Switching on and off of ultrastrong light-matter interaction: Photon statistics of quantum vacuum radiation

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(Received 15 April 2013; published 17 December 2013)

We study the extracavity quantum vacuum radiation generated by a sudden switch on and off of the vacuum Rabi frequency of a single two-level system coupled to a single cavity mode in the strong and ultrastrong coupling regime. Dissipation is described by including the qubit-resonator coupling in the derivation of the master equation. The output stream of photons and their second-order correlation functions are calculated by exploiting a recently developed input-output formulation of resonators valid for arbitrary light-matter couplings. The obtained output photon correlations provide direct information on the quantum correlations among the virtual excitations of the dressed vacuum.

DOI: 10.1103/PhysRevA.88.063829 PACS number(s): 42.50.Pq, 03.70.+k

I. INTRODUCTION

One of the most surprising predictions of modern quantum theory is that the vacuum of space is not empty but filled with a sea of virtual particles. A direct evidence of the existence of such virtual particles is provided by the dynamical Casimir effect [1]. It predicts that vacuum amplification effects, resulting in the creation of real particles out of vacuum fluctuations, are induced by rapidly modulating the boundary conditions of a quantum field. This phenomenon is closely related to other pair creation mechanisms [2] such as the parametric amplification, the Schwinger process [3], the Unruh effect [4], and the Hawking radiation [5] by which vacuum fluctuations are amplified into real photons. For many years, almost all of these effects could not be experimentally verified due to the extreme conditions under which these dynamical phenomena become appreciable. Specifically, to observe the dynamical Casimir effect, a rapid modulation of the boundary conditions of the electromagnetic field with peak velocities close to the speed of light was required. This fact led to a great variety of proposals [6–9] to create physical systems able to generate the desired amplification effects, for instance using surface acoustic waves, nanomechanical resonators, or modulation of the electrical properties of a cavity. Recently, due to the experimental progress in the development of circuit QED systems [10–13], the dynamical Casimir effect has been observed in superconducting circuits [14,15]. Actually, by modulating the inductance of a superconducting quantum interference device (SQUID) connected to the extremity of a coplanar transmission line, it is effectively possible to change the electrical length of the circuit with a very fast change rate (a substantial fraction of the speed of light) providing the necessary time-dependent boundary conditions for real photons to be emitted. The dynamical Casimir effect has also been demonstrated using a Josephson metamaterial embedded in a microwave cavity [16]. In this case, the modulation of the effective length of the cavity is obtained by flux biasing the metamaterial consisting of an array of SQUIDs, which form the effective signal line of a superconducting coplanar waveguide.

As superconducting circuits based on Josephson junctions can behave like artificial atoms, thanks to the recent technological advances it was also possible to exploit them for the implementation of a wide variety of atomic-physics and quantum-optics experiments on a chip [13]. Because these artificial atoms display very high coupling rates with microwave cavity photons, it is easy to obtain the strong coupling regime even with a single artificial atom embedded in a coplanar waveguide resonator [17,18]. In this regime the atom coherently exchanges energy with one special cavity mode more rapidly than it decays into the continuum of other noncavity modes. This coherent coupling may provide a basis for future applications in quantum information processing or schemes for coherent control [19–23]. Moreover, a very interesting feature of superconducting quantum circuits relies on the possibility to achieve the so-called ultrastrong-coupling regime [24–27], where the strength of the interaction between an atom and one cavity photon becomes comparable to the transition frequency of the atom or the resonance frequency of the cavity mode. Recently this regime has also been achieved with photochromic molecules [28], by using inter-sub-band transitions in semiconductor structures [29–31] or by coupling the cyclotron transition of a high-mobility two-dimensional electron gas to the photonic modes of an array of electronic split-ring resonators [32]. The ultrastrong-coupling regime presents indeed a great variety of phenomena [24,27,33–35] that cannot be observed in the conventional weak- and strong-coupling regimes. In this regime the light-matter interaction can no longer be described by the standard Jaynes-Cummings model (JC). In this model the antiresonant terms in the interaction Hamiltonian, describing the processes in which the atom and the field are excited or deexcited simultaneously, are neglected (rotating wave approximation). Conversely, when the coupling strength of the interaction becomes comparable to the cavity resonance frequency, in order to correctly describe the most important effects arising in the ultrastrong coupling regime, the antiresonant terms of the light-matter coupling must be included [36]. One of the puzzling consequences of this fact is that the ground state of these systems is a squeezed vacuum containing pairs of cavity photons [33,37]. These photon pairs are, however, virtual and cannot escape the cavity [33]. It has recently been shown that a harmonic temporal modulation of the vacuum Rabi frequency of a single two-level system strongly coupled to a single cavity mode leads to the release of these bound photons into extracavity quantum vacuum radiation [33]. In this case, the presence of the antiresonant terms in the Rabi Hamiltonian is essential to describe the extracavity emission. Recently, an experimental design for the ultrafast switch of the coupling between a superconducting flux qubit and a microwave transmission line has been proposed [38]. Other switching mechanisms have been realized and proposed by exploiting cascade quantum systems [30,39–41]. Using these setups, the mutual influence between artificial atoms and the resonator can be completely suppressed or turned on in very short times, much faster than the atom-resonator dynamics.

In the present work, we focus our attention on the particular case in which the interaction between a two-level system (qubit) and a single cavity mode is abruptly switched on and off. Even if any real experiment requires a finite switching time, such assumption can be regarded as a good approximation when the switching times are short as compared to the characteristic times of the system. In both cases we study the temporal evolution of the system observing that a nonadiabatic modulation of the light-matter interaction effectively leads to the emission of extracavity quantum vacuum radiation. Most importantly, we obtain conspicuous information on the ongoing physics by calculating the temporal evolution of the equal-time normalized second-order correlation functions for the output photons. In order to describe a realistic system, our theoretical model explicitly takes into account dissipation induced by the coupling of the system with its environment. It is worth pointing out a substantial difference between the two cases above considered. In the first case the two-level system and the resonator are initially in the ultrastrong-coupling regime with the system prepared in its dressed ground state. Once the interaction is abruptly switched off, the time evolution of the intracavity mean photon number as well as the output photon statistics can be studied by using the standard electric field operators. Analogously, the cavity losses can be correctly described by the standard quantum optics master equation. In the second case, we consider that the qubit and the resonator are initially decoupled with the system prepared in the bare ground state so that, after the abrupt switch on of the interaction, the system instantaneously enters the ultrastrong-coupling regime. From a theoretical point of view, one of the most inconvenient issues arising in this regime is that the usual normal order correlation functions fail to describe the output photon emission rate and photon statistics [33,42]. This problem has been recently solved and the correct input-output relations valid for arbitrary degrees of light-matter interaction have been obtained [43]. In addition, in the ultrastrong-coupling regime the quantum optical master equation fails to describe the dissipative processes of the system. To avoid these annoyances, the qubit-resonator coupling must be included in the treatment of dissipation in the derivation of the master equation [44]. Taking into account these two aspects, we properly calculate the extracavity emission rate as a function of time. We also calculate the dynamics of the equal-time second-order correlation function and of the equal-time cross-correlation function describing the quantum correlations between cavity photons and the qubit.

II. THEORETICAL MODEL

The theoretical description of a two-level system coupled with a single cavity mode is based on the Rabi Hamiltonian that takes the form ($\hbar = 1$)

$$H_{\mathrm{R}} = H_0 + \Omega_{\mathrm{R}}(a + a^\dagger)(\sigma_- + \sigma_+), \tag{1}$$

where $H_0 = \omega_c a^\dagger a + \omega_{eg}\sigma_+\sigma_-$ is the free Hamiltonian of the system with ω_c and ω_{eg} describing the frequency of the cavity mode and the transition frequency of the two level system, respectively; a and $a^\dagger$ are the creation and annihilation operators for the cavity photons, while σ_+ and σ_- represent the raising and lowering operators describing the two level system excitations ($\sigma_+|g\rangle = |e\rangle$; $\sigma_-|e\rangle = |g\rangle$). The term proportional to the coupling strength Ω_{R} describes the interaction between the two-level system and the cavity mode and explicitly contains the fast oscillating, nonrotating terms $\hbar g(a\sigma_- + a^\dagger\sigma_+)$ that are usually neglected in the so-called rotating wave approximation (RWA). This approximation, that leads to the JC Hamiltonian [45]

$$H_{\mathrm{JC}} = \omega_c a^\dagger a + \omega_{eg}\sigma_+\sigma_- + \Omega_{\mathrm{R}}(a\sigma_+ + a^\dagger\sigma_-), \tag{2}$$

while providing a good description of the system in the strong coupling regime when $\Omega_{\mathrm{R}} \ll \omega_c,\omega_{eg}$, becomes inaccurate in the so-called ultrastrong-coupling regime where the coupling strength Ω_{R} is comparable with the resonance frequencies ω_c and ω_{eg}. In contrast with the JC Hamiltonian, which can be easily diagonalized analytically, the Rabi Hamiltonian requires a numerical solution [46]. Actually, the analytical spectrum of H_{R} has recently been found by Braak [47] and it is defined in terms of the power series of a transcendental function. However, this derivation does not provide direct access to the eigenstates of the Rabi Hamiltonian, which are indeed essential to describe the dynamics of the system. In the present work, the spectrum and the eigenstates of H_{R} have been obtained by standard numerical diagonalization in a truncated, finite-dimensional Hilbert space. Specifically, we consider the Hilbert space resulting from the tensor product of the qubit basis $\{|g\rangle,|e\rangle\}$, and the basis constituted by the $N+1$ photonic Fock states up to the N-photon state $|N\rangle$. The truncation number N is chosen in order to ensure that the lowest M energy eigenvalues and corresponding eigenvectors of interest are not affected when increasing N. Figure 1 displays the first $M = 14$ eingenstates of H_{R} as function of the effective coupling strength $\Omega_{\mathrm{R}}/\omega_c$ for the Jaynes-Cummings model (dashed red line) and the Rabi Hamiltonian (solid blue line). These results have been obtained with a truncation number $N = 30$, even if numerical stability is already reached with $N = 20$.

One of the most important features of the ultrastrong light-matter regime, where the antiresonant terms in H_{R} become relevant, is that the dressed ground state $|\tilde{G}\rangle$ can be expressed in terms of the bare states as [44]:

$$|\tilde{G}\rangle = c_{g0}|g0\rangle + c_{e1}|e1\rangle + c_{g2}|g2\rangle + \cdots. \tag{3}$$

This result clearly shows that the ground state is no longer the simple H_{JC} ground state $|g0\rangle$, but contains indeed a finite population of excitations. It has been shown that unless a time-dependent perturbation of the light-matter coupling is applied to the system these excitations have to be considered virtual

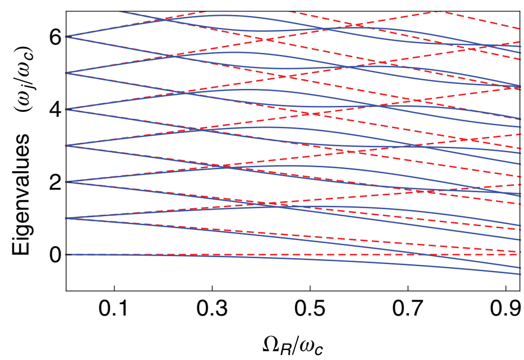

FIG. 1. (Color online) Energy spectrum of H_R as function of the coupling strength in the nondispersive regime ($\Delta = \omega_c - \omega_{eg} = 0$) for the Jaynes-Cummings model (dashed red line) and the Rabi Hamiltonian (solid blue line) containing the nonrotating terms.

as they cannot be experimentally detected [33]. Equation (3) can be simplified in the so-called Bloch-Siegert regime where the coupling strength Ω_R is small compared to $\omega_c + \omega_{eg}$ with the system still being in the ultrastrong-coupling regime. In this case, the expansion of $|\tilde{G}\rangle$ can be truncated up to the $|g2\rangle$ state and the eigenstates of H_R (dressed states) can be obtained analytically by simply applying a unitary transformation to the JC ladders $|n,\pm\rangle = (|g,n\rangle \pm |e,n-1\rangle)/\sqrt{2}$. It is important to note that although the antiresonant terms bring only slight deviations respect to the JC ladders in the Bloch-Siegert regime (Fig. 1) their presence is of fundamental importance in the description of the dynamical Casimir effect following a modulation of the Rabi frequency Ω_R or an abrupt switch on (off) of the light-matter interaction. In particular, if the system is initially prepared in the vacuum state no photons can be excited if the antiresonant terms are neglected while, as we will show in this paper, the presence of these terms leads to the emission of a significant amount of quantum vacuum radiation.

It is clear that in order to describe the dynamics of the quantum system in the ultrastrong-coupling regime, we have properly to consider its interaction with the environment through the appropriate master equation. Moreover, we have to take in account that the standard quantum optical correlation functions fail in correctly describing the extracavity photon emission rate as well as the output photon correlations. In the following subsections we will give a brief review of the correct input-output relations and correlation functions that apply to an arbitrary degree of light-matter interaction, followed by the appropriate treatment of dissipation in the ultrastrong-coupling regime.

A. Photodetection

One of the most inconvenient issues arising in the ultrastrong-coupling regime is that the usual normal order correlation functions fail to describe the output photon emission rate and photon statistics. A clear evidence of this fact is that the standard input-output relations would for example predict, for a vacuum input, a finite output photon flux proportional to the average number of cavity photons [33,43,44], i.e., $\langle A^-_{\rm out}(t)A^+_{\rm out}(t)\rangle \propto \langle a^\dagger(t)a(t)\rangle$, where $A^+_{\rm out}(t)$ and $A^-_{\rm out}(t)$ are, respectively, the positive and negative components of the output field operator. An incautious application of these standard relations for a system in its ground state, which now contains a finite number of photons due to the counter-rotating terms in the Rabi Hamiltonian, would therefore predict an unphysical stream of output photons so that:

$$\langle\tilde{G}|a^\dagger a|\tilde{G}\rangle \neq 0. \tag{4}$$

A solution to this problem has been recently proposed by Ridolfo *et al.* [43]. By expressing the cavity-electric field operator X in the atom-cavity dressed basis it is possible to derive the correct output photon emission rate and correlation functions. In both cases, once the cavity-electric field operator has been expressed in the dressed basis, it has to be separated in its positive and negative frequency components, X^+ and X^-; expanding the X operator in terms of the energy eigenstates $|j\rangle$of H_R one finds the relations:

$$X^+ = \sum_{j,k>j} X_{jk}|j\rangle\langle k|; \quad X^- = (X^+)^\dagger, \tag{5}$$

where $X_{jk} \equiv \langle j|a^\dagger + a|k\rangle$. Two aspects of these results are noteworthy: first of all, we note that in the ultrastrong-coupling regime, one correctly obtains $X^+|\tilde{G}\rangle = 0$ for the system in its ground state $|\tilde{G}\rangle$ in contrast to $a|\tilde{G}\rangle \neq 0$. Moreover, one finds that the positive frequency component of X, according to its actual dynamics, is not simply proportional to the photon annihilation operator a. As a consequence, for arbitrary degrees of light-matter interaction, the steady-state intracavity photon number can be calculated as:

$$N_c \equiv \langle X^- X^+\rangle. \tag{6}$$

According to the input-output theory for resonators, the output field operator can be related through a boundary condition to the intracavity field and the input field operators [48]. In order to be specific, we consider the case of a resonator coupled to a semi-infinite transmission line [48]. We point out that while the resonator is ultrastrongly coupled with the qubit, its interaction with the reservoir (e.g., the transmission line) is weak. By the Markovian approximation, the input-output relation can be written as

$$A_{\rm out}(t) = A_{\rm in}(t) - \sqrt{\frac{\hbar\gamma_c}{2\omega_c v}}X(t), \tag{7}$$

where

$$A_{\rm out} = A^+_{\rm out} + A^-_{\rm out}, \tag{8}$$

with

$$A^+_{\rm out}(t) = \frac{1}{2}\int_0^\infty d\omega\sqrt{\frac{\hbar}{\pi\omega_c v}}a(\omega,t_1)e^{-i\omega(t-t_1)}, \tag{9}$$

and $A^-_{\rm out} = [A^+_{\rm out}]^\dagger$. Here, v is the phase velocity of the transmission line, the operators $a(\omega,t_1)$ are the output bosonic annihilation operators associated to the external (transmission line) continuous modes and $t_1 > t$ can be assumed in the remote future. The rate γ_c represents the cavity damping due to the interaction of the single cavity mode with the continuum of the external modes. An analogous expression can be written for $A^+_{\rm in}(t)$, just replacing in Eq. (9) t_1 with $t_0 < t$ in the remote past. By taking the Fourier transform of Eq. (7) and summing up only the components at positive frequency, we obtain,

$$A_{\mathrm{out}}^{+} = A_{\mathrm{in}}^{+} - \sqrt{\frac{\hbar\gamma_{\mathrm{c}}}{2\omega_{\mathrm{c}} v}} X^{+}. \tag{10}$$

The extracavity emission rate $\langle A_{\mathrm{out}}^{-} A_{\mathrm{out}}^{+}\rangle$, for the case of an input in the vacuum state, can be readily obtained by applying the input-output relation (10),

$$\frac{2\omega_{\mathrm{c}} v}{\hbar}\langle A_{\mathrm{out}}^{-} A_{\mathrm{out}}^{+}\rangle = \gamma_{\mathrm{c}}\langle X^{-} X^{+}\rangle \tag{11}$$

In the same way, the quantum mechanical nth order coherence function, for an input field in the ground state, can now be calculated as:

$$g^{(n)}(\tau = 0) \equiv \frac{\langle [X^{-}(t)]^{n}[X^{+}(t)]^{n}\rangle}{\langle X^{-}(t)X^{+}(t)\rangle^{n}}. \tag{12}$$

It is important to remark that generally speaking the photodetection rate is not strictly proportional to the photon number operator. For example, in optics Eq. (6) should be rewritten in terms of the electric-field operators, while in circuit-QED it should be rewritten in terms of voltage operator. When the output comes from a single-mode cavity, if anharmonicity is not too strong, the resulting detection rate will differ from $\langle a_{\mathrm{out}}^{\dagger} a_{\mathrm{out}}\rangle$ only by a proportionality factor. In the present case, only minor corrections would result from replacing the photon operators by the positive and negative frequency electric-field operators or by corresponding voltage operators and for the sake of simplicity we will just use photon operators. However, correlation functions based on different electromagnetic operators depending on the specific measurement setup can be calculated without additional problems within the present approach.

B. Dissipation

In order to correctly describe the quantum dynamics of the system, dissipation induced by its coupling to the environment needs to be considered. Assuming that the cavity and the two-level system are weakly coupled with two different baths of harmonic oscillators, the standard approach where the coupling Ω_{R} is ignored while obtaining the dissipative part of the master equation would lead, for a $T = 0$ reservoir, to the result:

$$\dot{\rho}(t) = -i[H_{\mathrm{S}},\rho(t)] + \gamma_{\mathrm{a}}\mathcal{D}[a]\rho(t) + \gamma_{\mathrm{x}}\mathcal{D}[\sigma_{-}]\rho(t), \tag{13}$$

where $\mathcal{D}[O]\rho = \frac{1}{2}(2O\rho O^{\dagger} - \rho O^{\dagger}O - O^{\dagger}O\rho)$, γ_{a} is the rate of the cavity losses, and γ_{x} is the atom relaxation rate. H_{S} is, in general, the system Hamiltonian in the absence of the thermal baths. In the present case, H_{S} will be equal to H_{R} or H_0 according to the specific investigated dynamics. This standard quantum optical master equation can be safely used to describe the dynamics of the system in the weak- and strong-coupling regimes, in which the ratio $\Omega_{\mathrm{R}}/\omega_{\mathrm{c}}$ is small enough for the RWA approximation to be still valid [45]. In the ultrastrong-coupling regime however, owing to the high ratio $\Omega_{\mathrm{R}}/\omega_{\mathrm{c}}$, Eq. (13) fails to correctly describe the dissipation processes and leads to unphysical results as well. In particular, it predicts that even at $T = 0$, relaxation would drive the system out of its ground state $|\tilde{G}\rangle$ generating photons in excess to those already present. The proper procedure that solves such issues consists in taking into account the atom-cavity coupling when deriving the master equation after expressing the Hamiltonian of the system in a basis formed by the eigenstates $|j\rangle$ of H_{R}, whose energy eigenvalues are denoted by $\hbar\omega_j$, i.e., $H_{\mathrm{R}}|j\rangle = \hbar\omega_j|j\rangle$. If we choose the labeling of the states $|j\rangle$ such that $\omega_k > \omega_j$ for $k > j$ and treat the dissipation baths in the Born-Markov approximation [49] (this can always be done when the coupling of the system and the environment is weak enough so that the evolution of the system is not significantly affected by the coupling itself), we thus arrive at the master equation in the dressed picture [44] for a $T = 0$ reservoir:

$$\dot{\rho}(t) = -i[H_{\mathrm{S}},\rho(t)] + \mathcal{L}_{\mathrm{a}}\rho(t) + \mathcal{L}_{\mathrm{x}}\rho(t). \tag{14}$$

Here $\mathcal{L}_{\mathrm{a}}$ and $\mathcal{L}_{\mathrm{x}}$ are the Liouvillian superoperators correctly describing the losses of the system where $\mathcal{L}_{\mathrm{c}}\rho(t) = \sum_{j,k>j}\Gamma_{\mathrm{c}}^{jk}\mathcal{D}[|j\rangle\langle k|]\rho(t)$ for $c = a,\sigma_{-}$. In the limit $\Omega_{\mathrm{R}} \to 0$, standard dissipators are recovered. The relaxation rates $\Gamma_c^{jk} = 2\pi d_c(\Delta_{kj})\alpha_c^2(\Delta_{kj})|C_{jk}^{c}|^2$ depend on the density of states of the baths $d_{\mathrm{c}}(\Delta_{kj})$ and the system-bath coupling strength $\alpha_{\mathrm{c}}(\Delta_{kj})$ at the respective transition frequency $\Delta_{kj} \equiv \omega_k - \omega_j$ as well as on the transition coefficients $C_{jk} = \langle j|c + c^{\dagger}|k\rangle (c = a,\sigma_{-})$. In the Born-Markov approximation the density of states of the baths can be considered a slowly varying function of the transition frequencies Δ_{kj}, so that we can safely assume it to be constant as well as the coupling strength α_{c}. Equation (14) can be easily extended to take into account $T \neq 0$ reservoirs [44].

In the following sections we will present results based on the numerical solution of the master equation. In order to solve numerically Eqs. (13) and (14), we first diagonalize H_{S} in a finite dimensional Hilbert space generated by the tensor product of the qubit basis $\{|g\rangle,|e\rangle\}$ and the basis constituted by the $N + 1$ photonic Fock states up to the N-photon state $|N\rangle$. We then calculate the matrix elements of Eq. (13) or (14) in the basis of the eigenstates of H_{S} (notice that when $H_{\mathrm{S}} = H_0$ the initial basis is already diagonal). In this way the operatorial Eqs. (13) and (14) are converted into linear systems of first-order differential equations, which can be solved numerically with standard techniques. The obtained time-dependent matrix elements of the density operator are then employed to calculate the desired expectation values. The truncation order N is chosen in order to ensure that the expectation values of interest are not affected when increasing N.

III. SWITCH OFF OF THE INTERACTION

We start our analysis considering the case in which the two-level system and the resonator are initially in the ultrastrong-coupling regime so that the Hamiltonian describing the whole system is trivially given by Eq. (1). We consider the system to be prepared in its dressed ground state $|\tilde{G}\rangle$, which contains a certain amount of virtual light-matter excitations. It is clear that in the absence of any time-dependent perturbations, virtual photons cannot be released from the cavity and no emission of quantum vacuum radiation can be observed. At the time $t = t_0$ we completely and instantly switch off the light-matter

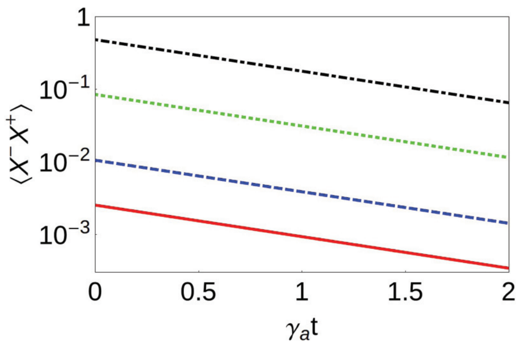

FIG. 2. (Color online) Logarithmic plot of time evolution of the intracavity mean photon number $\langle X^- X^+ \rangle$ calculated for different coupling strengths $\Omega_R/\omega_c = 0.1$ (solid red line), 0.2 (dashed blue line), 0.5 (dotted green line), 0.9 (dot-dashed black line). Larger signals correspond to larger couplings. The typical exponential decay is due to the cavity losses described by the standard master equation.

interaction [50] so that $\Omega_R(t)$ can be taken of the form:

$$\Omega_R(t) = \Omega_0 \Theta(t_0 - t), \tag{15}$$

where $\Theta(t_0 - t)$ is the Heaviside step function. From an experimental point of view, an ideal abrupt switch is very difficult to realize. However, when the switching time is much shorter that the inverse of the resonance frequencies of the system, the Heaviside step function represents a very good approximation. Recently, novel experimental circuit QED designs where the qubit is ultrastrongly coupled to a transmission line (open or not) have been introduced. In all these experimental setups the coupling can be tuned in strength and kind in a versatile way by simply applying an external flux bias [38]. In addition, the ultrastrong coupling can be switched on and off very rapidly by employing three-level cascade quantum systems where the uppermost two levels are those coupled with the resonator. The interaction can be switched by sending ultrafast π pulses resonant with the lowest energy transition [30,39]. Since this change is nonadiabatic, the system will be, at the time t_0^+, still in the state $|\tilde{G}\rangle$ that now contains a finite and real photon population. Precisely, we can observe that before the switch off $\langle \tilde{G}|X^- X^+|\tilde{G}\rangle = 0$ while, in the absence of interaction, $\langle \tilde{G}|X^- X^+|\tilde{G}\rangle = \langle \tilde{G}|a^\dagger a|\tilde{G}\rangle \neq 0$. Figure 2 shows the mean intracavity photon number as a function of time for different values of Ω_R/ω_c, considering that the interaction is switched off at $t_0 = 0$ (for $t < t_0$ we trivially find that there is no emission of quantum vacuum radiation). It can be observed that the photon emission immediately following the switch off strongly depends on the coupling strength and also presents the typical exponential decay owing to the dissipation processes described by Eq. (13). Looking at the energy spectrum of H_R and H_{JC} (see Fig. 1), it results that the energy of the dressed vacuum is lower than that of the JC ground state $|g0\rangle$. Hence one may ask: Where does the energy released after the switch off in the form of quantum vacuum radiation come from? The explanation relies on the Ritz theorem. Just after the switch off, the initial state $|\tilde{G}\rangle$ remains unchanged [50] while $H_R \rightarrow H_0$ so that $|\tilde{G}\rangle$ is no longer the ground state of the system Hamiltonian. Hence, according to the Ritz theorem $\langle \tilde{G}|H_0|\tilde{G}\rangle > \langle g0|H_0|g0\rangle \equiv E_g$, and the system can relax to the actual ground state by emitting the exceeding energy as quantum vacuum radiation. In the Schrödinger picture the time evolution of the ground state $|\tilde{G}\rangle$ in the bare states basis for $t \rightarrow t_0^+$ is given by:

$$\begin{aligned} |\tilde{G}\rangle_t = {} & c_{g0} e^{-i\omega_{g0}t}|g0\rangle + c_{e1} e^{-i\omega_{e1}t}|e1\rangle + c_{g2} e^{-i\omega_{g2}t}|g2\rangle \\ & + c_{e3} e^{-i\omega_{e3}t}|e3\rangle + c_{g4} e^{-i\omega_{g4}t}|g4\rangle + \cdots, \end{aligned} \tag{16}$$

where the contribution of the coefficients $c_{(g,e)n}$ to the expansion of $|\tilde{G}\rangle$ vary for different values of the ratio Ω_R/ω_c. Specifically, the probability that in the ultrastrong-coupling regime the ground state $|\tilde{G}\rangle$ contains an n-photon state is given by:

$$P_n = \mathrm{Tr}[\rho(t_0)|n\rangle\langle n|], \tag{17}$$

where $\rho(t_0)$ is the density matrix of the system immediately after the interaction has been switched off. Figure 3 shows the probability P_n for different values of Ω_R/ω_c. It can be observed that in the Bloch-Siegert regime the main contributions to the ground state come from the bare states $|g0\rangle, |e1\rangle, |g2\rangle$, while the contributions from the states containing a higher photon number are very small and can be neglected. However, when the value of the coupling strength becomes larger, these terms become significant giving rise to the observed increasing value of $\langle X^- X^+ \rangle$ in this regime.

It is interesting to notice that, although the main contribution to the ground state generally comes from the $|g0\rangle$ state (with values of P_0 close to unity) for a wide range of values of Ω_R/ω_c, the system presents indeed a different behavior for very high values of the coupling ($\Omega_R/\omega_c \geqslant 1.3$). In this case, the probability P_n exhibits a peak for $n = 2$ (for the parity of the system to be conserved, the only contribution to P_2 essentially comes from the $|g2\rangle$ state) so that the main contribution to the expansion of $|\tilde{G}\rangle$ is no longer given by the bare ground state $|g0\rangle$. Once the interaction is abruptly switched off, the probability distribution P_n of virtual photons in the ground state $|\tilde{G}\rangle$ converts into physical photons that can be detected. Recently, it has been shown that the quantum state of an electromagnetic superconducting resonator consisting in a superposition of states with different number of photons can be fully characterized [51,52], and the probability distribution P_n can be experimentally obtained. In particular, this can be done using a quantum circuit in which a phase qubit is capacitively coupled to a superconducting coplanar waveguide resonator; once the qubit and the resonator are brought into resonance for a certain time, the subsequent measurement of the probability P_e for the qubit to be in the excited state allows to determine the n-photon probabilities P_n. Since this measurement gives only information about the probabilities P_n so that the relative phases of the Fock states are lost, Wigner tomography is needed to univocally characterize the quantum state [51].

It is important to consider that a larger signal can be obtained in a system consisting in a chain of N qubits inductively coupled to a transmission line resonator described by the Hamiltonian:

$$H_R = \omega_c a^\dagger a + \sum_{i=1}^{N} \omega_{eg}^{(i)} \sigma_i^+ \sigma_i^- + (a + a^\dagger) \sum_{i=1}^{N} \Omega_R^{(i)} (\sigma_i^- + \sigma_i^+). \tag{18}$$

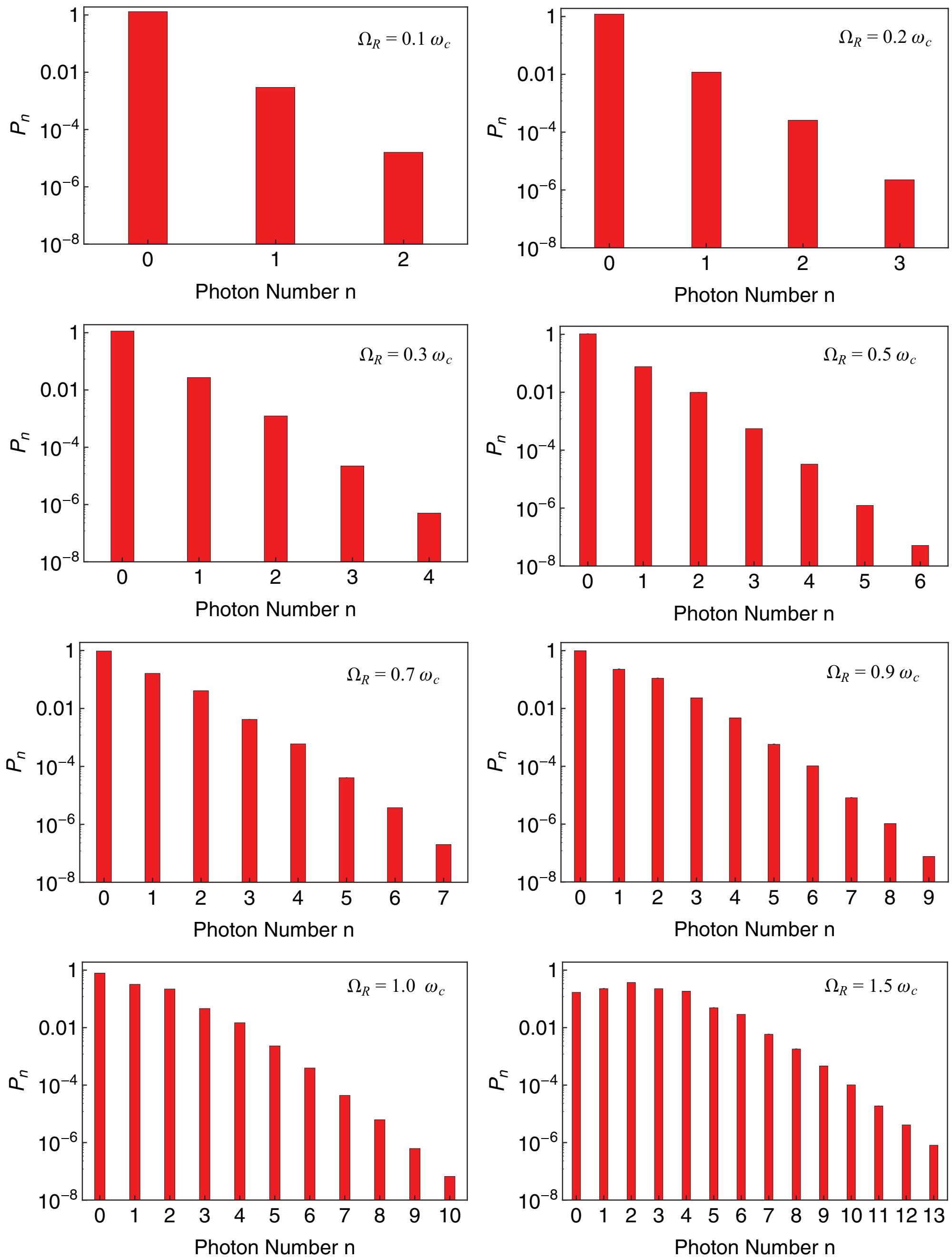

FIG. 3. (Color online) Probability distribution P_n for the ground state $|\tilde{G}\rangle$ to contain an n-photon state calculated for different coupling strengths Ω_R/ω_c. It can be observed that the main contribution to the dressed ground state generally comes from the $|g0\rangle$ state (with values of P_0 close to unity) for a wide range of values of Ω_R/ω_c. For very high values of the coupling strength, however, the system presents indeed a different behavior and the main contribution to the ground state $|\tilde{G}\rangle$ comes from the two-photon state $|g2\rangle$. Once the interaction is switched off, the probability distribution P_n of virtual photons in the ground state $|\tilde{G}\rangle$ converts to physical photons that can be detected.

In the particular case $N = 3$ with $\Omega_R^{(1)} = \Omega_R^{(2)} = \Omega_R^{(3)} = 0.4\omega_c$, we obtain for $t = t_0^+$ an intracavity mean photon number $\langle X^- X^+\rangle_{t=t_0^+} \cong 0.7$.

Another interesting feature of the switch-off process is the possibility to study the output photon correlations considering that the field operators X^- and X^+ reduce to the standard photon operators $a^\dagger, a$ in the limit $\Omega_R \to 0$. The time-dependent zero-delay normalized second-order correlation function is then given by:

$$g^{(2)}(t,t) = \frac{\langle a^{\dagger 2}(t) a^2(t)\rangle}{\langle a^\dagger(t) a(t)\rangle^2}. \tag{19}$$

Considering that in the expansion(16) only the coefficients up to the $|g4\rangle$ state are relevant, the analytic expression of $g^{(2)}(t,t)$ for $0.1 < \Omega_R/\omega_c < 0.3$ can be easily calculated and leads to

TABLE I. Calculated values of $g^{(2)}(t,t)$ for different values of the effective coupling rate Ω_R/ω_c.

Ω_R/ω_c	0.1	0.2	0.3	0.5	0.7	0.9	1	1.5
$g^{(2)}(t,t)$	3.943	3.778	3.524	2.854	0.529	0.724	1.102	1.420

the result:

$$g^{(2)}(t,t) = \frac{2(|c_{g2}|^2 + 3|c_{e3}|^2 + 6|c_{g4}|^2)}{(|c_{e1}|^2 + 2|c_{g2}|^2 + 3|c_{e3}|^2 + 4|c_{g4}|^2)^2}. \quad (20)$$

Equation (20) can be generalized to all values of the coupling strength by the expression:

$$g^{(2)}(t,t) = \frac{\sum_{n=1}^{\infty} 2n[(2n-1)|c_{g,2n}|^2 + (2n+1)|c_{e,2n+1}|^2]}{\left[\sum_{n=1}^{\infty}(2n-1)|c_{e,2n-1}|^2 + 2n|c_{g,2n}|^2\right]^2}. \quad (21)$$

In general, it can be observed that the second-order correlation function is time independent for all values of the effective coupling constant and this can be explained considering that after the switch off of the interaction the free photons inside the cavity exhibit the typical dissipative dynamics according to the standard master equation (13). In this case, it can be shown that the time evolution of the numerator of the second-order correlation function is given by:

$$\frac{d}{dt}\langle a^{\dagger 2}(t)a^2(t)\rangle = -2\gamma_a\langle a^{\dagger 2}(t)a^2(t)\rangle, \quad (22)$$

while the decay of the cavity mode is described by the standard relation:

$$\frac{d}{dt}\langle a^{\dagger}(t)a(t)\rangle = -\gamma_a\langle a^{\dagger}(t)a(t)\rangle, \quad (23)$$

so that $\langle a^{\dagger 2}(t)a^2(t)\rangle$ and $\langle a^{\dagger}(t)a(t)\rangle^2$ present the same decay rate giving rise to the observed constant value of $g^{(2)}(t,t)$. Numerical values of $g^{(2)}(t,t)$ for different ratios Ω_R/ω_c are reported in Table I. Although, as shown in Fig. 3, there is a non-negligible probability for photons to be emitted in pairs, a strong bunching effect is not observed due to the presence of the one-photon state $|e1\rangle$. The resulting values of $g^{(2)}(t,t)$ shown in Table I are mainly a consequence of the competition between one and two-photon events.

IV. SWITCH ON OF THE INTERACTION

In this section we consider the case in which the two-level atom and the resonator are initially decoupled ($\Omega_R = 0$) with the system prepared in the $|g0\rangle$ state. The interaction is switched on in a nonadiabatic way at $t = t_0$ so that the system immediately enters the so-called ultrastrong-coupling regime. The Hamiltonian of the system can then be expressed by the following compact expression:

$$H = \omega_c a^{\dagger}a + \omega_{eg}\sigma_+\sigma_- + \Omega_R(t)(a + a^{\dagger})(\sigma_- + \sigma_+), \quad (24)$$

where in this case $\Omega_R(t) = \Omega_0\Theta(t - t_0)$ and $\Theta(t - t_0)$ is the Heaviside step function. In fact, according to Eq. (24), the system is described by the free Hamiltonian $H_0 = \omega_c a^{\dagger}a + \omega_{eg}\sigma_+\sigma_-$ for $t < t_0$, while for $t > t_0$ the Rabi Hamiltonian H_R is correctly recovered. Once the interaction is switched on, the time evolution of $|g0\rangle$ will then be determined by the Rabi Hamiltonian, which explicitly contains the antiresonant terms. After expressing the $|g0\rangle$ state in the dressed basis ($|0\rangle \equiv |\tilde{G}\rangle, |1\rangle, |2\rangle, \ldots$) its time evolution will be given by ($\hbar = 1$)

$$|g0\rangle_t = c_0 e^{-i\omega_0 t}|\tilde{G}\rangle + \sum_{n=1}^{\infty} c_n e^{-i\omega_n t}|n\rangle. \quad (25)$$

It is important to observe that unlike the previous case, in which the photodetection after the switch off of the interaction could be studied using the standard photon operators, now the cavity-electric field operator X has to be expressed in the dressed basis and subsequently separated in its positive and negative frequency components X^+ and X^- given by Eq. (5). In particular, the expressions for X^+ and X^- contains the matrix elements X_{jk} of the operator $X \equiv a^{\dagger} + a$. These elements, which can be evaluated analytically in the Bloch-Siegert regime [44], require numerical evaluation for higher values of the coupling strength Ω_R/ω_c. Numerical results for the temporal evolution of the mean cavity number of physical photons $\langle X^- X^+\rangle$ are displayed in Fig. 4. For $t < t_0$ no emission is observed, the system being in its ground state $|g0\rangle$, while for $t > t_0$, once the interaction has been switched on, we observe a finite emission of physical photons corresponding to an actual output photon rate $\gamma_a\langle X^- X^+\rangle$, which can be experimentally detected. The interesting feature in the mean number

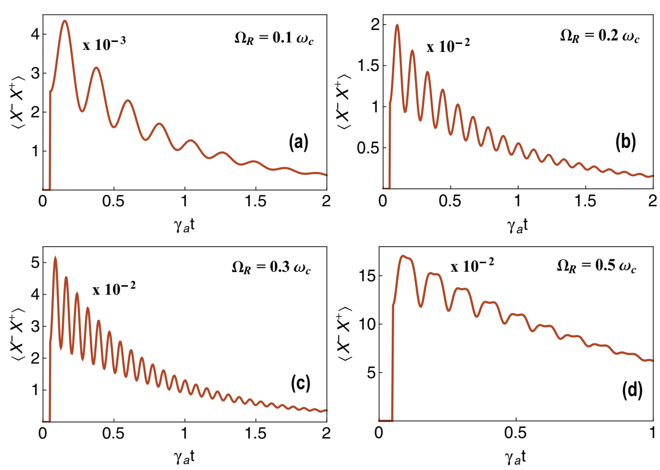

FIG. 4. (Color online) Temporal evolution of the mean cavity number of physical photons $\langle X^- X^+\rangle$ for different values of the coupling strength $\Omega_R/\omega_c = 0.1$ (a), 0.2 (b), 0.3 (c), 0.5 (d). Parameters are: $\omega_c = \omega_{eg}$; $\gamma_a = \gamma_x = 0.01\omega_c$. For $t < t_0$ no emission is observed, the system being in its ground state $|g0\rangle$. Once the interaction has been switched on, we observe a finite emission of physical photons corresponding to an actual output photon rate $\gamma_a\langle X^- X^+\rangle$, which can be experimentally detected. In the Bloch-Siegert regime (a) and (b), dynamics of released cavity photons will originate from the superposition of states $|3\rangle$ and $|4\rangle$, which will give rise to typical Rabi oscillations. As shown in Figs. 4(c) and 4(d), for higher values of the coupling strength ($\Omega_R \geqslant 0.3\omega_c$) a more complex dynamics can be observed. As contributions from higher states transitions become relevant, $\langle X^- X^+\rangle_t$ does not exhibit an exact periodicity at the transition frequency $\Delta\omega_{43} \equiv \omega_4 - \omega_3$ but is now a sum of oscillatory terms with different frequencies. These contributions become more relevant as the coupling strength increases.

of cavity photons is the presence of dumped oscillations [according with the dissipation processes correctly described by the master equation (14)] with a frequency corresponding (for $0 < \Omega_R \leqslant 0.25\omega_c$) to the transition frequency between the dressed states $|3\rangle$ and $|4\rangle$.

In order to better understand the numerical results, we present a semianalytical analysis in the range $0 < \Omega_R \leqslant 0.25\omega_c$. Once the interaction has been switched on, the time evolution of $|g0\rangle$ is given by Eq. (25). As the Rabi Hamiltonian preserves the parity of the total excitation number (light plus matter), the (even) ground state $|g0\rangle$ does not contain contributions from the odd states $|1\rangle$ and $|2\rangle$. In this situation, $|g0\rangle$ can then be approximated by a superposition of the states $|\tilde{G}\rangle, |3\rangle, |4\rangle$

$$|g0\rangle_t = c_0 e^{-i\omega_0 t}|\tilde{G}\rangle + c_3 e^{-i\omega_3 t}|3\rangle + c_4 e^{-i\omega_4 t}|4\rangle. \quad (26)$$

As the dressed ground state $|\tilde{G}\rangle$ does not contribute to the intracavity mean photon number $\langle X^- X^+\rangle$, the dynamics of released cavity photons will then originate from the superposition of states $|3\rangle$ and $|4\rangle$, which will give rise to typical Rabi oscillations. Contributions from higher even-parity states are very small and can be neglected. Using Eqs. (5) and (26), we obtain a semianalytical expression for $\langle X^- X^+\rangle_t$ valid for $0 < \Omega_R \leqslant 0.25\omega_c$

$$\langle X^- X^+\rangle_t = |c_3|^2\left(A_{31}^2 + A_{32}^2\right) + |c_4|^2\left(A_{41}^2 + A_{42}^2\right) + 2c_3 c_4 (A_{31}A_{41} + A_{32}A_{42})\cos(\omega_4 - \omega_3)t. \quad (27)$$

This equation has been obtained by neglecting cavity losses. This result is in very good agreement with the corresponding numerical results as those shown in Figs. 4(a) and 4(b), but calculated in the absence of damping. As shown in Figs. 4(c) and 4(d), for higher values of the coupling strength ($\Omega_R \geqslant 0.3\omega_c$) a more complex dynamics can be observed. In particular, it can be observed that $\langle X^- X^+\rangle_t$ does not exhibit an exact periodicity at the transition frequency $\Delta\omega_{43} \equiv \omega_4 - \omega_3$ but, due to contributions from higher states transitions, this function is now a sum of oscillatory terms with different frequencies. These contributions become more relevant as the coupling strength increases.

Conspicuous additional information on the ongoing physics can be obtained by studying the statistics of the emitted photons. In particular, we focus our attention on the correlation function $\mathcal{G}(t,t) = g^{(2)}(t,t)\langle X^- X^+\rangle$, where $g^{(2)}(t,t) = \langle X^- X^- X^+ X^+\rangle / \langle X^- X^+\rangle^2$ is the equal-time normalized second-order correlation function. When photons are emitted in pairs, we expect that $\mathcal{G}(t,t) \approx 1$. It is interesting to compare the $\mathcal{G}(t,t)$ with the cross-correlation function

$$\mathcal{G}_{\text{cross}}^{\text{std}}(t,t) = \frac{\langle a^\dagger \sigma_+ \sigma_- a\rangle}{[\langle a^\dagger a\rangle\langle \sigma_+ \sigma_-\rangle]^{1/2}}, \quad (28)$$

describing the quantum correlations between the cavity photons and the emitted photons from the qubit into noncavity modes proportional to the qubit exited-state population. In circuit QED systems this emission can be detected by coupling the qubit with an additional microwave antenna [51]. Equation (28) is valid only for relatively small values of the coupling strength. The correct expression for $\mathcal{G}_{\text{cross}}(t,t)$ in the ultrastrong-coupling regime requires indeed the expansion of the atomic operators σ_+ and σ_- as well as the cavity-electric

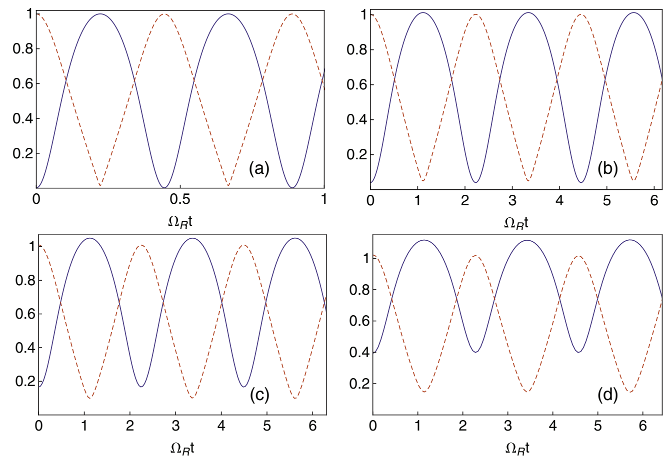

FIG. 5. (Color online) Temporal evolution of the correlation function $\mathcal{G}(t,t)$ (solid blue line) and the cross-correlation function $\mathcal{G}_{\text{cross}}(t,t)$ (dashed red line) for different values of the coupling strength $\Omega_R/\omega_c = 0.01$ (a), 0.1 (b), 0.2 (c), 0.3 (d). For weak values of the coupling strength (a) it can be observed that for $t \to t_0^+$ the cross-correlation function $G_{\text{cross}}(t,t) = 1$ while the corresponding value of $\mathcal{G}(t,t)$ is zero. Due to the perturbation $V \cong \hbar g(a\sigma_- + a^\dagger\sigma_+)$ the system, initially prepared in the bare ground state, undergoes the transition $|g0\rangle \to V|g0\rangle = |e1\rangle$. In this case, then, no pairs of photon can be released ($\mathcal{G}(t,t) = 0$) and the value $G_{\text{cross}}(t,t) = 1$ is the signature of the presence of $|e1\rangle$. As the system undergoes Rabi oscillations between the two states $|e1\rangle$ and $|g2\rangle$ we observe that when the system is exactly in the $|g2\rangle$ state $\mathcal{G}(t,t) = 1$ while the cross correlation goes to zero. Figures 5(b)–5(d) show that in the ultrastrong-coupling regime both correlation functions present a different behavior. As the contributions from the upper dressed states become more relevant, we observe that $\mathcal{G}(t,t) \neq 0$ even for $t \to t_0^+$ and reaches values larger than one. For the same reason the minima of the cross-correlation function never goes to zero due to the non-negligible contribution of the states $|7\rangle, |8\rangle$.

field operators $a^\dagger$ and a in the dressed basis. This leads to the following expression for $\mathcal{G}_{\text{cross}}(t,t)$ valid for arbitrary degrees of light-matter interaction

$$\mathcal{G}_{\text{cross}}(t,t) = \frac{\langle X^- C^- C^+ X^+\rangle}{[\langle X^- X^+\rangle\langle C^- C^+\rangle]^{1/2}}, \quad (29)$$

where $C^+ = \sum_{j,k>j} C_{jk}|j\rangle\langle k|; C^- = (C^+)^\dagger$ with $C_{jk} \equiv \langle j|\sigma_- + \sigma_+|k\rangle$. In the limit $\Omega_R \to 0$ Eq. (28) is recovered. Figure 5 displays the temporal evolution of the $\mathcal{G}(t,t)$ (solid blue line) and $\mathcal{G}_{\text{cross}}(t,t)$ (dashed red line) for different values of the coupling strength once the interaction has been switched on. Calculations have been performed at zero detuning by neglecting cavity and qubit losses. For weak values of the coupling strength [Fig. 5(a)] it can be observed that for $t \to t_0^+$ the cross-correlation function $G_{\text{cross}}(t,t) = 1$ while the corresponding value of $\mathcal{G}(t,t)$ is zero. This fact can be explained regarding the nonresonant terms in the Rabi Hamiltonian H_R as a perturbation. According to the first-order perturbation theory, the potential V can be then approximated by $V \cong \hbar g(a\sigma_- + a^\dagger\sigma_+)$. Once this perturbation is abruptly switched on at $t = t_0$ the system, initially prepared in the ground state $|g0\rangle$, undergoes the transition $|g0\rangle \to V|g0\rangle = |e1\rangle$. Thus, no pairs of photon can be released at $t = t_0^+$ so that

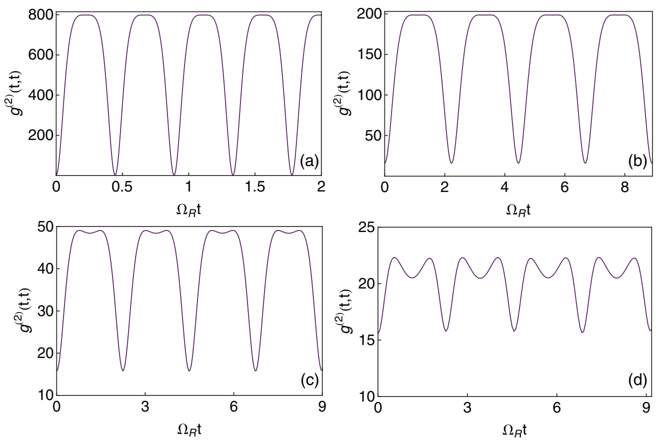

FIG. 6. (Color online) Temporal evolution of the equal-time normalized second-order correlation function $g^{(2)}(t,t)$ for different values of the coupling strength $\Omega_R/\omega_c = 0.01$ (a), 0.1 (b), 0.2 (c), 0.3 (d). When the ratio Ω_R/ω_c is small (a), $g^{(2)}(t,t)$ is exactly zero immediately after the switch on confirming the fact that the system is essentially in a superposition of the states $|g0\rangle,|e1\rangle$ so that no pairs of photons can be released at $t = 0$. A maximum is observed when the system is in the $|g2\rangle$ state where the second-order correlation function certifies a highly super-Poissonian statistics. Figures 6(b)–6(d) show that in the ultrastrong-coupling regime, where the contribution from the upper dressed states becomes more relevant, there is a finite possibility for photons to be emitted in pairs even immediately after the switch on of the interaction. The more complex dynamics observed for increasing value of the coupling strength originates from contributions from higher states transitions.

$\mathcal{G}(t_0^+,t_0^+) = 0$ and the value $G_{\text{cross}}(t_0^+,t_0^+) = 1$ is the signature of the excitation of the state $|e1\rangle$. After the switch-on of the interaction, the system undergoes Rabi oscillations between the two states $|e1\rangle$ and $|g2\rangle$ or, equivalently, between the dressed states $|3\rangle$ and $|4\rangle$ which, due to the relative small value of Ω_R/ω_c, can be identified with the doubly excited states $|2,\pm\rangle = (|g,2\rangle \pm |e,1\rangle)/\sqrt{2}$ of the JC ladder. When the system is exactly in the $|g2\rangle$ state, we observe that $\mathcal{G}(t,t) = 1$ while the cross correlation goes to zero. As shown in Figs. 5(b)–5(d), in the ultrastrong-coupling regime both correlation functions present a different behavior. In particular, as the contributions from the upper dressed states become more relevant when the coupling strength increases, we observe that $\mathcal{G}(t,t) \neq 0$ even for $t \to t_0^+$ and reaches values larger than one. For the same reason the minima of the cross-correlation function never goes to zero due to the non-negligible contribution in Eq. (25) of the states $|7\rangle,|8\rangle$.

Finally, numerical results for the equal-time normalized second-order correlation function $g^{(2)}(t,t)$ are displayed in Fig. 6. As expected, when the ratio Ω_R/ω_c is small, $g^{(2)}(t,t)$ is exactly zero immediately after the switch on [Fig. 6(a)] confirming the fact that the system is essentially in a superposition of the states $|g0\rangle,|e1\rangle$ so that no pairs of photons can be released at $t = t_0$. Very high values of $g^{(2)}(t,t)$ are observed at the maximum when the system is in the $|g2\rangle$ state and the second-order correlation function certifies a highly super-Poissonian statistics, evidencing that cavity photons are released in pairs. Figures 6(b)–6(d) show that in the ultrastrong-coupling regime the contribution from the upper dressed states leads to a finite value of $g^{(2)}(t,t)$ even at the minima. It can also be observed that the maxima of the second-order correlation function reach values lower than those displayed in Fig. 6(a) for the case of relatively small values of Ω_R/ω_c. Moreover, for $\Omega_R = 0.2\omega_c$ and $\Omega_R = 0.3\omega_c$, numerical results displayed in Figs. 6(c) and 6(d) show a more complex structure for $g^{(2)}(t,t)$ due to contributions from higher states transitions.

V. CONCLUSION

We have analyzed the dynamics of the extracavity quantum vacuum radiation that is generated by a sudden switch on and off of the interaction rate of a single two-level system coupled to a single cavity mode. The calculations include dissipation effects, which are described by taking into account the qubit-resonator coupling in the derivation of the master equation. In this way it is possible to obtain the Lindblad master equations valid for arbitrary light-matter interaction [44]. The output stream of photons and their second-order correlation functions have been calculated by exploiting a recently developed input-output formulation of resonators valid for arbitrary light-matter couplings [43]. They are crucial to provide an accurate picture of the switch-on problem.

We gathered conspicuous information on the ongoing physics by calculating the temporal evolution of the equal-time normalized second-order correlation functions for the output photons. The on and off switches have been discussed separately. Once the interaction is abruptly switched off, the probability distribution P_n of virtual photons in the ground state $|\tilde{G}\rangle$ converts into physical photons that can be detected. The reconstruction of such probability distribution provides direct information on the virtual multiphoton states in the dressed vacuum. The abrupt switch on at moderate and intermediate qubit-resonator coupling rates gives rise to an initial state with one photon and the qubit into the excited state that evolves under the typical strong-coupling dynamics as confirmed by the calculation of the photonic second-order correlation function and the cross-correlation (photon-qubit) function.

It would be interesting to apply the present analysis to extended media [53] such as planar optical cavities [28] and arrays of microcavities [54] in the ultrastrong-coupling regime.

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