Controllable optical bistability based on photons and phonons in a two-mode optomechanical system

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We explore theoretically the bistable behavior of the intracavity photon number in a two-mode cavity optomechanical system, where two cavity modes are coupled to a common mechanical resonator. When the two cavity modes are driven by two pump laser beams, respectively, we find that the optical bistability can be controlled by tuning the power and the frequency of the pump beams. The common interaction with a mechanical mode enables one to control the bistable behavior in one cavity by adjusting the pump laser beam driving another cavity. We also show theoretically that both branches of optical bistability at photon numbers below unity can exist in this two-mode optomechanical system. This phenomenon can find potential applications in a controllable optical switch.

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I. INTRODUCTION

Cavity optomechanics [1–3] explores the interaction between a mechanical resonator and photons in a driven electromagnetic cavity via radiation pressure force. In the past decade, remarkable progress has been made in this emerging field, including quantum ground state cooling of the nanomechanical resonators [4,5], optomechanically induced transparency (OMIT) [6–8], and coherent photon-phonon conversion [9–11] and quantum-state transfer [12–15]. The force exerted by a single photon on a macroscopic mechanical resonator is typically weak and intrinsically nonlinear. Experiments to date have focused on the regime of strong optical driving, where the optomechanical coupling can by enhanced by a factor $\sqrt{n}$, where n is the mean photon number in the cavity [16,17]. But such enhancement comes at the cost of losing the nonlinear character of the photon-photon interaction. Recently, several theoretical studies of the single-photon strong-coupling regime, where the single-photon optomechanical coupling rate g exceeds the cavity decay rate κ, have been reported in single-mode [18–24] or two-mode optomechanical systems [25–27]. In this regime, the inherently nonlinear optomechanical interaction is significant at the level of single photons and phonons.

Among all the nonlinear phenomena in a cavity optomechanical system, optical bistability is one of the focuses of research interest. Recently, the bistable behavior of the mean intracavity photon number in optomechanical systems with a Bose-Einstein condensate (BEC) [28–30], ultracold atoms [31–33], and a quantum well [34] has been extensively studied. The photon number in the optical cavity with a BEC or ultracold atoms to allow bistable behavior is usually low and even below unity due to the collective atomic motion. However, in the generic optomechanical system consisting of an empty optical cavity with one movable end mirror, optical bistability typically occurs at high photon numbers. In this Brief Report, we theoretically investigate the bistable behavior of the intracavity photon number in a two-mode optomechanical system in the simultaneous presence of two strong pump laser beams and a weak probe laser beam. This Brief Report is organized as follows. Section II gives the theoretical model and method. Results and discussion are given in Sec. III. A summary is presented in Sec. IV.

II. MODEL AND THEORY

The system under consideration is shown in Fig. 1. Two optical cavity modes are coupled to a common mechanical mode via an interaction Hamiltonian $H_I = \sum_{k=1,2}\hbar g_k a_k^\dagger a_k(b^\dagger + b)$, where a_k and b are the annihilation operators of the cavity and mechanical mode, respectively, and g_k is the single-photon coupling rate between the mechanical mode and the kth cavity mode. Physically, g_k represents the frequency shift of cavity mode k due to the zero-point motion of the mechanical resonator. The left cavity is driven by a strong pump laser beam E_L of frequency ω_L and a weak probe laser beam E_p of frequency ω_p simultaneously, and the right cavity is only driven by a strong pump laser beam E_R of frequency ω_R. In a rotating frame at the pump frequency ω_L and ω_R, the Hamiltonian of the two-mode optomechanical system reads [11]

$$H = \sum_{k=1,2}\hbar\Delta_k a_k^\dagger a_k + \hbar\omega_m b^\dagger b - \sum_{k=1,2}\hbar g_k a_k^\dagger a_k(b^\dagger + b) + i\hbar\sqrt{\kappa_{e,1}}E_L(a_1^\dagger - a_1) + i\hbar\sqrt{\kappa_{e,2}}E_R(a_2^\dagger - a_2) + i\hbar\sqrt{\kappa_{e,1}}E_p(a_1^\dagger e^{-i\delta t} - a_1 e^{i\delta t}). \quad (1)$$

The first term represents the energy of the two optical cavity modes with resonance frequency $\omega_k (k = 1,2)$, where $\Delta_1 = \omega_1 - \omega_L$ and $\Delta_2 = \omega_2 - \omega_R$ are the corresponding cavity-pump field detunings. The second term gives the energy of the mechanical mode with resonance frequency ω_m and effective mass m. The last three terms describe the interaction between the input fields and the cavity fields, where E_L, E_R, and E_p are related to the power of the applied laser fields by $|E_L| = \sqrt{2P_L\kappa_1/\hbar\omega_L}$, $|E_R| = \sqrt{2P_R\kappa_2/\hbar\omega_R}$, and $|E_p| = \sqrt{2P_p\kappa_1/\hbar\omega_p}$ (κ_k is the linewidth of the kth cavity mode), respectively. Each optical cavity is coupled not only to

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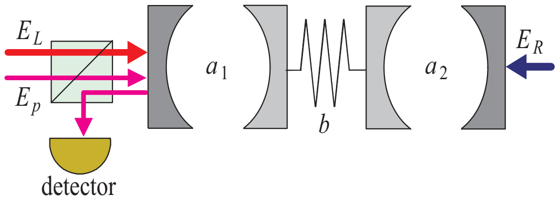

FIG. 1. (Color online) Schematic of a two-mode optomechanical system where two optical cavity modes, a_1 and a_2, are coupled to the same mechanical mode b. The left cavity is driven by a strong pump beam E_L in the simultaneous presence of a weak probe beam E_p, while the right cavity is only driven by a pump beam E_R.

a shared mechanical mode but also to an optical bath at rate $\kappa_{i,k}$ and to an external photonic waveguide at rate $\kappa_{e,k}$. Therefore, the total cavity linewidth $\kappa_k = \kappa_{i,k} + \kappa_{e,k}$. Here, $\delta = \omega_p - \omega_L$ is the detuning between the probe laser beam and the left pump laser beam.

According to the Heisenberg equations of motion and the commutation relation $[a_k, a_k^\dagger] = 1$ and $[b, b^\dagger] = 1$, the temporal evolutions of operators a_1, a_2, and Q [which is defined as $Q = b^\dagger + b$] can be obtained. Introducing the corresponding damping and noise terms for the mechanical and cavity modes, we derive the quantum Langevin equations as follows:

$$\dot{a_1} = -i(\Delta_1 - g_1 Q)a_1 - \kappa_1 a_1 + \sqrt{\kappa_{e,1}}(E_L + E_p e^{-i\delta t}) + \sqrt{2\kappa_1} a_{in,1}, \tag{2}$$

$$\dot{a_2} = -i(\Delta_2 - g_2 Q)a_2 - \kappa_2 a_2 + \sqrt{\kappa_{e,2}} E_R + \sqrt{2\kappa_2} a_{in,2}, \tag{3}$$

$$\ddot{Q} + \gamma_m \dot{Q} + \omega_m^2 Q = 2g_1\omega_m a_1^\dagger a_1 + 2g_2\omega_m a_2^\dagger a_2 + \xi. \tag{4}$$

The cavity modes decay at the rate κ_k $(k = 1,2)$ and are affected by the input vacuum noise operator $a_{in,k}$ with zero mean value, and the mechanical mode is affected by a viscous force with damping rate γ_m and by a Brownian stochastic force with zero mean value ξ [35].

Setting all the time derivatives to zero, we derive the steady-state solution to Eqs. (2)–(4):

$$a_{s,1} = \frac{\sqrt{\kappa_{e,1}} E_L}{\kappa_1 + i\Delta_1'}, \quad a_{s,2} = \frac{\sqrt{\kappa_{e,2}} E_R}{\kappa_2 + i\Delta_2'},$$
$$Q_s = \frac{2}{\omega_m}(g_1|a_{s,1}|^2 + g_2|a_{s,2}|^2), \tag{5}$$

where $\Delta_1' = \Delta_1 - g_1 Q_s$ and $\Delta_2' = \Delta_2 - g_2 Q_s$ are the effective cavity detunings including radiation pressure effects. The stability condition for the coupled system can be derived by means of the Routh-Hurwitz criterion [36], whose general form is somewhat cumbersome. However, in the large cooperativity limit [i.e., $C_k \equiv G_k^2/(\kappa_k\gamma_m) \gg 0$, where $G_k = g_k a_{s,k}$], the explicit expression can be approximated as [37]

$$\tilde{G}^2 > \bar{C}\gamma_m \max\left[\kappa_1 - \kappa_2, \frac{\kappa_2^2 - \kappa_1^2}{2\gamma_m + \kappa_1 + \kappa_2}\right], \tag{6}$$

where $\tilde{G} \equiv \sqrt{G_1^2 - G_2^2}$ and $\bar{C} \equiv (G_1^2 + G_2^2)/[\gamma_m(\kappa_1 + \kappa_2)]$. The mean intracavity photon numbers $n_{pk} = |a_{s,k}|^2$ can be determined by the following coupled equations:

$$n_{p1} = \frac{\kappa_{e,1} E_L^2}{\kappa_1^2 + [\Delta_1 - 2g_1/\omega_m(g_1 n_{p1} + g_2 n_{p2})]^2}, \tag{7}$$

$$n_{p2} = \frac{\kappa_{e,2} E_R^2}{\kappa_2^2 + [\Delta_2 - 2g_2/\omega_m(g_1 n_{p1} + g_2 n_{p2})]^2}. \tag{8}$$

This form of coupled cubic equation is characteristic of the optical multistability [31,32]. It is clearly seen from Eqs. (7) and (8) that intracavity photon numbers n_{p1} and n_{p2} are interconnected, which can be tuned by the power and frequency of the pump laser beams via changing parameters E_L, E_R, Δ_1, and Δ_2. This enables us to control the intracavity photon numbers in a more diverse way. For example, the photon number in the right cavity n_{p2} can be controlled by the right pump beam directly or by the left pump beam indirectly, which will be discussed in the following.

III. NUMERICAL RESULTS AND DISCUSSION

We consider for illustration an experimentally realized two-mode optomechanical system. The parameters used are [11] $\omega_1 = 2\pi \times 205.3$ THz, $\omega_2 = 2\pi \times 194.1$ THz, $\kappa_1 = 2\pi \times 520$ MHz, $\kappa_2 = 1.73$ GHz, $\kappa_{e,1} = 0.2\kappa_1$, $\kappa_{e,2} = 0.42\kappa_2$, $g_1 = 2\pi \times 960$ kHz, $g_2 = 2\pi \times 430$ kHz, $\omega_m = 2\pi \times 4$ GHz, $Q_m = 87 \times 10^3$, where Q_m is the quality factor of the nanomechanical resonator and the damping rate γ_m is given by $\frac{\omega_m}{Q_m}$. Substituting the numerical value of κ_1, κ_2, and γ_m into the stability condition (6), we find that $G_1 > 1.36 G_2$ is required for the system to be stable. The stability condition is always satisfied in the following chosen parameter regime.

The two-mode optomechanical system we consider here enables more controllability in the bistable behavior of the intracavity photon number. Figure 2(a) plots the mean intracavity photon number in the left cavity as a function of the left cavity-pump detuning $\Delta_1 = \omega_1 - \omega_L$ for various pump powers. When the power of the left pump beam is $P_L = 0.1$ μW, the curve is nearly Lorentzian. However, when the power increases above a critical value, the system exhibits

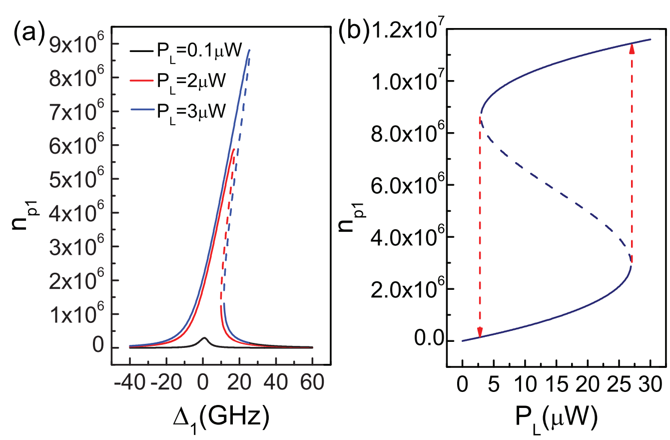

FIG. 2. (Color online) Mean intracavity photon number of the left cavity as a function of (a) the cavity-pump detuning $\Delta_1 = \omega_1 - \omega_L$ for left pump power P_L equal to 0.1, 2, and 3 μW (from bottom to top) and (b) the left pump power P_L for $\Delta_1 = \Delta_2 = \omega_m$. The right pump power is kept equal to 0.1 μW.

bistable behavior, as shown in the curves for $P_L = 2\ \mu$W and $P_L = 3\ \mu$W, where the initially Lorentzian resonance curve becomes asymmetric. In this case, the coupled cubic equations (7) and (8) for the mean intracavity photon number yield three real roots. The largest and smallest roots are stable, and the middle one is unstable, which is represented by the dashed lines in Fig. 2(a). Furthermore, we can see that larger cavity-pump detuning is necessary to observe the optical bistable behavior with the increasing pump beam power. The bistable behavior can also be seen from the hysteresis loop for the mean intracavity photon number versus the pump-power curve shown in Fig. 2(b). Here, both the cavities are pumped on their respective red sidebands, i.e., $\Delta_1 = \Delta_2 = \omega_m$. Consider that the left pump power increases from zero gradually; the mean intracavity photon number n_{p1} initially lies in the lower stable branch (corresponding to the smallest root). When the pump power P_L increases to a critical value, about 27 μW in our case, n_{p1} approaches the end of this branch. The hysteresis then follows the arrow and jumps to the upper branch. If the P_L is increased further, n_{p1} remains on the upper branch. If P_L is decreased, n_{p1} gradually approaches the beginning of the second stable branch. If P_L is decreased even further, the hysteresis follows the arrow and switches back to the lower stable branch.

In the following, we mainly investigate the optical bistability in the right cavity by controlling the frequency and power of the left pump beam. Mean intracavity photon number n_{p2} in the right cavity as a function of the left cavity-pump beam detuning Δ_1 is plotted in Fig. 3. When the coupling between the left cavity and the mechanical resonator turns off, i.e., $g_1 = 0$, the two-mode optomechanical system becomes the generic single-mode optomechanics, and the pump beam driving the left cavity cannot have an impact on the photon numbers in the right cavity via the mechanical mode. In this case, if the right cavity is pumped on its red sideband, it can be seen clearly from the middle straight line in Fig. 3 that the mean intracavity photon number n_{p2} remains constant when the left cavity-pump detuning Δ_1 changes. However, if the coupling between the left cavity and the mechanical resonator turns on, bistable behavior of the mean intracavity photon number in the right cavity will appear. When $\Delta_2 = \omega_m$, the average photon number is larger than the constant value obtained before. However, if the right cavity is pumped on its blue sideband, i.e., $\Delta_2 = -\omega_m$, the average photon number is smaller than the above constant value. The underling physical mechanism for this phenomenon can be explained as follows. When $g_1 = 0$ and $\Delta_2 = \omega_m$, the hybrid system turns to the typical single-mode optomechanical system, and the intracavity photon number is directly related to the pump power P_R and the square of the cavity-pump detuning Δ_2^2 [17]. Therefore, the photon number remains constant when $g_1 = 0, P_R = 0.1\ \mu$W, and $\Delta_2 = \pm\omega_m$. However, when $g_1 \neq 0$, the intracavity photons in the left cavity will have an effect on the common mechanical resonator and, subsequently, the photon number in the right cavity. The simultaneous presence of the left pump and probe beam induces a radiation pressure force at the beat frequency $\delta = \omega_p - \omega_L$, which drives the mechanical mode to oscillate coherently. When $\Delta_2 = \omega_m$, the highly off-resonant Stokes scattering at the frequency $\omega_R - \omega_m$ is strongly suppressed and only the anti-Stokes scattering at the frequency $\omega_R + \omega_m$ builds up within the right cavity, leading to the up-conversion of the pump photons to the cavity photons at the frequency ω_2. Therefore, the average photon number in the right cavity is larger than the constant value without the effect of the left cavity. Consequently, by adjusting the left cavity-pump beam detuning Δ_1, one can observe the bistable behavior of the intracavity photon number in the right cavity. Optical bistability in the right cavity can also be seen from the hysteresis loop for the mean intracavity photon number versus the left pump power when $\Delta_2 = \omega_m$ and $\Delta_2 = -\omega_m$, as shown in Fig. 4. Here we have taken left cavity-pump beam detuning to be $\Delta_1 = \omega_m$ and $P_R = 0.1\ \mu$W. Similarly, the mean intracavity photon number when $\Delta_2 = \omega_m$ is larger than the situation when $\Delta_2 = -\omega_m$.

In our previous discussions, we have demonstrated optical bistability in both cavities, and the mean intracavity photon number is usually very large, at least thousands of photons in the right cavity (see Figs. 3 and 4). The single-mode optomechanical system ($g_2 = 0$) would need many more photons in the cavity in order to reach the bistable regime

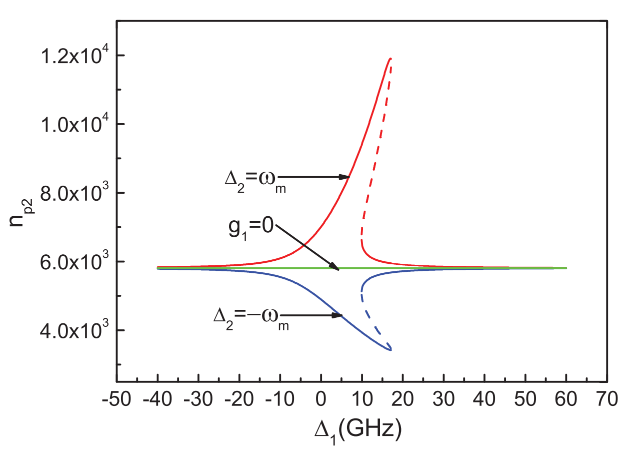

FIG. 3. (Color online) Mean intracavity photon number of the right cavity vs the left cavity-pump detuning Δ_1 with $\Delta_2 = \omega_m$ and $\Delta_2 = -\omega_m$, respectively. The left pump power P_L equals 2 μW, and the right pump power P_R equals 0.1 μW.

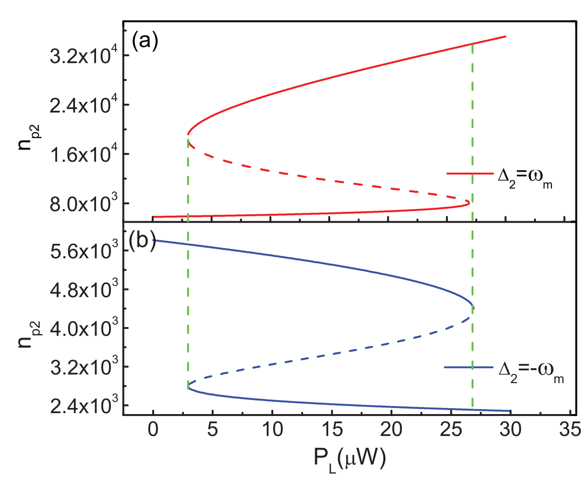

FIG. 4. (Color online) Mean intracavity photon number of the right cavity as a function of the left pump power for $P_R = 0.1\ \mu$W with (a) $\Delta_2 = \omega_m$ and (b) $\Delta_2 = -\omega_m$. The left cavity-pump detuning Δ_1 is kept equal to ω_m.

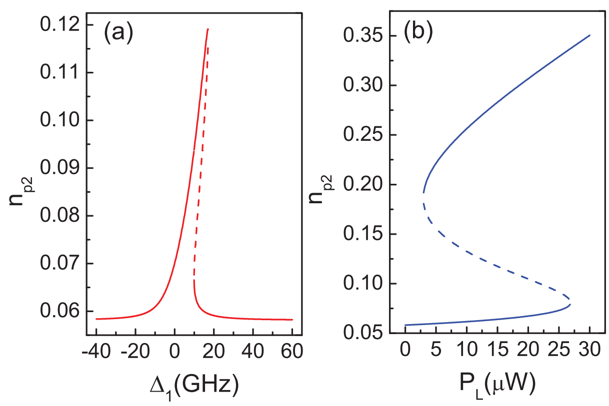

FIG. 5. (Color online) Mean intracavity photon number of the right cavity vs (a) the left cavity-pump detuning Δ_1 for $P_L = 2\ \mu$W and (b) the left pump power P_L for $\Delta_1 = \omega_m$. The other parameters used are $P_R = 1$ pW and $\Delta_2 = \omega_m$.

(see Fig. 2 for an illustration) [34]. In what follows, we will show that the two-mode optomechanical system allows for optical bistability at extremely low cavity photon numbers. Figures 5(a) and 5(b) plot the mean photon number in the right cavity as a function of left cavity-pump beam detuning Δ_1 for $P_L = 2\ \mu$W and left pump power P_L for $\Delta_1 = \omega_m$, respectively. The parameters of the right pump beam are $P_R = 1$ pW and $\Delta_2 = \omega_m$. Due to the low pump power, the intracavity photon number in the right cavity is very small, i.e., $n_{p2} \leqslant 1$. Generally, such low photon numbers cannot exhibit bistable behavior in empty cavity optomechanical systems. However, in the two-mode optomechanics we consider here, when the left cavity is driven by a strong pump power, because the two cavities are coupled to a common nanomechanical resonator, optical bistability can still exist in the right cavity at the extremely low intracavity photon numbers. This behavior can be understood as follows. The radiation pressure force exerted by the left cavity field induces the vibration of the mechanical resonator, which in turn modifies the optical path length of the two cavities, and thus, a position-dependent phase shift on the cavity field is generated. The bistability at low photon numbers results from this nonlinear feedback between the photons and phonons. This phenomenon signifies the strong nonlinear effects in the weak-coupling regime [31], which is enabled by the long lifetime of the mechanical mode and the strong pump on the left cavity. Recently, two related works by Lü *et al.* [38] and Kuzyk *et al.* [39] have also shown that the strong nonlinearities can be obtained in two-mode optomechanical systems in the weak-coupling regime. In addition, the bistable behavior of intracavity photon numbers in the two-mode optomechanical system under consideration also provides a candidate for realizing a controllable optical switch. For this, the two stable branches of photon numbers in the right cavity act as the optical switch. When the frequency and power of the left pump beam are fixed, the switch between the lower stable branch and the upper stable branch can easily be realized by controlling the frequency and power of the right pump beam. Furthermore, the left pump beam can be used as a control parameter to enable or disable this switch.

IV. CONCLUSION

In conclusion, we have investigated the optical bistability in a two-mode optomechanical system. Compared with the generic single-mode cavity optomechanics, such a two-mode optomechanical system allows one to control the optical bistability in a much more flexible way. The bistable behavior of the mean intracavity photon number in one cavity can be tuned by the power and frequency of the pump laser beam driving another cavity. Furthermore, bistability at low photon numbers below unity should be possible in such a coupled system.

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