangle + \sin(\theta_j)|e\rangle$, with $\theta = \frac{\pi}{4} + (-1)^j \frac{\delta}{2}$ and $\sin(\delta) = \epsilon$, the mean populations of $|A\rangle$ and $|D\rangle$ are maximally separated, and for large N ,

$$\text{Size}_{P_g}^{(3)} = N\epsilon \sqrt{1 - \frac{2(\text{Erf}^{-1}(2P_g - 1))^2}{N \frac{\epsilon^2}{1 - \epsilon^2}}}, \quad (7)$$

which tends to $N\epsilon$ in the asymptotic limit.

Figure 2 shows the size of the states that we considered for the guessing probability $P_g = 2/3$. The typical behavior of the size as a function of P_g is shown in Fig. 3.

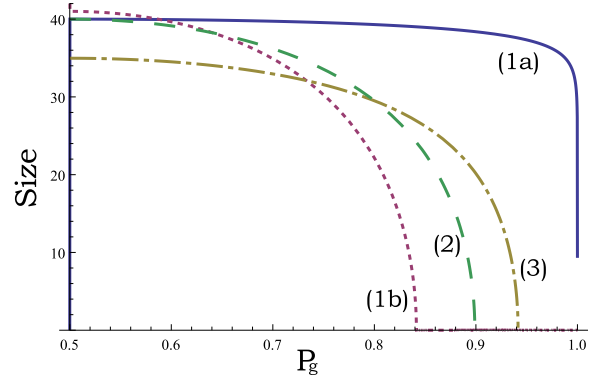


FIG. 3. (Color online) Size of several states for a fixed “particle number” N as a function of the guessing probability P_g . (1a) $|0\rangle$ and $|\beta\rangle$ with $|\beta|^2 = 40$. (1b) $|\alpha\rangle$ and $\mathcal{D}(\alpha)|\beta\rangle$ with $|\alpha|^2 = 400$ and $|\beta|^2 = 1$. (2) $\mathcal{D}(\alpha)|+\rangle$ and $\mathcal{D}(\alpha)|-\rangle$ with $|\alpha|^2 = 400$. (3) $|\phi_1\rangle^{\otimes N}$ and $|\phi_2\rangle^{\otimes N}$ with $N = 500$ and $\delta = 0.07$.

D. A comment on the size for several copies

An interesting question is how the size of the superposition in our definition behaves when several copies of the state are available. Can one predict what happens when two copies of the states are provided, going from $\{|A\rangle, |D\rangle\}$ to $\{|A\rangle|A\rangle, |D\rangle|D\rangle\}$? Unfortunately, this is impossible with any definition based on the guessing probability P_g governed by the trace distance. The problem appears already on the classical level: when two copies are measured they give a couple of outcomes (x, y) spanned by $p_A(x)p_A(y)$ or $p_D(x)p_D(y)$. But for the trace distance no general relation between $D[p_A, p_D]$ and $D[p_A^{\otimes 2}, p_D^{\otimes 2}]$ exists, and the optimal partition of the outcome plane (x, y) depends on the particular shape of the distributions p_A and p_D . A good example illustrating this is the task of guessing between two biased coins with face-tail probabilities $p_A = \{p, 1-p\}$ and $p_D = \{1-p, p\}$. One easily verifies that the probability of making a correct guess does not increase after the second throw. Remark that the fidelity between two distributions $F[p_A, p_D] = \int dx \sqrt{p_A(x)p_D(x)}$ behaves nicely with respect to the number of copies $F[p_A^{\otimes N}, p_D^{\otimes N}] = F^N[p_A, p_D]$. But it does not have a nice interpretation in terms of the probability of discriminating between the two states, so a fidelity-based definition of the size is not physically motivated, contrary to (4).

IV. PHASE RESOLUTION AND ENTANGLEMENT

So far we have presented an approach to determining whether the components on side B of state (1) are macroscopically distinct, assuming throughout that they are in a superposition. A certified way of ensuring that this is the case, i.e., that components $|\uparrow\rangle_A|A\rangle_B$ and $|\downarrow\rangle_A|D\rangle_B$ are indeed superposed and not mixed, is to reveal entanglement between A and B. To do so, measurements of the number of photons used for macroscopicity are not sufficient on their own; one also needs at least one measurement in another basis. In the single-mode case any such measurement will imply the use of a local oscillator providing phase information, since the underlying POVM necessarily involves coherences between different Fock components $|n\rangle\langle m|$. In

practice, any measurement involving a local oscillator will suffer from a limited phase resolution $\Delta\varphi$. This limitation can be equivalently pictured as a degradation of the local oscillator phase or as a quantum channel injecting a random phase in the system

$$\mathcal{E}_{\Delta\varphi}(\rho) = \int d\varphi \tilde{p}(\varphi) e^{-i\varphi a^\dagger a} \rho e^{i\varphi a^\dagger a}, \quad (8)$$

with a normally distributed random variable φ characterized by the standard deviation $\Delta\varphi$. The entanglement in state (1), which is experimentally accessible with measurements having a limited phase resolution $\Delta\varphi$, equals the algebraic entanglement in $\mathcal{E}_{\Delta\varphi}^B(|\uparrow\rangle_A |A\rangle_B + |\downarrow\rangle_A |D\rangle_B)$.

The usual phase noise channel, (8), admits a representation by a unitary evolution of the system plus the environment, which is delightful in the present context. Consider an environmental pointer state $|E_0\rangle$ interacting with the system ρ via $U = e^{-i\hat{p}a^\dagger a}$. The propagator U shifts the position of the pointer in a controlled way, $|E_0(x)\rangle \rightarrow |E_0(x - a^\dagger a)\rangle$, similarly to the detector defined above. For a pure state $|E_0(x)\rangle$ with a Gaussian envelope and spread Δx , what we have described is nothing other than a weak measurement of the photon number performed by the environment. The state of the system after such an interaction is $\rho' = \text{tr}_E U \rho |E_0\rangle \langle E_0| U^\dagger$. Using $\mathbb{1}_E = \int dp |p\rangle \langle p|$ one finds

$$\rho' = \int dp \tilde{p}(p) e^{-ipa^\dagger a} \rho e^{ipa^\dagger a} = \mathcal{E}_{\Delta p}(\rho), \quad (9)$$

where $\tilde{p}(p) = |\langle p|E_0\rangle|^2 = |\tilde{E}_0(p)|^2$ is a Gaussian with standard deviation Δp . The probability amplitude $\tilde{E}_0(p)$ in the momentum space is the Fourier transform of the amplitude $E_0(x)$ in the position space, so the following relation holds: $\Delta x = \frac{1}{2\Delta p}$. Therefore, a standard phase noise channel with fluctuation $\Delta\varphi$ corresponds to a weak photon number measurement of the state by the environment, with a pointer of spread $\frac{1}{2\Delta\varphi}$.

The entanglement in state $\mathcal{E}_{\Delta\varphi}(|\uparrow\rangle_A |A\rangle_B + |\downarrow\rangle_A |D\rangle_B)$ degrades when the ‘‘which-path’’ information ($|A\rangle$ or $|D\rangle$) available to the environment increases. The probability that after the weak measurement the environment will correctly guess between $|A\rangle$ and $|D\rangle$ is lower-bounded by $P^{\frac{1}{2\Delta\varphi}}[|A\rangle, |D\rangle]$ (it is not an equality because measuring in the x basis might not be the optimal choice for the environment). To put it more quantitatively, to experimentally reveal a fraction E of the initial entanglement in state (1), one needs measurements with a phase resolution smaller than

$$\Delta\varphi = \frac{\sqrt{2}\text{erf}^{-1}(2P - 1)}{\text{Size}_P[|A\rangle, |D\rangle]}, \quad (10)$$

where $P = \frac{1}{2}(1 + \sqrt{1 - E^2})$. In other words, for a fixed phase resolution, quantum features are washed out as the size of state (1) increases: any trace of entanglement progressively disappears from the measurement results.

V. CONCLUSION

We have proposed a measure of the size of macroscopic quantum superpositions. Our criterion relies on the intuition that what makes a property macroscopic is the possibility

of observing it with the simplest device. Accordingly, we define two components as being macroscopically distinct if they can be distinguished in a single shot with a noisy photon number measurement (‘‘classical’’ detector). The size of a superposition of these components is determined by first quantifying the maximal amount of noise that still allows one to distinguish them with a fixed probability and then comparing this to a superposition of Fock states $|M\rangle$ and $|M + N\rangle$, which we calibrate to be of size N . We applied our measure to several examples and extended our criteria to spin ensembles. We further showed that any phase fluctuation can be seen as a noisy (weak) measurement of the photon number. Therefore, any single-mode superposition state will only reveal its quantum features under measurements with a phase resolution inversely proportional to the size. Although there is a large variety of criteria for macroscopicity, reflecting the complexity of the concept of macroscopic quantum states, we contend that our way of defining macroscopicity via measurements is not only very natural but also explains why it is so hard to observe quantum features in macro systems. An interesting perspective would be to apply our approach to other detectors that can reasonably be called ‘‘classical,’’ e.g., detection in the phase space using noisy quadrature measurement. It would also be interesting to extend our approach to states with more than two components, $\lambda_0 |0\rangle_A |A\rangle_B + \lambda_1 |1\rangle_A |D\rangle_B + \lambda_2 |2\rangle_A |S\rangle_B + \dots$, etc. There the guessing probability could be replaced with the information obtained by the detector in a single shot. Such a measure would apply to continuous variable entangled states, which typically have large Schmidt numbers. Also, it will be interesting to compare our measure to those defined previously.

ACKNOWLEDGMENTS

We thank B. Sanguinetti and W. Dür for stimulating discussions. This work was supported by Swiss SNSF project CR23I2 127118.

APPENDIX: MEASUREMENTS WITHOUT PHASE REFERENCE AND PASSIVE MEASUREMENT

When phase information is disregarded any element M_k from a POVM becomes a mixture of projectors on energy eigenstates $M_k = \sum_n p_n^{(k)} |E_n\rangle \langle E_n|$ (it is a measurement of the energy with a random choice of n). A passive device (not actively prepared), by definition, starts in a stationary state, i.e., a mixture of energy eigenstates $\rho_0 = \sum_n p_n |E_n\rangle \langle E_n|$ of the Hamiltonian of the measurement device H_M (the Hamiltonian of the system is H_S). The outcome state $|a\rangle$ of the measurement device also has to be stationary in time, as it carries the record of the measurement result, so it is also an eigenstate of H_M . The fact that the total energy is conserved during the evolution U is expressed by the commutation relation $[U, H_S + H_M] = 0$ (or $[U, H_S] = [H_M, U]$). Finally, the probability of an outcome $|a\rangle$ of the measurement device is given by $\text{tr}_{S\&M} \rho_0 \otimes \rho_S U^\dagger |a\rangle \langle a| \otimes \mathbb{1}_S U = \text{tr}_S \rho_S (\text{tr}_M \rho_0 \otimes \mathbb{1}_S U^\dagger |a\rangle \langle a| \otimes \mathbb{1}_S U)$ and corresponds to the operator $M_a = \text{tr}_M U \rho_0 \otimes \mathbb{1}_S U^\dagger |a\rangle \langle a| \otimes \mathbb{1}_S$ acting on the system. Using the

three previous observations one gets

$$[M_a, H_S] = \text{tr}_M[H_M, U]\rho_0 \otimes \mathbb{1}_S U^\dagger |a\rangle \langle a| \otimes \mathbb{1}_S + \text{tr}_M U \rho_0 \otimes \mathbb{1}_S [H_M, U^\dagger] |a\rangle \langle a| \otimes \mathbb{1}_S = 0. \quad (\text{A1})$$

Therefore M_a is diagonal in the eigenbasis of the system's energy; in other words, it corresponds to a measurement of energy.

-
- [1] A. J. Leggett, *Prog. Theor. Phys. Suppl.* **69**, 80 (1980).
 [2] W. Dür, C. Simon, and J. I. Cirac, *Phys. Rev. Lett.* **89**, 210402 (2002).
 [3] G. Bjorn and P. Mana, *J. Opt. B* **6**, 429 (2004).
 [4] J. I. Korbakken, K. B. Whaley, J. Dubois, and J. I. Cirac, *Phys. Rev. A* **75**, 042106 (2007).
 [5] F. Marquardt, B. Abel, and J. von Delft, *Phys. Rev. A* **78**, 012109 (2008).
 [6] C. W. Lee and H. Jeong, *Phys. Rev. Lett.* **106**, 220401 (2011).
 [7] F. Fröwis and W. Dür, *New J. Phys.* **14**, 093039 (2012).
 [8] S. Nimmrichter and K. Hornberger, *Phys. Rev. Lett.* **110**, 160403 (2013).
 [9] S. Aaronson, *Quantum Computing Since Democritus* (Cambridge University Press, Cambridge, 2013).
 [10] A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and P. Grangier, *Science* **312**, 83 (2006).
 [11] J. S. Neegaard-Nielsen, B. M. Nielsen, C. Hettich, K. Mølmer, and E. S. Polzik, *Phys. Rev. Lett.* **97**, 083604 (2006).
 [12] K. Wakui, H. Takahashi, A. Furusawa, and M. Sasaki, *Opt. Express* **15**, 3568 (2007).
 [13] P. Sekatski, N. Sangouard, M. Stobinska, F. Bussières, M. Afzelius, and N. Gisin, *Phys. Rev. A* **86**, 060301 (2012).
 [14] N. Bruno, A. Martin, P. Sekatski, N. Sangouard, R. Thew, and N. Gisin, *Nature Phys.* **9**, 545 (2013).
 [15] A. I. Lvovsky, R. Ghobadi, A. Chandra, A. S. Prasad, and C. Simon, [arXiv:1212.3713](https://arxiv.org/abs/1212.3713).