

**Electromagnetically-induced-transparency-like phenomenon with two atomic ensembles in a cavity**Yusuf Turek,<sup>1</sup> Yong Li,<sup>2,\*</sup> and C. P. Sun<sup>1,2,†</sup><sup>1</sup>*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, and University of the Chinese Academy of Sciences, Beijing 100190, China*<sup>2</sup>*Beijing Computational Science Research Center, Beijing 100084, China*

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We study the spectra of collective low excitations of two atomic ensembles which couple with a single-mode cavity field. When the left ensemble is driven with an external optical field, its corresponding response spectrum to the incident optical light shows an electromagnetically-induced-transparency-like (EIT-like) phenomenon when the layers are arranged in the sequence of node antinode but not in the sequence of antinode node. In the case of antinode-antinode sequence, the response spectrum shows an EIT-like phenomenon with two transparent windows. We also investigate the fluctuation spectra of the atomic collective excitation modes and show similar EIT-like phenomena.

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**I. INTRODUCTION**

It is well known that a photon system is a prior candidate for quantum information processing such as quantum computing or quantum cryptography due to its fast and easily available advantages. Since direct coupling between photons is absent according to the theory of quantum electrodynamics (QED), people proposed to store the information of photons into an atomic-ensemble-based quantum memory so that one could indirectly manipulate photon by photon through the atomic ensemble based on electromagnetically induced transparency (EIT). Therein EIT is used to effectively overcome the strong absorption of a propagating beam of electromagnetic radiation by an atomic medium [1,2]. EIT plays a role in manipulating the photons by ensembles of atoms for quantum information storage and slow-light phenomena [3–8]. Actually, the EIT phenomenon is a result of the Fano interference between transitions of atomic internal energy states [9,10] and induces many strange optical phenomena in the dispersion medium [11,12].

Recently, EIT phenomena were also observed in cavity systems [13–15]. Even if there is only one or a few atoms in the cavity [13,14], cavity EIT phenomena still appear due to the strong atom-light coupling provided by the cavity quantum electrodynamics. Interestingly, the cavity EIT phenomenon can also happen in the case of few photons or even for the electromagnetic vacuum field in the cavity. This is the so-called vacuum-induced transparency as given in Ref. [15], where the cavity field acts as the control field to control the transmission of the probe light through an atomic medium of  $\Lambda$ -type configuration.

Conventional EIT (or cavity EIT) phenomena were implemented for the three-level or four-level systems which look “dark” for the probe light [16,17]. Recently, the EIT analog in quantum optomechanical systems, i.e., optomechanically induced transparency, was suggested [18] and confirmed experimentally [19]. Most recently, it was also discovered that there may also exist EIT-like phenomena for the reflectivity

spectrum of x rays in systems of two layers [20] inside a cavity. Here, the two layers consist of the Mössbauer isotope <sup>57</sup>Fe nuclei, which are exactly modeled as two-level systems with a resonant transition of 14.4 keV for Mössbauer effect. Actually the scheme in this experiment have some resemblance to vacuum-induced transparency [15]. It was observed that the EIT-like phenomenon appears for the cavity configuration where the layers are arranged in the sequence of node antinode and disappears in the antinode-node sequence. Such observation was explained with a proposed three-level configuration, which was usually required for obtaining the EIT effect, but actually the <sup>57</sup>Fe nuclei for the considered problems are only modeled as two-level systems rather than three-level systems.

On the other hand, the EIT effect may have a classical analog that is referred to the three-level configuration directly: A system of two coupled harmonic oscillators (HOs) can exhibit the EIT-like effect [21] wherein a transparency window exists as the coupling induces a split in the absorption spectrum. Furthermore, two of the authors of the present paper (Sun and Li) [22,23] even used the coupled bosonic modes to describe the low excitations of the atomic ensemble with an EIT configuration. These studies actually gave a description of two coupled HOs for the EIT effect with atomic ensembles. With these considerations, it is possible to qualitatively understand the experiment [20] as an EIT-like phenomenon; that is, as a classical analog with coupled bosonic modes formed by the single-mode cavity field and collective excitation modes of two layers of nuclear ensembles with two-level systems rather than the three-level configuration.

In this paper, we consider a model system similar to the two-layer system in the x-ray quantum optics experiment [20]. However, in contrast to the latter cases, in our model the two HOs, which are realized by atomic collective excitation modes, are indirectly coupled by a quantized single-mode cavity field. We first bosonize the low excitations of two trapped atomic ensembles inside the single-mode cavity. The quantized single-mode cavity field provides a coupling between these two bosonic modes. From the quantum Langevin equations of our system’s variables for the two bosonic modes (with one of them driven by the external field) and cavity mode, we find that the steady-state response intensities of two atomic

\*liyong@csrc.ac.cn

†cpsun@csrc.ac.cn; <http://www.csrc.ac.cn/suncp>

ensembles show a transparency window in some conditions, e.g., when the layers are arranged in the sequence of node antinode (instead of the sequence of antinode node).

Indeed, the response spectra of two collective excitation modes to the external driving show that if the two atomic ensembles are placed in the proper positions, e.g., in the antinode-node sequence, the EIT-like window would appear for the driven ensemble and can be explained as a classical analog of EIT for two coupled HOs in Ref. [21]. In the exchanged node-antinode sequence, the EIT-like window disappears. These could reflect qualitatively the basic spirit hidden in the x-ray scattering experiment in Ref. [20], where the reflectivity spectrum shows the EIT phenomenon in the antinode-node sequence and not in the opposite sequence. If we put both the atomic ensembles to the antinodes of the cavity field, the spectra of both the ensembles would appear with two EIT-like windows. This is very similar to the ac Stark effect in atomic optics [24]. To confirm the above predictions based on a simple model, we also calculate the fluctuation spectra, which display similar EIT-like phenomena.

This paper is organized as follows: In Sec. II, we describe our model with an effective Hamiltonian in terms of the collective excitation operators of atomic ensembles. In Sec. III, we calculate the response spectra of the atomic collective excitation modes to the external driving field to show that EIT-like phenomena occurred in our scheme. In Sec. IV, we calculate the fluctuation spectra of system to confirm the results of Sec. III. Finally, we make conclusions and give some remarks in Sec. V.

## II. MODEL AND HAMILTONIAN OF TWO ENSEMBLES IN A CAVITY

As shown in Fig. 1, the model under consideration consists of two trapped ensembles of two-level atoms coupling with a single-mode cavity field. The left atomic ensemble is driven by a classical external field of frequency  $\omega_f$ . The model

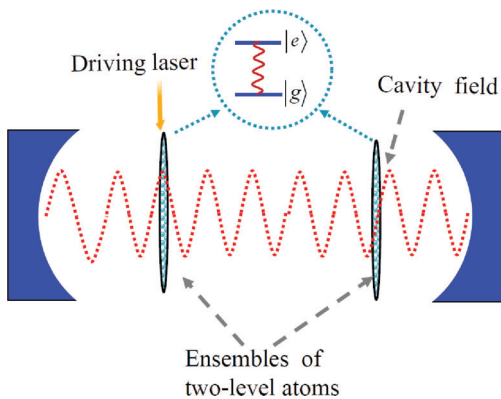


FIG. 1. (Color online) Schematic of two atomic ensembles coupling with a single-mode cavity field: Two atomic ensembles consisting of two-level atoms are trapped in proper positions of the cavity. A classical laser field with frequency  $\omega_f$  drives the left atomic ensemble.

Hamiltonian reads (hereafter we take  $\hbar = 1$ )

$$H = \omega_c c^\dagger c + \frac{\omega_a}{2} \sum_{i=1}^{N_a} \sigma_{z,a}^{(i)} + \frac{\omega_b}{2} \sum_{j=1}^{N_b} \sigma_{z,b}^{(j)} + \left[ g_a c \sum_{i=1}^{N_a} \sigma_{+,a}^{(i)} + g_b c \sum_{j=1}^{N_b} \sigma_{+,b}^{(j)} + \Omega e^{-i\omega_f t} \sum_{i=1}^{N_a} \sigma_{+,a}^{(i)} + \text{H.c.} \right]. \quad (1)$$

Here,  $c$  ( $c^\dagger$ ) is the annihilation (creation) operator of the cavity field of frequency  $\omega_c$ . And  $\sigma_{z,s}^{(l)} = |e\rangle_s^{(l)} \langle e| - |g\rangle_s^{(l)} \langle g|$ ,  $\sigma_{+,s}^{(l)} = |e\rangle_s^{(l)} \langle g|$ , and  $\sigma_{-,s}^{(l)} = |g\rangle_s^{(l)} \langle e|$  ( $s = a, b$ ) are the Pauli matrices for the  $l$ th atom in the left ensemble ( $s = a$ ) or the right ensemble ( $s = b$ ) with the same energy-level spacing  $\omega_a = \omega_b$ , the number of atoms  $N_a/N_b$ , and the excited and ground states of atoms  $|e\rangle_a/|g\rangle_a$ , respectively. It is pointed out that each ensemble is arranged in a thin layer whose size in the direction of the cavity axis has been assumed to be much smaller than the wavelength of the cavity field. Thus all the atoms in the left or right ensemble couple to the single-mode cavity field with the identical coupling strength  $g_a/g_b$ . For the same reason, the coupling coefficient between the external driving field and the atoms in the left ensemble,  $\Omega$ , is also identical.

In order to simplify the model Hamiltonian we introduce the following operators of atomic collective excitation modes [23,25,26] for the two atomic ensembles:

$$A^\dagger = \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} \sigma_{+,a}^{(i)}, \quad A = (A^\dagger)^\dagger, \quad (2)$$

and

$$B^\dagger = \frac{1}{\sqrt{N_b}} \sum_{j=1}^{N_b} \sigma_{+,b}^{(j)}, \quad B = (B^\dagger)^\dagger. \quad (3)$$

In the low-excitation limit with large  $N_a$  and  $N_b$ , the above operators satisfy the standard bosonic commutation relations

$$[A, A^\dagger] \approx [B, B^\dagger] \approx 1, \quad [A, B] = [A, B^\dagger] = 0. \quad (4)$$

And we can also have

$$\sum_{i=1}^{N_a} \sigma_{z,a}^{(i)} = 2A^\dagger A - N_a, \quad (5)$$

$$\sum_{j=1}^{N_b} \sigma_{z,b}^{(j)} = 2B^\dagger B - N_b. \quad (6)$$

Then, Hamiltonian (1) can be rewritten in terms of the atomic collective operators  $A$  ( $A^\dagger$ ) and  $B$  ( $B^\dagger$ ) as

$$H = \omega_c c^\dagger c + \omega_a A^\dagger A + \omega_b B^\dagger B + (G_A c A^\dagger + G_B c B^\dagger + \chi A^\dagger e^{-i\omega_f t} + \text{H.c.}), \quad (7)$$

with  $G_A \equiv \sqrt{N_a} g_a$ ,  $G_B \equiv \sqrt{N_b} g_b$ , and  $\chi \equiv \sqrt{N_a} \Omega$ . For simplicity here we have assumed all these coupling strengths are real. In the interaction picture with respect to  $H_0 = \omega_f (c^\dagger c + A^\dagger A + B^\dagger B)$ , the interaction Hamiltonian is given in time-independent form as

$$H_I = \Delta_c c^\dagger c + \Delta_a A^\dagger A + \Delta_b B^\dagger B + (G_A c A^\dagger + G_B c B^\dagger + \chi A^\dagger + \text{H.c.}), \quad (8)$$

where the detunings  $\Delta_r \equiv \omega_r - \omega_f$  for  $r = a, b, c$ . We note that during the derivation of Eq. (7) we have neglected the constant terms  $-(1/2)\omega_a N_a$  and  $-(1/2)\omega_b N_b$  since they have no effect on our results in the context. We would also like to remind the reader that although here we assumed the layer thickness is much less than the optical wavelength, the methods of low-excitation approximation and bosonization for collective operators still work when the layer thickness is comparable with the optical wavelength only if the atomic energy levels are identical [27,28].

### III. RESPONSE SPECTRUM FOR TWO ATOMIC ENSEMBLES IN A CAVITY

In our model the external optical driving field can be considered as a probe field which is incident from the upper side (normal to the cavity axis) as shown in Fig. 1. Thus the driving laser field does not disturb the cavity mode so much since the spatial extent of atomic ensemble is smaller than the wavelength of optical cavity field and only drives the left atomic ensemble. Let us first study the response spectra of atomic collective excitation modes to the driving. To this end, we investigate the steady-state solution of variables resorting to the quantum Langevin equations from Eq. (8)

$$\dot{c} = -i\Delta_c c - iG_A A - iG_B B - \frac{\kappa}{2}c + \sqrt{\kappa}c_{\text{in}}(t), \quad (9)$$

$$\dot{A} = -i\Delta_a A - iG_{AC} - i\chi - \frac{\gamma_A}{2}A + \sqrt{\gamma_A}A_{\text{in}}(t), \quad (10)$$

$$\dot{B} = -i\Delta_b B - iG_{BC} - \frac{\gamma_B}{2}B + \sqrt{\gamma_B}B_{\text{in}}(t). \quad (11)$$

Here  $\kappa$  is the decay rate of the cavity and  $\gamma_A$  and  $\gamma_B$  are the decay rates of collective modes  $A$  and  $B$ , the operators  $c_{\text{in}}(t)$ ,  $A_{\text{in}}(t)$  and  $B_{\text{in}}(t)$  denote the corresponding noises with the vanishing average values, i.e.,  $\langle c_{\text{in}} \rangle = \langle A_{\text{in}} \rangle = \langle B_{\text{in}} \rangle = 0$ . We note that  $\gamma_A$  and  $\gamma_B$  are the collective decay rate of the atomic ensemble (instead of the single-atom decay rate), scaling linearly with the numbers of the atoms in the layers [29–31], respectively. These noise operators satisfy the following fluctuation relations:

$$\langle c_{\text{in}}(t)c_{\text{in}}^\dagger(t') \rangle = [N(\omega_c) + 1]\delta(t - t'), \quad (12a)$$

$$\langle A_{\text{in}}(t)A_{\text{in}}^\dagger(t') \rangle = [N(\omega_a) + 1]\delta(t - t'), \quad (12b)$$

$$\langle B_{\text{in}}(t)B_{\text{in}}^\dagger(t') \rangle = [N(\omega_b) + 1]\delta(t - t'), \quad (12c)$$

where

$$N(\omega_r) = \frac{1}{\exp\left(\frac{\omega_r}{k_B T}\right) - 1}, \quad (r = a, b, c) \quad (13)$$

are, respectively, the average thermal excitation numbers of the cavity mode and atomic collective modes at temperature  $T$ .

The steady-state values of the atomic ensemble-cavity system are given by

$$A_s \equiv \langle A \rangle = -\frac{\chi F_A}{\Delta_a - i\frac{\gamma_A}{2}}, \quad (14)$$

$$B_s \equiv \langle B \rangle = \frac{\chi f_a f_b}{\Delta_{\text{eff}}^{(0)} - i\frac{1}{2}\kappa_{\text{eff}}^{(0)}}, \quad (15)$$

$$c_s \equiv \langle c \rangle = \frac{\chi f_a}{\Delta_{\text{eff}}^{(0)} - i\frac{1}{2}\kappa_{\text{eff}}^{(0)}}, \quad (16)$$

where

$$F_A = 1 + \frac{G_A f_a}{\Delta_{\text{eff}}^{(0)} - i\frac{1}{2}\kappa_{\text{eff}}^{(0)}} \quad (17)$$

is the modified factor of the coupling coefficient between the left atomic ensemble and external driving field, and

$$f_a = \frac{G_A}{\Delta_a - i\frac{1}{2}\gamma_A}, \quad f_b = \frac{G_B}{\Delta_b - i\frac{1}{2}\gamma_B}. \quad (18)$$

Here the effective decay rate and detuning between the cavity and external driving field are given by

$$\kappa_{\text{eff}}^{(0)} = \kappa + \frac{G_A^2 \gamma_A}{\Delta_a^2 + \frac{1}{4}\gamma_A^2} + \frac{G_B^2 \gamma_B}{\Delta_b^2 + \frac{1}{4}\gamma_B^2}, \quad (19)$$

and

$$\Delta_{\text{eff}}^{(0)} = \Delta_c - \frac{G_A^2 \Delta_a}{\Delta_a^2 + \frac{1}{4}\gamma_A^2} - \frac{G_B^2 \Delta_b}{\Delta_b^2 + \frac{1}{4}\gamma_B^2}, \quad (20)$$

respectively.

Seen from Eqs. (14), (15), and (16), the steady-state values of all the three bosonic modes are proportional to the driving strength of the external probe field  $\chi$ . In what follows in this section, we will investigate the steady-state response spectra (mean excitation populations  $|A_s|^2$  and  $|B_s|^2$ ) of the two collective excitation modes of atomic ensembles.

Let us first consider the case of antinode-node sequence that the left atomic ensemble is placed at an antinode and the right one close to a node of the single-mode cavity field. That is, the left (right) ensemble is strongly (weakly) coupled to the cavity field. Seen from Fig. 2(a), in this case the response of the left ensemble appears with an EIT window, which is similar to the case of two coupled HOs in Ref. [21], although our system behaves as a system of three bosonic modes. This is expected since the present system will reduce to the model of two coupled bosonic modes when the right ensemble is placed close to the node with a very small coupling to the cavity and leads to negligible contribution. As discussed in Ref. [21], this effect is similar to the ac Stark splitting in quantum optics [24].

Note that the above EIT-like phenomenon for the left driven ensemble happens when its coupling strength to the cavity field is larger than the decay rate of the field:  $G_A > \kappa$ , even when  $\gamma_A > \kappa, G_A$ . If we exchange the positions of the two atomic

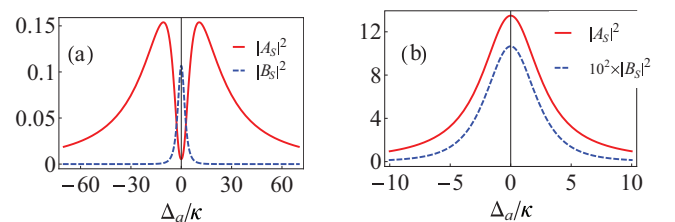


FIG. 2. (Color online) The response intensities of atomic ensembles  $|A_s|^2$  (red solid curve) and  $|B_s|^2$  (blue dashed curve) in arbitrary units according to Eqs. (14) and (15) vs the detuning  $\Delta_a$ . Here, the atomic ensembles are arranged in (a) the antinode-node sequence with  $G_A = 10, G_B = 1, \gamma_A = 50$ , and  $\gamma_B = 5$ , or in (b) the node-antinode sequence with  $G_A = 1, G_B = 10, \gamma_A = 5, \gamma_B = 50$ . All the frequencies are in units of  $\kappa$ . And we assumed the degenerate case  $\omega_a = \omega_b = \omega_c = 10^7$ ; that is,  $\Delta_a = \Delta_b = \Delta_c$ .

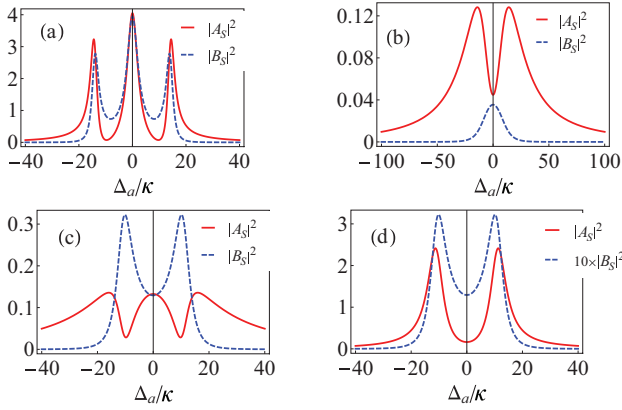


FIG. 3. (Color online) Similar to Fig. 2 except the atomic ensembles are arranged in the antinode-antinode sequence with  $G_A = G_B = 10$  but with different decay rates: (a)  $\gamma_A = 5, \gamma_B = 5$ ; (b)  $\gamma_A = 50, \gamma_B = 50$ ; (c)  $\gamma_A = 50, \gamma_B = 5$ ; (d)  $\gamma_A = 5, \gamma_B = 50$ .

ensemble to make it in the node-antinode sequence so that  $G_A \lesssim \kappa, \gamma_A$ , the EIT-like phenomenon for the left ensemble would disappear, as in Fig. 2(b). This phenomenon results from the weak coupling of the driven ensemble to the cavity compared with the corresponding decay rates. It is noted that in both the above cases the response of the right ensemble is still a Lorentz-type peak without EIT window due to the weak coupling of the right ensemble to the cavity field [see the blue dashed line in Fig. 2(a)] or the weak coupling of the left driven ensemble to the cavity field [see the blue dashed line in Fig. 2(b)].

Now let us consider the response spectra of the ensembles when both the atomic ensembles position at (or near) the antinodes of the cavity field, e.g.,  $G_A = G_B = 10\kappa$  in Fig. 3. If both the decay rates of the atomic collective excitation modes are not so large, e.g.,  $\gamma_A = \gamma_B = 5\kappa$  ( $< G_A, G_B$ ) as shown in Fig. 3(a), the response spectra of both the ensembles appear with two pronounced EIT-like windows which are expected to occur in a system of three coupled HOs. This is a simple generalization of the classical analog of EIT-like mechanism in a system of two coupled HOs [21]. However, when the decay rates of the ensembles are very large, e.g.,  $\gamma_A = \gamma_B = 50\kappa$  ( $> G_A, G_B$ ) in Fig. 3(b), the response of the driven ensemble appears with only one EIT-like window and the one of the right ensemble appears without any EIT window. In the case of  $\gamma_A = 50\kappa$  and  $\gamma_B = 5\kappa$  in Fig. 3(c), the driven ensemble responds with a two-window EIT-like phenomenon and the right one with a one-window EIT-like phenomenon. As shown in Fig. 3(d) with  $\gamma_A = 5\kappa$  and  $\gamma_B = 50\kappa$ , due to the fact that the strong decay rate of the right ensemble destroys its action on the other modes, the response of the driven ensemble happens with one EIT-like window, similar to the case of two coupled HOs. At the same time, the strong coupling of the right ensemble to the cavity field makes it have a weak EIT-like response to the external field driving on the left ensemble.

We also consider in Fig. 4 the effect of the different coupling strengths  $G_A$  and  $G_B$  on the response for the cases of fixed decay rates. In Fig. 4(a) where we fix  $G_B/\kappa = 1$  ( $\gamma_A = \gamma_B = 10\kappa$ ), we find the response intensity  $|A_s|^2$  appears with EIT-like phenomenon where the dip of the EIT window approaches

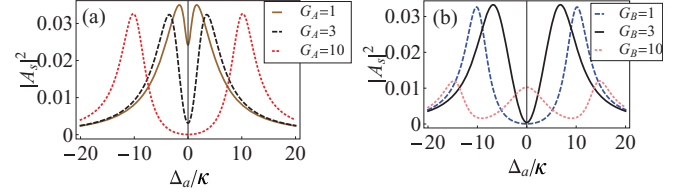


FIG. 4. (Color online) The response intensities of atomic ensemble  $|A_s|^2$  vs the detuning  $\Delta_a$  for (a)  $G_B = 1$  and different coupling strengths  $G_A$ , or (b)  $G_A = 10$  and different coupling strengths  $G_B$ . Here  $\gamma_A = \gamma_B = 10$ . For the other parameters, see Fig. 2.

more to zero and the corresponding width becomes wider with increasing coupling strength of driven left atomic ensemble to the single-mode cavity field,  $G_A$ . For the case of fixed  $G_A/\kappa = 10$  and  $\gamma_A = \gamma_B = 10\kappa$  as in Fig. 4(b), one can find that the EIT-like phenomenon of the response intensity  $|A_s|^2$  appears with one window in the case of weak-coupling strength  $G_B$  ( $G_B = \kappa$  or  $G_B = 3\kappa$ ) but with two windows in the case of strong-coupling strength  $G_B$  (e.g.,  $G_B = 10\kappa$ ). These agree with the previous analysis in Figs. 2 and 3.

We would like to remark that some theoretical and experimental works [11,29,32–36] about enhancing the index of refraction with vanishing absorption of a probe field. This is somehow similar to the EIT phenomena with vanishing absorption but without enhancing the refractive index for a resonant probe field in controlled three-level systems. In particular, Macovei *et al.* [29] and Proite *et al.* [36] studied theoretically and experimentally, respectively, the enhancement of the refractive index with vanishing absorption in two two-level atomic ensembles, which is similar to our present model and also stimulates our investigation in this work. The difference between these two models is the following: (1) In their works [29,36], the two two-level atomic ensembles with *different* spices are overlapped distributed spatially and are both driven by the control beam; however in the present work, the two two-level atomic ensembles of the same spice located separately in different places in the cavity. (2) Instead of focusing on enhancing the refractive index with vanishing absorption in their works, we focus on the EIT phenomena, as mentioned above.

#### IV. FLUCTUATION SPECTRUM FOR TWO ATOMIC ENSEMBLES IN A CAVITY

In this section, we consider the fluctuation spectra of the atomic ensembles in the cavity when the left ensemble is driven. To account for the effects of the quantum fluctuations we decompose each bosonic operator in the Langevin equations (9), (10), and (11) as the sum of its steady-state value and a small fluctuation, e.g.,  $c = c_s + \delta c$ . Substituting these quantities into the Langevin equations and linearizing the resulting equations for the fluctuations, one has

$$\delta \dot{c} = -i\Delta_c \delta c - iG_A \delta A - iG_B \delta B - \frac{\kappa}{2} \delta c + \sqrt{\kappa} c_{\text{in}}(t), \quad (21)$$

$$\delta \dot{A} = -i\Delta_a \delta A - iG_A \delta c - \frac{\gamma_A}{2} \delta A + \sqrt{\gamma_A} A_{\text{in}}(t), \quad (22)$$

$$\delta \dot{B} = -i\Delta_b \delta B - iG_B \delta c - \frac{\gamma_B}{2} \delta B + \sqrt{\gamma_B} B_{\text{in}}(t). \quad (23)$$

For the experimental perspective the frequency domain is more useful. Thus by Fourier-transferring these equations into the frequency domain like

$$\tilde{y}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} y(t) e^{i\omega t} dt \quad (24)$$

for any operator  $y(t)$ , it is easy to find the following solutions:

$$\delta\tilde{c}(\omega) = \frac{i}{\omega - \Delta_{\text{eff}}(\omega) + i\frac{1}{2}\kappa_{\text{eff}}(\omega)} \left[ \sqrt{\kappa} \tilde{c}_{\text{in}}(\omega) + \frac{G_A \sqrt{\gamma_A} \tilde{A}_{\text{in}}(\omega)}{(\omega - \Delta_a) + i\frac{\gamma_A}{2}} + \frac{G_B \sqrt{\gamma_B} \tilde{B}_{\text{in}}(\omega)}{(\omega - \Delta_b) + i\frac{\gamma_B}{2}} \right], \quad (25)$$

$$\begin{aligned} \delta\tilde{A}(\omega) &= \frac{G_A \delta\tilde{c}(\omega) + i\sqrt{\gamma_A} \tilde{A}_{\text{in}}(\omega)}{(\omega - \Delta_a) + i\frac{\gamma_A}{2}}, \\ \delta\tilde{B}(\omega) &= \frac{G_B \delta\tilde{c}(\omega) + i\sqrt{\gamma_B} \tilde{B}_{\text{in}}(\omega)}{(\omega - \Delta_b) + i\frac{\gamma_B}{2}}, \end{aligned} \quad (26)$$

$$S_A(\omega) = \frac{G_A^2 S_c(\omega) + \gamma_A [N(\omega_a) + 1] [1 + 2G_A^2 K_A(\omega)]}{(\omega - \Delta_a)^2 + \frac{1}{4}\gamma_A^2}, \quad (30)$$

$$S_B(\omega) = \frac{G_B^2 S_c(\omega) + \gamma_B [N(\omega_b) + 1] [1 + 2G_B^2 K_B(\omega)]}{(\omega - \Delta_b)^2 + \frac{1}{4}\gamma_B^2}, \quad (31)$$

where

$$S_c(\omega) = \frac{[N(\omega_c) + 1]\kappa + \frac{G_A^2 \gamma_A}{(\omega - \Delta_a)^2 + \frac{\gamma_A^2}{4}} [N(\omega_a) + 1] + \frac{G_B^2 \gamma_B}{(\omega - \Delta_b)^2 + \frac{\gamma_B^2}{4}} [N(\omega_b) + 1]}{[\omega - \Delta_{\text{eff}}(\omega)]^2 + \frac{1}{4}\kappa_{\text{eff}}^2(\omega)} \quad (32)$$

is the fluctuation spectrum of the cavity field. Here we take

$$K_A(\omega) = \frac{(\omega - \Delta_a)[\omega - \Delta_{\text{eff}}(\omega)] - \frac{1}{4}\gamma_A \kappa_{\text{eff}}(\omega)}{([\omega - \Delta_{\text{eff}}(\omega)]^2 + \frac{1}{4}\kappa_{\text{eff}}^2(\omega))[(\omega - \Delta_a)^2 + \frac{1}{4}\gamma_A^2]}, \quad (33a)$$

$$K_B(\omega) = \frac{(\omega - \Delta_b)[\omega - \Delta_{\text{eff}}(\omega)] - \frac{1}{4}\gamma_B \kappa_{\text{eff}}(\omega)}{([\omega - \Delta_{\text{eff}}(\omega)]^2 + \frac{1}{4}\kappa_{\text{eff}}^2(\omega))[(\omega - \Delta_b)^2 + \frac{1}{4}\gamma_B^2]}. \quad (33b)$$

For simplicity, in this paper we just consider the simple resonant case of  $\omega_a = \omega_b = \omega_c$ ; that is,  $N(\omega_a) = N(\omega_b) = N(\omega_c)$ . Note that the thermal photon number for the optical field (like visible light or x ray) is approximately zero even at room temperatures. So we just consider this noise response spectrum with  $N(\omega_a) = N(\omega_b) = N(\omega_c) = 0$ .

Seen from Fig. 5(a), the fluctuation spectrum of the driven left ensemble appears with asymmetrical configuration: The EIT-like phenomenon appears in the case of antinode-node sequence (the red solid curve) and disappears in the opposite sequence (the blue dotted curve), similar to the response spectra given in Fig. 2(a). However, the fluctuation spectrum of the right atomic ensemble in Fig. 5(b) happens with some different features compared to the response spectrum of the right atomic ensemble shown in Fig. 2: The former will appear with EIT-like window in the case of node-antinode sequence and the latter will only have Lorentz-type spectrum for the same given parameters.

where

$$\Delta_{\text{eff}}(\omega) = \Delta_c + \frac{G_A^2(\omega - \Delta_a)}{(\omega - \Delta_a)^2 + \frac{\gamma_A^2}{4}} + \frac{G_B^2(\omega - \Delta_b)}{(\omega - \Delta_b)^2 + \frac{\gamma_B^2}{4}}, \quad (27)$$

$$\kappa_{\text{eff}}(\omega) = \kappa + \frac{\gamma_A G_A^2}{(\omega - \Delta_a)^2 + \frac{\gamma_A^2}{4}} + \frac{\gamma_B G_B^2}{(\omega - \Delta_b)^2 + \frac{\gamma_B^2}{4}}. \quad (28)$$

Now we calculate the fluctuation spectra of the cavity field and the atomic collective-excitation modes,  $S_{c,A,B}(\omega)$ , which are defined as [37]

$$S_y(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \langle \delta y(t - \tau) \delta y^\dagger(t) \rangle e^{i\omega\tau} d\tau \quad (y = c, A, B). \quad (29)$$

The explicit forms of the fluctuation spectra of the collective-excitation modes for the atomic ensemble are

It follows from Eqs. (30) and (31) that the fluctuation spectra of excitation for the right and the left atomic ensembles have similar expressions since they equally couple to a single-mode cavity field. From Fig. 5 we can see that the spectra have two symmetric peaks at  $\omega/\kappa = \pm G_A$  [see the red solid curve in Fig. 5(a)] for the left atomic ensemble and at  $\omega/\kappa = \pm G_B$  [see the blue dotted line in Fig. 5(b)] for the right one, respectively. When we further enhance the coupling coefficients  $G_A$  and  $G_B$ , the EIT-like windows which occur above would disappear. Thus the EIT-like windows are controlled by the amount coupling coefficients under certain conditions,

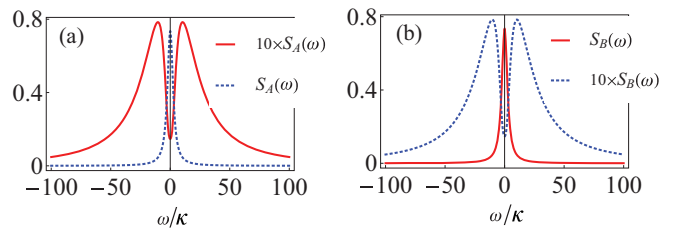


FIG. 5. (Color online) The fluctuation spectra of atomic ensembles, (a)  $S_A(\omega)$  in Eq. (30) and (b)  $S_B(\omega)$  in Eq. (31) in arbitrary units. Here,  $\omega_c = 10^7$ , and  $\Delta_c = \Delta_a = \Delta_b = 0$  (in units of  $\kappa$ ). The red solid curves correspond to the antinode-node sequence with the parameters  $G_A = 10$ ,  $G_B = 1$ ,  $\gamma_A = 50$ , and  $\gamma_B = 5$ , and the blue dotted curves correspond to the node-antinode sequence with the parameters  $G_A = 1$ ,  $G_B = 10$ ,  $\gamma_A = 5$ , and  $\gamma_B = 50$ .

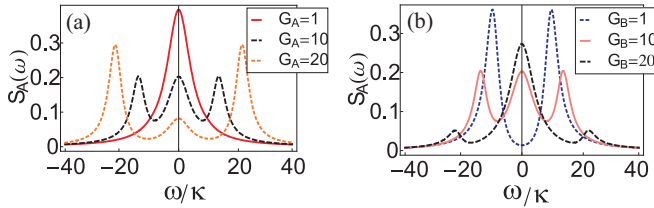


FIG. 6. (Color online) The fluctuation spectra of driven atomic ensemble,  $S_A(\omega)$ , for (a) different coupling strengths  $G_A$  with fixed  $G_B = 10$  or (b) different coupling strengths  $G_B$  with fixed  $G_A = 10$ . Here  $\gamma_A = \gamma_B = 10$ . Other parameters are the same as in Fig. 5.

similar to the response spectra of driven atomic ensemble seen in Fig. 4(a).

To further investigate the properties of the fluctuation spectra of the left atomic ensemble, we plot  $S_A(\omega)$  as a function of the coupling strength  $G_A$  or  $G_B$  for the fixed decay rates in Fig. 6. As shown in Fig. 6(a) where  $G_B = 10\kappa$  and  $\gamma_A = \gamma_B = 10\kappa$ , we find the fluctuation spectrum has the Lorentz form for weak  $G_A$  (e.g.,  $G_A = \kappa$  for the red solid curve), similar to the response spectrum of the red solid line in Fig. 2(b). When  $G_A$  is larger [e.g.,  $G_A = 10\kappa$  for the dashed black curve or  $G_A = 20\kappa$  for the dotted orange curve in Fig. 6(a)], there would occur two-window EIT-like phenomena similar to the red curve in Fig. 3(c), since in this case the model behaves like that of three-coupled HOs. Whereas, as shown in Fig. 6(b) for large fixed  $G_A = 10\kappa$  (as well as  $\gamma_A = \gamma_B = 10\kappa$ ), the fluctuation spectrum appears with the EIT-like phenomenon with one EIT window for weak  $G_B$  (corresponding to the model of two coupled HOs) and with

two windows for strong  $G_B$  (corresponding to the model of three coupled HOs).

## V. CONCLUSION

In conclusion, the EIT-like phenomenon was shown to happen for two two-level atomic ensembles inside a cavity when one of them is driven by a laser. The theoretical prediction was made for realistic systems without assuming a three-level configuration, which usually is used for the argument of the EIT mechanism. We attribute the EIT-like phenomena to a simplified model: the three coupled HOs consisting of the single cavity mode and two collective low-excitation modes of the two-level atomic ensembles.

Essentially, the EIT (or EIT-like) phenomenon we studied here for the two ensembles of two-level atoms inherently has the same mechanism as the EIT effect for a single ensemble with three-level configuration. This is because both could be modeled mathematically as the system of two coupled HOs. In this sense, it is possible to find such EIT-like phenomena in various hybrid systems, such as an atomic ensemble coupled to a nanomechanical resonator or a superconducting transmission line.

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