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Light sea fermions in electron-proton and muon-proton interactions

U. D. Jentschura

Department of Physics, Missouri University of Science and Technology, Rolla, Missouri, MO65409-0640, USA
and MTA–DE Particle Physics Research Group, P.O. Box 51, H-4001 Debrecen, Hungary

The proton radius conundrum [Pohl *et al.*, Nature **466**, 213 (2010) and Antognini *et al.*, Science **339**, 417 (2013)] highlights the need to revisit any conceivable sources of electron-muon nonuniversality in lepton-proton interactions within the standard model. Superficially, a number of perturbative processes could appear to lead to such a nonuniversality. One of these is a coupling of the scattered electron into an electronic vacuum-polarization loop as opposed to a muonic one in the photon exchange of two valence quarks, which is present only for electron projectiles as opposed to muon projectiles. However, we show that this effect actually is part of the radiative correction to the proton's polarizability contribution to the Lamb shift, equivalent to a radiative correction to double scattering. We conclude that any conceivable genuine nonuniversality must be connected with a nonperturbative feature of the proton's structure, e.g., with the possible presence of light sea fermions as constituent components of the proton. If we assume an average of roughly 0.7×10^{-7} light sea positrons per valence quark, then we can show that virtual electron-positron annihilation processes lead to an extra term in the electron-proton versus muon-proton interaction, which has the right sign and magnitude to explain the proton radius discrepancy.

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I. INTRODUCTION

The electromagnetic aspects of proton and neutron structures are somewhat elusive. It is well known that the mass difference between a proton and neutron is responsible for the stability of the universe (the hydrogen atom would otherwise be unstable against β decay into an electron-positron pair, a neutrino and a neutron). There have been attempts to explain the mass difference on the basis of the electromagnetic interaction among the constituent quarks [1–4]. *A priori*, one would think that the electrostatic interaction among the constituent quarks leads to an inversion of the mass hierarchy of proton versus neutron. Namely, the Coulomb interaction among valence quarks actually lowers the energy of the neutron as compared to the proton, as a naive counting argument shows. A hadron with valence quarks *uud* has interquark electromagnetic interactions proportional to the fractional charge numbers, $\frac{2}{3} \times (-\frac{1}{3}) + \frac{2}{3} \times (-\frac{1}{3}) + \frac{2}{3} \times \frac{2}{3} = 0$; whereas for the neutron with valence quarks *udd*, we have $\frac{2}{3} \times (-\frac{1}{3}) + \frac{2}{3} \times (-\frac{1}{3}) + (-\frac{1}{3}) \times (-\frac{1}{3}) = -\frac{1}{3}$. The latter expression, being negative, would suggest that the neutron is lighter than the proton if the mass difference were of electromagnetic origin and due to Coulomb exchange.

However, the radiative correction is not constrained to have any particular sign and warrants further investigation especially because the electromagnetic wave functions of the valence quarks bound in an MIT bag model [1] have a rather peculiar structure [4] and might give rise to significant radiative effects. The conclusion reached in Refs. [1–4] is that the electromagnetic self-energy of the quarks remains positive for all masses considered. Thus, the quantum electrodynamic (QED) radiative energy shift cannot explain the mass difference between a proton and neutron, where a negative self-energy would otherwise be required in view of the larger fractional charge of the up quarks as compared to the down quarks.

Still, the investigations [1–4] as well as the proton radius conundrum [5,6] highlight the need for a closer look at the internal structure of the proton if one is interested in its own "internal" electromagnetic interactions, as well as the interactions of the proton with the "outside world." If the interaction of the bound or scattered lepton with the proton is nonuniversal, then it is conceivable that the proton radius depends on the projectile particle. However, one can show that a number of perturbative higher-order effects which could appear to lead to such a nonuniversality of electron-proton versus muon-proton interactions are in fact absorbed into correction terms of known physical origin.

Let us consider electromagnetic interactions among the constituent particles of the proton, for example, a higher-order effect generated by a coupling of the scattered projectile (electron or muon) into a vacuum-polarization loop which in turn is inserted into a photon exchanged between two valence quarks. We here show that because Feynman propagators take care of all possible time orderings of virtual particle creation and annihilation processes, this effect actually constitutes a radiative correction to double scattering and is absorbed, in Lamb shift calculations, into the radiative correction to the proton's polarizability contribution to the Lamb shift. A quantitative parametric estimate for the order of magnitude of the effect is provided.

The second process is more speculative and conjectures the presence of light sea fermions as a nonperturbative physical property of the hadron, an admixture to the genuine particle content of the proton. We here show that the conceivable presence of these fermions would give rise to a Dirac-δ potential, in view of a virtual annihilation channel, with the right sign to explain the muonic hydrogen puzzle [5,6]. These two mechanisms are described in the following Secs. II and III, respectively. Units with $\hbar = c = \epsilon_0$ are used throughout this paper unless stated otherwise.

II. LEPTON-PROTON SCATTERING AND INTERNAL STRUCTURE OF THE PROTON

Let us first recall the relevant conventions. The leading-order process for the scattering of leptons (e.g., electrons or muons) off of a proton is depicted in Fig. 1(a). A virtual
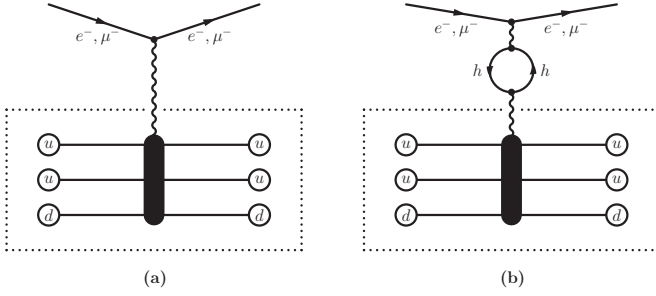
FIG. 1. Diagram (a) is the standard scattering process involving an incoming electron or muon, without radiative corrections. The hadronic vacuum-polarization contribution from diagram (b) is taken into account consistently in Lamb shift calculations and subtracted from scattering data as a radiative correction. The photon is emitted "collectively" by the quarks inside the proton. The proton as a component particle is encircled by dotted lines.

photon is emitted collectively by the proton, describing the electromagnetic interaction of lepton and hadron. Any insertions of virtual particles into the exchange photon are absorbed into the F_1 and F_2 form factors (or Sachs G_E and G_M form factors) of the proton, while the proton radius is defined as the slope of the Sachs G_E form factor, with all those terms [and radiative corrections, see Fig. 1(b)] subtracted from G_E. These would otherwise be ascribed to a point proton with the properties of a structureless spin-1/2 Dirac particle.

Let us briefly review the status (see also Ref. [7]). The proton interaction vertex is changed, in view of the nontrivial form factors, as

$$\gamma^\mu \to \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_p} F_2(q^2), \tag{1}$$

where F_1 and F_2 are the Dirac and Pauli form factors of the proton, respectively. The electric and magnetic G_E and G_M Sachs form factors are defined in terms of the F_1 and F_2 as follows:

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4(m_p c)^2} F_2(q^2), \tag{2a}$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2), \quad F_2(0) = \varkappa_p. \tag{2b}$$

One canonically separates the Sachs G_E form factor into a QED contribution $G_E^{\mathrm{QED}}(q^2)$, which captures all aspects of the point-particle QED nature of the proton, and a part $\overline{G}_E(q^2)$ which is due to the proton's internal structure [7],

$$G_E(q^2) = \overline{G}_E(q^2) + G_E^{\mathrm{QED}}(q^2). \tag{3}$$

The definition of the proton charge radius then reads as

$$\langle r^2 \rangle_p = 6\hbar^2 \frac{\partial \overline{G}_E(q^2)}{\partial q^2}\bigg|_{q^2=0}, \tag{4}$$

i.e., it measures the internal structure of the proton, after all QED contributions ("radiative corrections") have been subtracted (and that includes the infrared divergent slope of the QED one-loop contribution to the F_1 form factor). By definition, the subtraction of the QED contribution $G_E^{\mathrm{QED}}(q^2)$ also includes all virtual loop insertions into the exchange photon line that would otherwise affect the proton-lepton

interaction for a point proton. One of these is the hadronic vacuum-polarization loop in Fig. 1(b).

The proton radius is measured in the low-energy region, where one can use a dipole fit to $\overline{G}_E(q^2)$ to good approximation. Let us briefly recall the fundamental differences of low-energy elastic scattering, which mainly determines the proton's size, and high-energy deep inelastic scattering (DIS), which is relevant for momentum transfers $q^2 \gg m_p^2$. For an incoming lepton four-momentum ℓ_1 and an incoming proton momentum p (outgoing lepton momentum ℓ_2 and exchange photon four-momentum q), the Bjorken scaling law [8] is as follows. After the subtraction of radiative correction, one writes the deep inelastic cross section as

$$\sigma_{\mathrm{DIS}} = \sigma_0 \left[\frac{G_E^2(Q^2, \nu) + \tau\, G_M^2(Q^2, \nu)}{1 + \tau} \right.$$
$$\left. + 2\tau\, G_M^2(Q^2, \nu) \tan^2\left(\frac{\theta}{2}\right) \right]$$
$$\equiv \sigma_0 \left[W_2(Q^2, \nu) + 2\, W_1(Q^2, \nu) \tan^2\left(\frac{\theta}{2}\right) \right], \tag{5}$$

where σ_0 is the Mott scattering cross section, $\nu = q \cdot p/m_p$ is the energy loss of the lepton, $Q^2 \equiv -q^2$, $\tau = Q^2(4m_p^2)^{-1}$, and θ is the scattering angle of the lepton, i.e., the angle subtended by the spatial components of ℓ_1 and ℓ_2. The form factors $G_E(Q^2, \nu)$ and $G_M(Q^2, \nu)$ describe inelastic scattering (with energy loss), and the elastic counterparts are recovered in the limit $\nu \to 0$. Bjorken [8] observed that if the scattering in the high-energy region were to come from point-like constituents inside the proton, then the structure functions W_1 and W_2 should be consistent with scattering from asymptotically free constituents ("partons" or "quarks"),

$$\lim_{\substack{Q^2 \to \infty \\ Q^2/\nu \times \mathrm{const.}}} \nu W_2(Q^2, \nu) = m_p\, F_2(x), \tag{6a}$$

$$\lim_{\substack{Q^2 \to \infty \\ Q^2/\nu \times \mathrm{const.}}} W_1(Q^2, \nu) = F_1(x), \quad x \equiv \frac{Q^2}{2m_p \nu}, \tag{6b}$$

where the F_1 and F_2 are now structure functions instead of form factors; their argument is the Bjorken x variable. The Bjorken scaling was confirmed by the famous SLAC-MIT experiments [9–12]. However, in dealing with low-energy scattering processes and contributions to the Lamb shift, the proton's form factor can be approximated very well using a dipole fit [see, e.g., the discussion surrounding Eq. (74) of Ref. [13]].

Let us consider the diagram in Fig. 2(a), which could superficially be assumed to induce a nonuniversality of the electron-proton versus muon-proton interaction, on the level of higher-order corrections. Namely, the coupling of the projectile electron into the *electronic* vacuum-polarization loop of an electromagnetic interquark interaction is available only for an incoming electron (as opposed to an incoming muon). The Feynman propagators for the fermions and the leptons in Fig. 2(a) contain all possible time orderings, including scattering "backward in time" which leads to the vacuum-polarization loop. The diagram in Fig. 2(b) thus describes the same physical process as Fig. 2(a). Furthermore, it is necessary to remember what the "scattering off of a definite
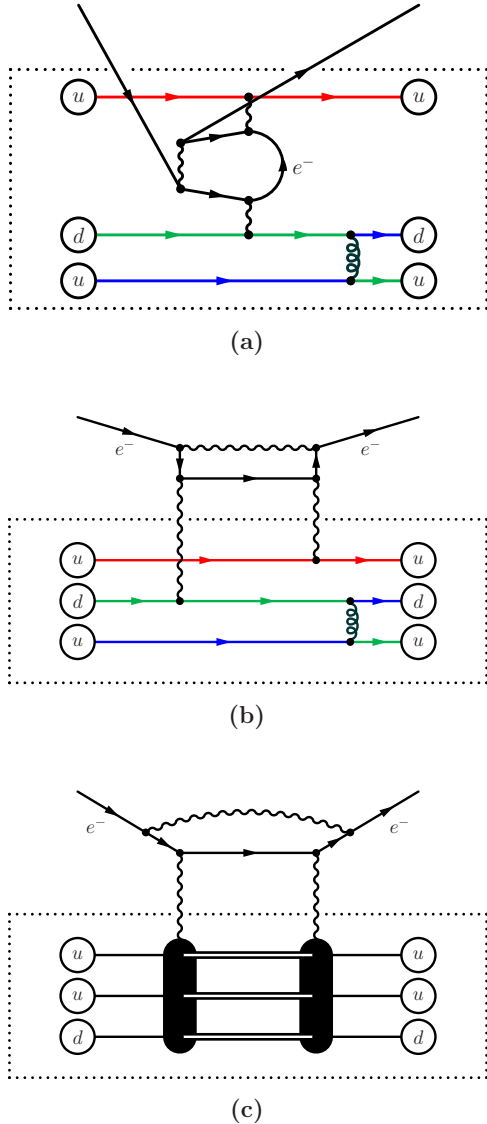
(a)

(b)

(c)

FIG. 2. (Color online) Diagram (a) describes the coupling of an incoming electron into the vacuum-polarization loop of an electromagnetic interquark interaction inside the proton. Feynman propagators describe all possible time orderings of the particle creation and annihilation processes and diagram (b) thus describes the same effect as (a). The gluon interaction in (b) is representative. The inelastic contribution to (b), with an "excited" quark in the virtual state, is identified in (c) as a radiative correction to the proton's polarizability contribution to the Lamb shift.

quark inside the proton" [see Fig. 2(a)] physically means in the characteristic momentum range of an electron or muon bound to the proton. It implies that the proton's internal state changes in between the two interactions of the virtual electron with the virtual photons emitted by the proton [see Fig. 2(c)]. Thus, the process in Fig. 2(a) can finally be identified as a radiative correction to the proton's polarizability contribution to the Lamb shift, as depicted in Fig. 2(c).

The contribution of double-scattering processes is canonically subtracted in the analysis of scattering experiments. In the context of bound states, the leading contribution from

two-photon exchange (without radiative corrections) gives rise to the so-called third Zemach moment term which is proportional to a convoluted charge distribution of the proton [7,14,15]. The elastic correction to the third Zemach moment due to the proton structure can be taken into account by inserting proton form factors into the two-photon-exchange forward-scattering amplitude [Eqs. (70)–(75) of Ref. [13]], and the inelastic correction to the third Zemach moment (due to an excited state of the proton in between the photon exchanges, also known as the proton polarizability correction) is numerically too small to explain the proton radius puzzle [13,16–18].

Finally, let us provide a parametric estimate for the contribution of the diagram in Fig. 2(c), based on the analogy with the two-Coulomb-vertex correction to the self-energy, as given by the calculation reported in Ref. [19]. The induced effective potential for the diagram in Fig. 2(c), by scaling arguments, can be estimated to be proportional to

$$H_{\rm vp} \propto \frac{\left[\alpha_{\rm QED}\left(m_{\rm eff}^2/m_e^2\right)\right]^3}{m_{\rm eff}^2} \delta^3(r) \,, \tag{7}$$

Here, $\alpha_{\rm QED}$ is the running QED coupling which is approximately equal to $1/137.036$ at zero momentum transfer, $\delta^3(r)$ is the three-dimensional Dirac-δ function, and $m_{\rm eff}$ is an effective mass or momentum scale entering the loop in Fig. 2(c). The latter can be estimated as follows. Let $\lambda \sim r_p$ be a characteristic de Broglie wavelength of the quarks inside the nucleus. Then, the associated momentum scale is $p \sim h/r_p$ where h is Planck's unit of action and the corresponding energy scale is obtained as $E \sim pc \sim 1.32 m_p$, which in turn is commensurate with the excitation energy of the proton into its first resonance, the Δ resonance at 1232 MeV. It is easy to check, based on the running of the QED coupling, that the effective coupling at the scale of the proton's momentum differs from the value of $\alpha_{\rm QED}$ at zero momentum transfer by less than 5%. The leading finite-size Hamiltonian is given as

$$H_{\rm fs} = \frac{2\pi\alpha_{\rm QED}}{3m_e^2} \left[m_e^2 \langle r_p^2 \rangle\right] \delta^3(r) \,. \tag{8}$$

The ratio is given as

$$R \propto \frac{\langle H_{\rm vp} \rangle}{\langle H_{\rm fs} \rangle} \sim \alpha_{\rm QED}^2 \frac{m_e^2}{m_{\rm eff}^2} \frac{1}{m_e^2 \langle r_p^2 \rangle} \sim 2.2 \times 10^{-6} \,, \tag{9}$$

where we take into account that $m_e^2 \langle r_p^2 \rangle \sim (1/386)^2$, and $m_e/m_{\rm eff} \sim m_e/m_p \sim 5.4 \times 10^{-4}$. The ratio R is too small to make a significant contribution to a solution of the proton radius puzzle. It is interesting to note that the simple-minded parametric estimate described above, with one radiative factor $\alpha_{\rm QED}$ from the self-energy loop excluded, gives the right order of magnitude for the leading-proton polarizability contribution evaluated in Ref. [15].

III. LIGHT SEA FERMIONS

Let us consider the possible presence of light sea fermions as nonperturbative contributions to the proton's structure, inspired by a possible importance of virtual electron-positron pairs in the lepton nonuniversality in interactions with protons. We consider a thought experiment: If we switched off the
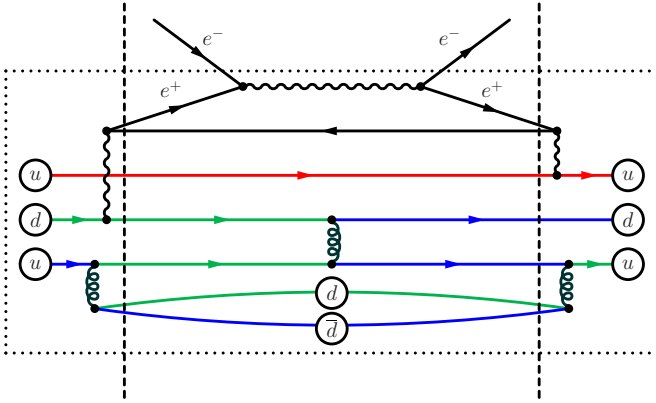
FIG. 3. (Color online) Typical Feynman diagram illustrating the virtual annihilation of a bound electron with a "light sea lepton" (positron) inside the proton. The up (u) and down (d) quarks, which carry noninteger charge numbers, interact electromagnetically. The dashed lines mark the formation of the asymptotic state of the proton in the distant past or future, with its valence and sea quark contents, and with a light sea lepton that annihilates with the bound electron. The given Feynman diagram is not included if the proton is treated perturbatively as a spin-1/2 particle with charge e. An exemplary QCD interaction via a blue-(anti)green gluon also is indicated in the figure.

electroweak interactions of quarks inside the proton, the resulting "proton" would of course be neutral but otherwise rather comparable in its mass and in its nuclear properties to a real proton with some nonperturbative quantum chromodynamic (QCD) "wave function." Now, if we include back the electroweak interactions of quarks, virtual photons and electron-positron pairs would backreact on the previous "wave function" leading to a reshaping and the actual "proton wave function" which now additionally contains photons and the electron-positron pairs. Due to the highly nonlinear nonperturbative nature of QCD, this reshaping can be much larger than the electromagnetic perturbation itself, and therefore there is room for the conceivable presence of electron-positron pairs inside the proton, which cannot be accounted for by perturbative QED considerations alone (see Fig. 3). This density (probability) of electron-positron pairs, because of the inherently nonperturbative nature of QCD, is difficult if not impossible to quantitatively estimate, but its presence is not excluded by any known experiments. In fact, a significant photon content of the proton is well confirmed in the so-called deep inelastic Compton scattering (DICS, see Refs. [20–25] and Fig. 3).

If the proton contains these electron-positron pairs, which are not accounted for in any perturbative higher-order QED term, then the interaction between the proton and electron is given by both photon exchange and annihilation diagrams. In natural units, the photon annihilation diagram in the case of positronium leads to the effective interaction [26]

$$\delta H = \frac{\pi \alpha_{\text{QED}}}{2m_e^2} \left(3 + \vec{\sigma}_+ \cdot \vec{\sigma}_- \right) \delta^3(r) . \quad (10)$$

This Hamiltonian gives a nonvanishing interaction of the bound electron and the light sea positron if their spins add

up to unity. Assuming that the electron-positron pairs within the proton are not polarized, we can replace $\vec{\sigma}_+ \cdot \vec{\sigma}_- \to 0$ after averaging over the polarizations of the sea leptons. For atomic (electronic) hydrogen, the additional interaction of the electron with the proton due to the annihilation channel therefore is of the form

$$H_{\text{ann}} = \epsilon_p \frac{3\pi \alpha_{\text{QED}}}{2m_e^2} \delta^3(r) , \quad (11)$$

where ϵ_p measures the amount of electron-positron pairs within the proton. For muonic hydrogen, the effect is expected to vanish because the dominant contribution to the sea leptons comes from the lightest leptons, namely, electron-positron pairs and thus the annihilation channel is not available. By comparison, the finite nuclear size effect is given by Eq. (8). For an S state, the ratio of the corresponding energy shifts is

$$\frac{\langle H_{\text{ann}} \rangle}{\langle H_{\text{fs}} \rangle} = \frac{9}{4} \frac{\epsilon_p}{m_e^2 \langle r_p^2 \rangle} \overset{!}{=} \frac{0.88^2 - 0.84^2}{0.88^2} = 0.089 . \quad (12)$$

The equality marked with the exclamation mark has to hold if we are to explain the discrepancy of the electronic and muonic hydrogen values of the proton charge radius, which are roughly 0.88 fm and 0.84 fm, respectively [5,6,16,27]. The parameter ϵ_p thus can be as low as

$$\epsilon_p = 2.1 \times 10^{-7} , \quad (13)$$

and still explain the effect the different proton radii obtained from electronic and muonic hydrogen. Per valence quark, one thus has a fraction of $\epsilon_p/3 = 0.7 \times 10^{-7}$ sea fermion pairs. The interaction due to the annihilation channel has the right sign; it enhances the nuclear size effect for electronic as opposed to muonic hydrogen and thus makes the proton appear larger for electronic systems.

IV. CONCLUSIONS

Let us include some historical remarks. In the 1970s, transition frequencies in muonic transitions were found to be in disagreement with theory [28]. After a sign error in the calculation of the two-loop vacuum-polarization correction [29] was eliminated [30,31] and a standard γ-ray spectrometer used in the experiments was recalibrated [32], other experiments later found agreement of theory and experiment in muonic systems (e.g., Refs. [33,34]). Nuclear radii of some carbon, nitrogen, and oxygen isotopes [35] were determined by analyzing muonic transitions, and the resulting radii were found to be in agreement with electron scattering radii to better than 5%. Later, the radius of ^{12}C was updated in Refs. [36,37], finally "converging" to a value of $r_C = 2.478(9)$ fm, in good agreement with the value from muonic x-ray studies. Muonic atom and ion spectroscopy is meanwhile regarded as an established tool for the determination of nuclear radii [38].

However, the light sea fermions discussed in Sec. III are distributed only inside the nucleons as opposed to the entire nucleus which is held together by meson exchange, because the local electromagnetic field is strongest inside the protons and neutrons. The size of the proton could be determined by the light sea fermions, among other things, but the size of a composed large nucleus is determined

by the nuclear meson-mediated force. Expressed differently, the muonic hydrogen experiments probe *one and only one* nucleon, whereas other experiments involving, say, a muon bound to a ^{12}C nucleus, probe the charge radius of an ensemble of nucleons, which is mainly determined by the arrangement of the nucleons inside the ^{12}C nucleus. Thus, the effect proposed in Sec. III should become smaller for larger nuclei. A rough estimate would entail the observation that the carbon nucleus is three times bigger than the proton. So, naively and classically, three nucleons fit into the diameter of the ^{12}C nucleus. If every one of these has its effective diameter reduced by 5%, then the overall radius is reduced by only 1.7%. While the tables of Ref. [37] suggest agreement to (slightly) better than 1% for independent experimental determinations of the ^{12}C charge radius, it is noteworthy that this agreement was achieved only after earlier discrepancies had been resolved.

Very interestingly, a possible electron-muon nonuniversality has been seen in a scattering experiment [39] some forty years ago, comparing the scattering of electrons versus muons off of protons, and was cautiously ascribed by the authors of Ref. [39] to an incorrect normalization of the scattering data. The observed 8% difference in the cross sections observed in Ref. [39] translates into a 4% difference in the proton radius, with the same sign and magnitude as that observed in muonic spectroscopy experiments [5,6]. If one ignores the possibility of an incorrect normalization of the data in Ref. [39], then the proton, "seen" with the "eyes" of a muon, appears to be 4%–5% smaller than its "appearance" when seen through the eyes of an electron [5,6,39]. The experiment [39] urgently needs to be confronted with an independent investigation.

According to Refs. [17,40] and other theoretical works which came to the same conclusion, it is hard to imagine any perturbative process (direct exchange of a virtual "subversive" particle, or insertion of a "subversive" particle into the exchange photon line) which could explain the muonic hydrogen discrepancy without seriously questioning the validity of other measurements and corresponding theory, such as the muon *g*-factor measurement. Furthermore, any other perturbative insertions into the photon-proton vertex, conceivably involving internal constituents of the proton, are absorbed in the definition of the proton radius and thus could not explain the discrepancy (see Sec. II). Without questioning the validity of the Maxwell equations or QED, and without introducing any additional virtual particles, it is perhaps permissible to speculate that a nonperturbative mechanism such as the one proposed in Sec. III might be a feasible candidate in the case of further experimental confirmations of the proton radius discrepancy [5,6,39] between electronic as opposed to muonic bound systems.

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