Stochastic coupling in two-mode systems: From weakly to strongly fluctuating coupling

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We investigate the weakly- to strongly-fluctuating-coupling transition of two linearly coupled systems under the influence of a phase-fluctuating coupling. In the weakly-fluctuating-coupling regime, the exponential decay of quantum properties is well known. A different scenario occurs in the strongly-fluctuating-coupling regime, namely the inhibition of the dynamics which tends to "freeze" as the ratio between coupling strength and average phase-fluctuation time increases. Exciton-polariton oscillations and the self-trapping phenomenon in the Bose-Einstein condensate qualitatively illustrate the weak and strong regimes, respectively.

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I. INTRODUCTION

Decoherence effects are now believed to be the essential ingredient which destroys most of the counterintuitive aspects of quantum mechanics. Such effects are at the same time an academic tool in the understanding of the classical limit of quantum mechanics as well as an important ingredient in the area of quantum computation. The dynamics of quantum open systems has therefore been studied extensively [1]. Of particular importance in this context is the Born-Markov approximation, which leads to master equations of various kinds [2], whose validity is limited by the weak-coupling approximation. The strong-coupling regime, however, has been less explored.

It is the purpose of the present contribution to shed some light onto the weakly- to strongly-fluctuating-coupling transition in the context of two linearly interacting systems under the influence of a phase-fluctuating coupling. In spite of its schematic character, the model has been shown in several instances and different areas to reflect and adequately describe experimental results. Examples include the description of exciton-polariton damped oscillations [3,4], predictions for the behavior of oscillations of two coupled modes in the context of microwave cavities [5,6], and the self-trapping phenomenon in the tunneling process of a Bose-Einstein condensate (BEC) [7–14].

We will show that in the weakly-fluctuating-coupling regime, the usual master equation results are recovered and the usual phenomenological damping constant is derived as a function of the model parameters. The strongly fluctuating coupling limit, however, leads to a completely different scenario, namely the "freezing" of the dynamics. We will illustrate these effects in the context of exciton-polariton oscillations and the self-trapping of a BEC in a devised laser potential.

II. THE MODEL

In this section, we give a detailed derivation of the stochastic time evolution of the following system:

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar[g(t)a^\dagger b + g^*(t)b^\dagger a], \tag{1}$$

where $a^\dagger$ (a) and $b^\dagger$ (b) are creation (annihilation) operators. ω_a and ω_b are the frequencies of the a-mode and b-mode, respectively. Since the operators a and b were considered as bosons, they will obey the commutation relation for bosons $[a,a^\dagger] = 1$ and $[b,b^\dagger] = 1$. In the third term, $g(t)$ stands for the interaction strength between the modes, which is assumed here to be time-dependent in the sense that $|g(t)|$ is constant but its phase is a stochastic variable. The Hamiltonian (1) can be written in matrix form,

$$H = \hbar(a^\dagger \quad b^\dagger)\begin{pmatrix} \omega_a & g(t) \\ g^*(t) & \omega_b \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}. \tag{2}$$

The time evolution operator for the system is given by

$$U(t) = e^{-iH_I t/\hbar} = \begin{pmatrix} \cos(|g|t) & -i\frac{g(t)}{|g|}e^{i\Delta t}\sin(|g|t) \\ -i\frac{g^*(t)}{|g|}e^{-i\Delta t}\sin(|g|t) & \cos(|g|t) \end{pmatrix}, \tag{3}$$

where H_I is the Hamiltonian of the system in the interaction picture and $\Delta = \omega_a - \omega_b$ is the detuning between the a-mode and the b-mode. Our intention here is to isolate solely the phase fluctuation effects which are present in the physical situations we have in mind. Also, we emphasize that our stochastic process can be simplified to $\Delta = 0$, since the inclusion of a nonstochastic time-dependent phase would require new considerations.

We will specify the noise by defining a stochastic process for $\phi(t)$. In our model, we assume that

$$g(t) = g_0 \exp[i\phi(t)], \tag{4}$$

where g_0 is the nonstochastic amplitude while the phase $\phi(t)$ is treated as a stochastic variable. Here, we will consider random phase telegraph noise where $\phi(t)$ itself fluctuates in the manner of jumps. In particular, the phase fluctuations were describe by a Wierner-Levy (phase-diffusion) process and the amplitude fluctuations by a colored Gaussian noise. An

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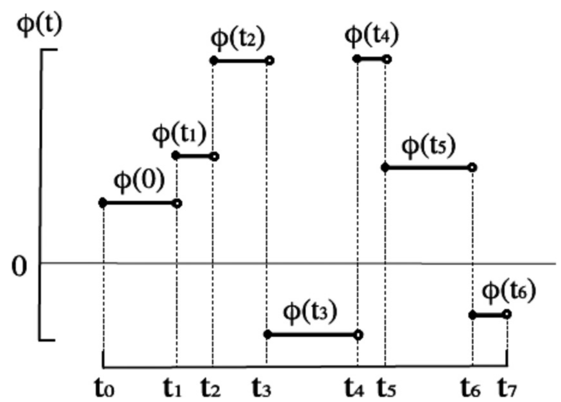

FIG. 1. Schematic representation of the phase distribution of coupling between the modes.

alternative model which represents noise by means of discrete jump processes was first introduced into quantum optics by Burshtein and Oseledchik [15]. A simple example of such a jump process is the two-state random telegraph. These models are very convenient and elegant to study the noise of the electromagnetic atom-field interaction in a nonperturbative manner. The random telegraph models, whether associated with phase, frequency, or amplitude fluctuations, lead to an equation for average responses in exact algebraic form. The model of random telegraph (jump-type) noise is physically very sound to describe the noise arising from electromagnetic field fluctuation or from collisions of various kinds or from other external sources. Indeed, that model, including the effects of stochastic phase and/or in amplitude, has been explicitly solved for the case of the Jaynes-Cummings model (JCM) by Joshi [16]. The fluctuations are modeled by the random telegraph process, and an equation for the density operator averaged over the fluctuations is obtained. The solution of these equations was used to study the decoherence effects in the dynamics of the system. Joshi's work was treated in a pedagogical form by Ospina [17]. We assume further that the change in $\phi(t)$ occurs instantaneously jumpwise and the jumps are separated by mean time intervals of the order τ_0 in which $\phi(t) = \text{const}$, as shown in Fig. 1.

There are two stochastic variables: the time interval τ between two jumps, and the value of the phase constant ϕ in each of these intervals. The variable $\tau = t_i - t_{i-1}$ follow the probability distribution

$$dQ(\tau) = \frac{1}{\tau_0} e^{-\tau/\tau_0} d\tau \tag{5}$$

with $t_0 = 0$. The above distribution specifies the probability of duration of each such jump interval and has an interval mean duration of τ_0. We consider only the case in which the phases $\phi(t)$ are uncorrelated. The probability distribution to the phase is given by

$$dq(\phi) = \frac{d\phi}{2\pi}, \tag{6}$$

with mean value $\langle \phi \rangle = \pi$. So, at any instant, the probability of finding a given ϕ remains the same and is equal to $dQ(t)$, and there is no limitation on the form of this distribution. In other words, $\phi(t)$ is undergoing random continuous change of Markov-type.

The dynamics of the system is given by the unitary transformation $U(\phi,t,t')$ such that

$$\rho(t;\phi) = U(\phi,t,t')\rho(t')U^{-1}(\phi,t,t'). \tag{7}$$

At the end of each (ith) interval, we find the density matrix $\rho(t)$ which is the initial condition for the next matrix, so, if in the interval $(0,t)$ there are k jumps in ϕ, then

$$\begin{aligned} &\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k) \\ &\quad = U(\phi_k;t,t_k)U(\phi_{k-1};t_k,t_{k-1}) \times \cdots \\ &\qquad \times U(\phi_1;t_2,t_1)U(\phi_0;t_1,0)\rho(0)U^{-1}(\phi_0;t_1,0)U^{-1} \\ &\qquad \times (\phi_1;t_2,t_1) \times \cdots \times U^{-1}(\phi_{k-1};t_k,t_{k-1})U^{-1}(\phi_k;t,t_k). \end{aligned} \tag{8}$$

The above expression is of a multiplicative nature and hence it is quite easy to average over. The probability [in the interval $(0,t)$] that k changes of ϕ have actually occurred at successive instants $t_1,t_2,\ldots,t_k$ and that a certain sequence of $\phi_1,\phi_2,\ldots,\phi_k$ [where $\phi_i = \phi(t_i)$] was realized between them is obviously equal to

$$\begin{aligned} &dP(t_1,t_2,\ldots,t_k;\phi_1,\phi_2,\ldots,\phi_k,t) \\ &\quad = \frac{1}{\tau_0^k} e^{-t/\tau_0} \left(\prod_{i=1}^{k} dt_i \right) \left(\prod_{i=1}^{k} dq(\phi_i) \right). \end{aligned} \tag{9}$$

The average density operator can thus be written as

$$\begin{aligned} \bar{\rho}(t) = \sum_{k=0}^{\infty} \int \int & dP(t_1,t_2,\ldots,t_k;\phi_1,\phi_2,\ldots,\phi_k,t) \\ & \times \rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k). \end{aligned} \tag{10}$$

Rewriting (10) with use of (9) (see the Appendix in this paper or the Appendix in Ref. [17]), we get

$$\begin{aligned} \bar{\rho}(t)e^{t/\tau_0} = \sum_{k=0}^{\infty} \frac{1}{\tau_0^k} & \int_0^t dt_k \int_0^{t_k} dt_{k-1} \cdots \int_0^{t_2} dt_1 \\ & \times \int dq(\phi_k) \int dq(\phi_{k-1}) \times \cdots \\ & \times \int dq(\phi_0)\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k). \end{aligned} \tag{11}$$

Note that the term with $k = 0$ [when ϕ does not change at all in the interval $(0,t)$] will not contain integrals with respect to time and thus is given by

$$\int dq(\phi_0)\rho(t,\phi_0). \tag{12}$$

Now using the recurrence relation (7), we can multiply both sides of Eq. (11) from the left (right) by $U^{-1}(\phi;\tau,t)$ $[U(\phi;\tau,t)]$, respectively, and also by $dq(\phi)dt/\tau_0$, and then integrate with respect to time from 0 to τ and eliminate the entire series using Eq. (11). After some simplifications [see (A18)–(A26) in the Appendix], we can show that

$$\begin{aligned}\bar{\rho}(\tau)e^{\tau/\tau_0} &= \int dq(\phi_0)U(\phi_0;\tau,0)\rho(0)U^{-1}(\phi_0;\tau,0)\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{t/\tau_0}\int dq(\phi)U(\phi;\tau,t)\bar{\rho}(t)U^{-1}(\phi;\tau,t).\end{aligned} \tag{13}$$

Now we will rewrite Eq. (13) above in terms of matrix elements of operators $\bar{\rho}(\tau)$ and $U(\phi;\tau,t)$,

$$\begin{aligned}\bar{\rho}(\tau)_{im}e^{\tau/\tau_0} &= \int dq(\phi_0)\sum_{k,l}U(\phi_0;\tau,0)_{ik}\rho(0)_{kl}U^{-1}(\phi_0;\tau,0)_{lm}\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{t/\tau_0}\\ &\quad\times\int dq(\phi)\sum_{k,l}U(\phi;\tau,t)_{ik}\bar{\rho}(t)_{kl}U^{-1}(\phi;\tau,t)_{lm}.\end{aligned} \tag{14}$$

Define the conjunct of matrix $\{\mathbf{G}^{im}(\tau,t)\}$ with elements given by

$$G^{im}(\tau,t)_{lk} = \int dq(\phi)U_{ik}(\phi;\tau,t)U^{-1}_{lm}(\phi;\tau,t), \tag{15}$$

so

$$\begin{aligned}\bar{\rho}(\tau)_{im}e^{\tau/\tau_0} &= \sum_l\left[\sum_k G^{im}(\tau,0)_{lk}\rho(0)_{kl}\right]\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{t/\tau_0}\sum_l\left[\sum_k G^{im}(\tau,t)_{lk}\bar{\rho}(t)_{kl}\right]\\ &=\sum_l[\mathbf{G}^{im}(\tau,0)\rho(0)]_{ll}\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{t/\tau_0}\sum_l[\mathbf{G}^{im}(\tau,t)\bar{\rho}(t)]_{ll}.\end{aligned} \tag{16}$$

Using the trace definition $\mathrm{Tr}A=\sum_l a_{ll}$, we get

$$\begin{aligned}\bar{\rho}(\tau)_{im} &= e^{-\tau/\tau_0}\mathrm{Tr}[\mathbf{G}^{im}(\tau,0)\rho(0)]\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{-(\tau-t)/\tau_0}\mathrm{Tr}[\mathbf{G}^{im}(\tau,t)\bar{\rho}(t)].\end{aligned} \tag{17}$$

This is the statistical average over the random variable $\phi(t)$. To determine the dynamical evolution of the system, one has to determine $\mathbf{G}$. The problem is now simplified because we have to deal with an interval in which ϕ (or alternatively g) is constant and the change in ρ is perfectly regular. Thus, knowing $\mathbf{G}$, we can find the average variation of the system during the relaxation process [15]. To determine $\mathbf{G}$, we will use the time evolution operator $U(t)$ given by (3) with $\Delta=0$ and $g(t)$ defined in (4),

$$U(t)=\begin{pmatrix}\cos(|g|t) & -ie^{i\phi}\sin(|g|t)\\ -ie^{-i\phi}\sin(|g|t) & \cos(|g|t)\end{pmatrix}. \tag{18}$$

The elements to be averaged in the calculations of $\mathbf{G}$ are those containing the factors $e^{\pm i\phi}$. Since the phases are equally probable, most of the terms vanish after averaging. The remaining (relevant for our purposes) nonvanishing elements of $\mathbf{G}$ are

$$\begin{aligned}G^{11}_{11}&=G^{22}_{22}=\cos^2[g_0(\tau-t)],\\ G^{11}_{22}&=G^{22}_{11}=\sin^2[g_0(\tau-t)],\\ G^{12}_{21}&=G^{21}_{12}=\cos^2[g_0(\tau-t)].\end{aligned} \tag{19}$$

We have thus

$$\begin{aligned}G^{11}(\tau,t)&=\begin{pmatrix}\cos^2[g_0(\tau-t)] & 0\\ 0 & \sin^2[g_0(\tau-t)]\end{pmatrix},\\ G^{12}(\tau,t)&=\begin{pmatrix}0 & 0\\ \cos^2[g_0(\tau-t)] & 0\end{pmatrix}=[G^{21}(\tau,t)]^T,\\ G^{22}(\tau,t)&=\begin{pmatrix}\sin^2[g_0(\tau-t)] & 0\\ 0 & \cos^2[g_0(\tau-t)]\end{pmatrix},\end{aligned} \tag{20}$$

and using Eqs. (17) and (20) we obtain

$$\begin{aligned}\bar{\rho}(\tau)_{11}e^{\tau/\tau_0} &= \rho(0)_{11}+[\rho(0)_{22}-\rho(0)_{11}]\sin^2(g_0\tau)\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{t/\tau_0}\{\bar{\rho}(t)_{11}+[\bar{\rho}(t)_{22}-\bar{\rho}(t)_{11}]\\ &\quad\times\sin^2[g_0(\tau-t)]\},\\ \bar{\rho}(\tau)_{22}e^{\tau/\tau_0} &= \rho(0)_{22}+[\rho(0)_{11}-\rho(0)_{22}]\sin^2(g_0\tau)\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dt\, e^{t/\tau_0}\{\bar{\rho}(t)_{22}+[\bar{\rho}(t)_{11}-\bar{\rho}(t)_{22}]\\ &\quad\times\sin^2[g_0(\tau-t)]\}.\end{aligned} \tag{21}$$

III. DYNAMICS OF THE AVERAGE NUMBER OF A MODE

The average boson number is given by

$$n_a(t)=\mathrm{Tr}[\rho(t)a^\dagger a] \tag{22}$$

and can be evaluated using Eqs. (21):

$$\begin{aligned}\langle\overline{n_a(\tau)}\rangle e^{\tau/\tau_0} &= \langle n_a(0)\rangle[1-2\sin^2(g_0\tau)]+\langle N\rangle\sin^2(g_0\tau)\\ &\quad+\frac{1}{\tau_0}\int_0^\tau e^{t/\tau_0}\langle\overline{n_a(t)}\rangle\{1-2\sin^2[g_0(\tau-t)]\}dt\\ &\quad+\frac{\langle N\rangle}{\tau_0}\int_0^\tau e^{t/\tau_0}\sin^2[g_0(\tau-t)]dt,\end{aligned} \tag{23}$$

where $\langle N\rangle=\langle n_a(0)\rangle+\langle n_b(0)\rangle$ is the initial excitation number, with $\langle n_a(0)\rangle$ and $\langle n_b(0)\rangle$ being the average excitations in each mode. This equation describes the relaxation of the intensity of the mode a and can be solved using the Laplace transform. To solve (23), we will define

$$\begin{aligned}f(t)&=\langle\overline{n_a(t)}\rangle e^{t/\tau_0},\\ g(t)&=1-2\sin^2(g_0t)=\cos(2g_0t),\\ h(t)&=e^{t/\tau_0},\\ j(t)&=\sin^2(g_0t).\end{aligned}$$

Inserting the functions defined above into (23), we obtain

$$\begin{aligned}f(\tau)&=\langle n_a(0)\rangle g(\tau)+\frac{1}{\tau_0}\int_0^\tau f(t)g(\tau-t)dt\\ &\quad+\langle N\rangle j(\tau)+\frac{\langle N\rangle}{\tau_0}\int_0^\tau h(t)j(\tau-t)dt.\end{aligned} \tag{24}$$

Now applying the Laplace transform on both sides of (24), we obtain

$$\hat{f}(s) = \langle n_a(0)\rangle \hat{g}(s) + \frac{1}{\tau_0}\hat{f}(s)\hat{g}(s) + \langle N\rangle \hat{j}(s) + \frac{\langle N\rangle}{\tau_0}\hat{h}(s)\hat{j}(s)$$
$$= \frac{\langle n_a(0)\rangle \hat{g}(s) + \langle N\rangle \hat{j}(s) + \frac{\langle N\rangle}{\tau_0}\hat{h}(s)\hat{j}(s)}{\left[1 - \frac{1}{\tau_0}\hat{g}(s)\right]}. \quad (25)$$

Calculating the Laplace transform of the functions $\hat{g}(s)$, $\hat{h}(s)$, and $\hat{j}(s)$ and substituting it into (25), we obtain

$$\hat{f}(s) = \frac{\langle n_a(0)\rangle s}{\left(s - \frac{1}{2\tau_0}\right)^2 + \Omega^2} + \frac{\langle N\rangle (2g_0)^2}{2\left(s - \frac{1}{\tau_0}\right)} \frac{1}{\left[\left(s - \frac{1}{2\tau_0}\right)^2 + \Omega^2\right]}, \quad (26)$$

where we defined $\Omega = \sqrt{(2g_0)^2 - \frac{1}{(2\tau_0)^2}}$. The inverse Laplace transform of Eq. (26) yields the following expression for the time evolution of the relaxation:

$$\overline{\langle n_a(t)\rangle} = \langle n_a(0)\rangle e^{-t/2\tau_0}\left[\cos(\Omega t) + \frac{\sin(\Omega t)}{2\tau_0\Omega}\right] + \frac{\langle N\rangle e^{-t/2\tau_0}}{2}\left\{[e^{t/2\tau_0} - \cos(\Omega t)] - \frac{\sin(\Omega t)}{2\tau_0\Omega}\right\}. \quad (27)$$

In the limit when the average time τ_0 between phase jumps is large as compared with the oscillation period ($\tau_0 \to \infty$), a pure oscillatory regime with frequency $2g_0$ is obtained. As the average time between frequency jumps increases, one obtains an envelope limiting the oscillation amplitudes, and the oscillation frequency is only slightly altered. However, in the limit where the average time between jumps decreases and assumes the value $\tau_0 = \frac{1}{4g_0}$, the oscillation ceases and an overdamped limit sets in. Figures 2 and 3 illustrate the weakly fluctuating coupling regime ($\tau_0 \gg \frac{1}{4g_0}$) and the strongly fluctuating coupling regime ($\tau_0 > \frac{1}{4g_0}$), respectively. Figure 4 illustrates the "freezing" of the dynamics as τ_0 decreases beyond the limit $\tau_0 = \frac{1}{4g_0}$. The fluctuating interaction records information of the system. The information transfer plays the role of an unobserved detection process [18]. In our case, small values of τ_0 would imply according to this reasoning that the system is being "measured" with increasing frequency, freezing out as a Zeno-like effect.

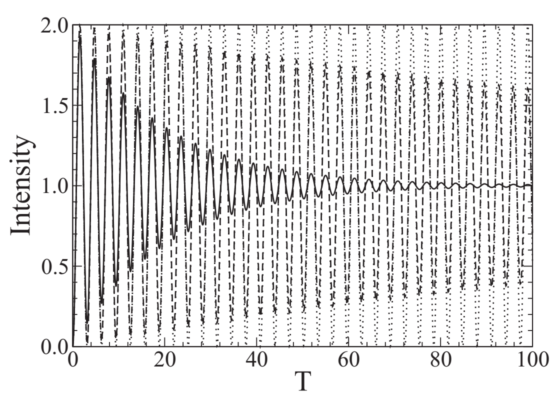

FIG. 2. Light intensity as a function of dimensionless time $T = g_0 t$ for the case in which the excitons are initially in a number state $N = 2$ for $g_0\tau_0 \to \infty$ (dotted line), $g_0\tau_0 = 100$ (dashed line), and $g_0\tau_0 = 10$ (solid line). The intensity is in arbitrary units.

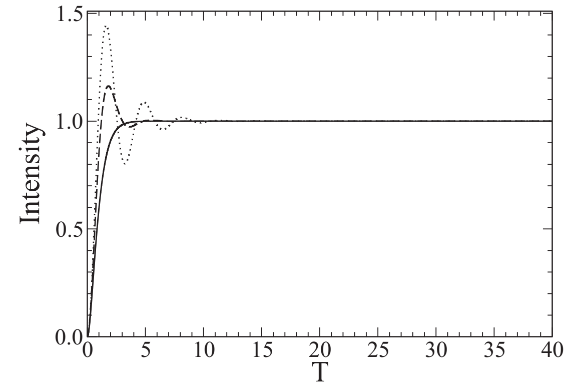

FIG. 3. Light intensity as a function of dimensionless time $T = g_0 t$ for the case in which the excitons are initially in a number state $N_b = 2$ for $g_0\tau_0 = 1.0$ (dotted line), $g_0\tau_0 = 0.50$ (dashed line), and $g_0\tau_0 = 0.25$ (solid line). The intensity is in arbitrary units.

IV. APPLICATIONS

A. The weakly-fluctuating-coupling regime: Exciton-polariton oscillations

Let us consider the weakly-fluctuating-coupling regime (WCR), where $\tau_0 > \frac{1}{4g_0}$, and consequently Ω is a real number. Note that, as $\tau_0 \to \infty$, the expression for $\overline{\langle n_a(t)\rangle}$ reduces to the usual result without fluctuations. The effects of phase fluctuation in the intensity of the mode a, e.g., for an initial number state $|\varphi(0)\rangle_b = |N_b\rangle$, and the mode a in a vacuum state are given, according to (27), by

$$\overline{\langle n_a(t)\rangle} = \frac{N_b e^{-t/2\tau_0}}{2}\left\{[e^{t/2\tau_0} - \cos(\Omega t)] - \frac{\sin(\Omega t)}{2\tau_0\Omega}\right\}. \quad (28)$$

The result above shows that the intensity of the mode a contains two parts: (i) Rabi oscillations with frequency Ω; (ii) a comparatively slowly varying part $e^{-t/2\tau_0}$. To see this more clearly, we plot Eq. (28) in Figs. 2 and 3 for $N_b = 2$. It is

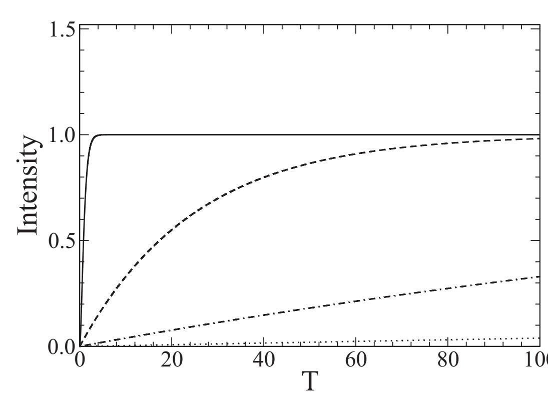

FIG. 4. Light intensity as a function of dimensionless time $T = g_0 t$ for the case in which the excitons are initially in a number state $N_b = 2$ for $g_0\tau_0 = 0.25$ (solid line), $g_0\tau_0 = 0.01$ (dashed line), $g_0\tau_0 = 0.001$ (dashed-dotted line), and $g_0\tau_0 = 0.0001$ (dotted line). The intensity is in arbitrary units.

clear that the damping of Rabi oscillations is more pronounced when the mean time interval τ_0 between phase jumps becomes shorter and shorter until $\tau_0 = \frac{1}{4g_0}$ (see Figs. 2 and 3). In other words, the decoherence mechanism is faster for shorter-phase jump intervals.

In this section, we compare the results obtained in Ref. [4] that describe two coupled harmonic (the exciton-polariton) oscillations, which are damped by couplings to reservoirs. In a real cavity, the modes of exciton and photon are coupled to a continuum of modes, which leads to dissipation. The coupling can be scattering of phonons in the case of an exciton or cavity damping in the case of a photon. In both cases, the result is to dampen the mode of interest. The result obtained in Ref. [4] can be seen in Eq. (28), where the damping coefficient corresponds to $\tau_0 = \gamma^{-1} = (\gamma_p + \gamma_{ex})^{-1}$, where γ_{ex} and γ_p are exciton and photon damping from the reservoirs. Using the values of γ adopted in Ref. [4], we may estimate the order of magnitude of τ_0 as 10^{-10} s. The model has been shown to reproduce experimental results [19,20]. In this situation, Eq. (28) can be written as

$$\overline{\langle n_a(t)\rangle} = \frac{N_b}{2}[1 - e^{-\gamma t/2}\cos(2g_0 t)]. \tag{29}$$

The results obtained here describe the decoherence process in the system. However, the fluctuations introduce a finite width in the transmission spectrum even in a lossless cavity. On the other hand, in recent work, Schneider *et al.* [21] also included fluctuations in intensity and phase in the exciting laser pulse to explain the effects of decoherence for a single trapped ion. In Schneider's model, the intensity and phase fluctuations define a stochastic Schrödinger equation in the Ito formalism [22], or more appropriately a stochastic Liouville–von Neumann equation. The results are in good qualitative agreement with recent ion experiments [23].

B. An alternative self-trapping mechanism

Now we analyze the strongly-fluctuating-coupling regime (SCR), where $\tau_0 < \frac{1}{4g_0}$. In this case, Ω is purely imaginary and Eq. (27) can be written as

$$\overline{\langle n_a(t)\rangle} = \frac{N_b e^{-t/2\tau_0}}{2}\left\{[e^{t/2\tau_0} - \cosh(|\Omega| t)] - \frac{\sinh(|\Omega| t)}{2\tau_0|\Omega|}\right\}. \tag{30}$$

The SCR may be investigated by looking at the intensity of the mode a. As observed above, when τ_0 becomes shorter and shorter as compared to $\frac{1}{4g_0}$, the fluctuation effects are larger, and the fluctuations prevail over the oscillation between mode a and mode b. In this case, the SCR modifies the picture. The inhibition of the transition of excitations between the modes is induced by the fluctuations in the coupling. This can be interpreted as an environment-induced "quantum Zeno-like effect (QZLE)" [6,24–28]. In the regime $\tau_0 \ll \frac{1}{4g_0}$, the interaction between mode a and mode b is not able to absorb or release energy and therefore stays put. The fluctuations in the interaction strength between mode a and mode b inhibit the excitation of mode a (in Fig. 4, we exemplify this effect). When $\tau_0 \to 0$, $\overline{\langle n_a(t)\rangle} \to 0$; when $\langle n_a(0)\rangle = 0$, the dynamics is frozen. In Ref. [7], a self-trapping mechanism of BEC in a laser potential has been reported. Two explanations have been given: (i) the one using a nonlinear Gross-Pitayesty equation [9,11], and (ii) a schematic many-body system [14]. In the present contribution, one might view modes a and b as the two sides of the well, and the self-trapping mechanism as the freezing out of the dynamics due to uncontrollable fluctuations in the experiment.

V. CONCLUSION

We studied a system of two linearly coupled oscillators and the effects of a phase-fluctuating coupling. The model can be solved analytically and displays the weakly- to strongly-fluctuating-coupling transition. We show that this transition is a function of a dimensionless parameter $g_0\tau_0$ and occurs at $g_0\tau_0 = 0.25$. In the weakly-fluctuating-coupling regime, we provide for an analytical expression for the damping parameter and compare with that of Ref. [4] in the context of exciton-polariton oscillations. The weakly- to strongly-fluctuating-coupling regime leads to a "freezing" of the dynamics and may qualitatively provide for yet a third explanation for the self-trapping phenomenon in the BEC (the first two are given in Refs. [9,11] and [14]).

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APPENDIX: AVERAGE DENSITY OPERATOR WITH A STOCHASTIC PHASE VARIABLE

The random telegraph models, whether associated with phase, frequency, or amplitude fluctuations, lead to an equation for average responses in exact algebraic form. Indeed, that model, including the effects of stochastic phase and/or in amplitude, has been explicitly solved for the case of the Jaynes-Cummings model (JCM) by Joshi [16]. The fluctuations are modeled by the random telegraph process, and an equation for the density operator averaged over the fluctuations is obtained. In this Appendix, we will show the deduction of the average density operator obtained in Ref. [16] and used in this paper. A more detailed deduction can be found in Ref. [17]. Here we consider only the effect of stochastic phase. So, there are two stochastic variables: the time interval τ between 1 and the next jump, and the value of the phase constant ϕ in each of these intervals (see Fig. 1). The variable $\tau = t_i - t_{i-1}$ follows the probability distribution

$$dQ(\tau) = \frac{1}{\tau_0} e^{-\tau/\tau_0} d\tau \tag{A1}$$

with $t_0 = 0$. The above distribution specifies the probability of duration of each such jump interval and has an interval mean duration of τ_0. We consider only the case in which the phases $\phi(t)$ are uncorrelated. The probability phase distribution is given by

$$dq(\phi) = \frac{d\phi}{2\pi}, \tag{A2}$$

with a mean value $\langle\phi\rangle = \pi$. So, at any instant, the probability of finding a given ϕ remains the same and equal to $dQ(t)$, and there is no limitation on the form of this distribution. In other words, $\phi(t)$ is undergoing random continuous change of Markov-type.

The dynamics of the system is given by the unitary transformation $U(\phi,t,t')$ such that

$$\rho(t;\phi) = U(\phi,t,t')\rho(t')U^{-1}(\phi,t,t'). \tag{A3}$$

If the interval $(0,t)$ occurs k-jumps in ϕ, which will be ϕ_0 in $[0,t_1)$, ϕ_1 in $[t_1,t_2)$, ϕ_2 in $[t_2,t_3)$, etc., until ϕ_k in $[t_k,t)$ with $0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < t$, at the first time interval the dynamics is given by

$$\rho(t_1;\phi_0) = U(\phi_0;t_1,0)\rho(0)U^{-1}(\phi_0;t_1,0), \tag{A4}$$

and so for the second interval we obtain

$$\begin{aligned}\rho(t_2;t_1;\phi_0,\phi_1) &= U(\phi_1;t_2,t_1)\rho(t_1;\phi_0)U^{-1}(\phi_1;t_2,t_1)\\ &= U(\phi_1;t_2,t_1)U(\phi_0;t_1,0)\rho(0)\\ &\quad\times U^{-1}(\phi_0;t_1,0)U^{-1}(\phi_1;t_2,t_1),\end{aligned} \tag{A5}$$

and for the third,

$$\begin{aligned}&\rho(t_3;t_1,t_2,\phi_0,\phi_1,\phi_2)\\ &\quad= U(\phi_2;t_3,t_2)\rho(t_2;t_1,\phi_0,\phi_1)U^{-1}(\phi_2;t_3,t_2)\\ &\quad= U(\phi_2;t_3,t_2)U(\phi_1;t_2,t_1)U(\phi_0;t_1,0)\rho(0)U^{-1}(\phi_0;t_1,0)\\ &\qquad\times U^{-1}(\phi_1;t_2,t_1)U^{-1}(\phi_2;t_3,t_2),\end{aligned} \tag{A6}$$

where the t_i's and the ϕ_i's that appear at the right of the ";" are random variables already chosen. Finally, at time t,

$$\begin{aligned}&\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k)\\ &\quad= U(\phi_k;t,t_k)U(\phi_{k-1};t_k,t_{k-1})\times\cdots U(\phi_1;t_2,t_1)\\ &\qquad\times U(\phi_0;t_1,0)\rho(0)U^{-1}(\phi_0;t_1,0)U^{-1}(\phi_1;t_2,t_1)\\ &\qquad\times\cdots\times U^{-1}(\phi_{k-1};t_k,t_{k-1})U^{-1}(\phi_k;t,t_k).\end{aligned} \tag{A7}$$

The probability [in the interval $(0,t)$] that k changes of ϕ have actually occurred at successive instants $t_1,t_2,\ldots,t_k$ and that a certain sequence of $\phi_1,\phi_2,\ldots,\phi_k$ [where $\phi_i = \phi(t_i)$] was realized between them is obviously equal to

$$\begin{aligned}&dP(t_1,t_2,\ldots,t_k;\phi_0,\phi_1,\phi_2,\ldots,\phi_k,t)\\ &\quad= \text{Prob}\{\phi(0)=\phi_0\}\times\text{Prob}\{\tau_1 = t_1-0\}\\ &\qquad\times\text{Prob}\{\phi(t_1)=\phi_1\}\times\text{Prob}\{\tau_2 = t_2-t_1\}\\ &\qquad\times\text{Prob}\{\phi(t_2)=\phi_2\}\times\text{Prob}\{\tau_3 = t_3-t_2\}\times\cdots\\ &\qquad\times\text{Prob}\{\phi(t_{k-1})=\phi_{k-1}\}\times\text{Prob}\{\tau_k = t_k-t_{k-1}\}\\ &\qquad\times\text{Prob}\{\phi(t_k)=\phi_k\}\times\text{Prob}\{\tau_{k+1} > t-t_k\},\end{aligned} \tag{A8}$$

where $t_0 = 0$. According to (A1), $\text{Prob}\{\tau_i = t_i - t_{i-1}\} = dQ(t_i - t_{i-1})$ for $i = 1,\ldots,k$, and from (A2) we have $\text{Prob}\{\phi(t_i) = \phi_i\} = dq(\phi)$ for $i = 0,\ldots,k$. If we impose that in the interval $[t_k,t]$ the $(k+1)$th jump does not occur, then $\tau_{k+1} > t - t_k$. Integrating over a range

$$\begin{aligned}\text{Prob}\{\tau_{k+1} > t-t_k\} &= \int_{t-t_k}^{\infty} dQ(\tau_{k+1})\\ &= \frac{1}{\tau_0}\int_{t-t_k}^{\infty} e^{-\tau_{k+1}/\tau_0}d\tau_{k+1}\\ &= [-e^{-\tau_{k+1}/\tau_0}]_{t-t_k}^{\infty} = e^{(t-t_k)/\tau_0},\end{aligned} \tag{A9}$$

then

$$\begin{aligned}&dP(t_1,\ldots,t_k,\phi_0,\ldots,\phi_k;t)\\ &\quad= dq(\phi_0)\frac{1}{\tau_0}e^{-(t_1-0)/\tau_0}d\tau_1 dq(\phi_1)\frac{1}{\tau_0}e^{-(t_2-t_1)/\tau_0}d\tau_2\times\cdots\\ &\qquad\times dq(\phi_{k-1})\frac{1}{\tau_0}e^{-(t_k-t_{k-1})/\tau_0}d\tau_k dq(\phi_k)e^{-(t-t_k)/\tau_0}\\ &\quad= \frac{1}{\tau_0^k}e^{-t/\tau_0}\left(\prod_{i=1}^{k} d\tau_i\right)\left(\prod_{i=1}^{k} dq(\phi_i)\right).\end{aligned} \tag{A10}$$

The sample space includes only the region where $\tau_i > 0$ with $i = 1,\ldots,k$ and $\tau_1 + \tau_2 + \cdots \tau_k < t$.

To change the integrals in the variables $\tau_1,\tau_2,\ldots,\tau_k$ to integrals in the variables $t_1,t_2,\ldots,t_k$, we will use the Jacobian matrix

$$\frac{\partial(\tau_1,\tau_2,\ldots,\tau_k)}{\partial(t_1,\ldots,t_k)} = \begin{pmatrix}\frac{\partial\tau_1}{\partial t_1} & \frac{\partial\tau_1}{\partial t_2} & \cdots & \frac{\partial\tau_1}{\partial t_k}\\ \frac{\partial\tau_2}{\partial t_1} & \frac{\partial\tau_2}{\partial t_2} & \cdots & \frac{\partial\tau_2}{\partial t_k}\\ \vdots & \vdots & \cdots & \vdots\\ \frac{\partial\tau_k}{\partial t_1} & \frac{\partial\tau_k}{\partial t_2} & \cdots & \frac{\partial\tau_k}{\partial t_k}\end{pmatrix} \tag{A11}$$

$$= \begin{pmatrix}1 & 0 & 0 & \cdots & 0 & 0\\ -1 & 1 & 0 & \cdots & \vdots & \vdots\\ 0 & -1 & 1 & \cdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 1 & 0\\ 0 & 0 & 0 & \cdots & -1 & 1\end{pmatrix}, \tag{A12}$$

as the determinant of a triangular matrix is the product of the diagonal elements,

$$\det\left[\frac{\partial(\tau_1,\tau_2,\ldots,\tau_k)}{\partial(t_1,\ldots,t_k)}\right] = 1, \tag{A13}$$

therefore

$$\begin{aligned}&dP(t_1,\ldots,t_k,\phi_0,\ldots,\phi_k;t)\\ &\quad= \frac{1}{\tau_0^k}e^{-t/\tau_0}\left(\prod_{i=1}^{k} dt_i\right)\left(\prod_{i=1}^{k} dq(\phi_i)\right).\end{aligned} \tag{A14}$$

In a similar form, the sample space includes only the region where $0 < t_1 < t_2 < \cdots < t_k < t$. Therefore, the average value of the matrix density at time t, assuming that the change in the phase ϕ occurs in instantaneous stochastic jumps, is given by

$$\begin{aligned}\bar{\rho}(t) = \sum_{k=0}^{\infty}\int_\Gamma\int_\Lambda &dP(t_1,\ldots,t_k,\phi_0,\ldots,\phi_k;t)\\ &\times\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k)\end{aligned} \tag{A15}$$

as $\Gamma = 0 < t_1 < t_2 < \cdots < t_k < t$ and $\Lambda = 0 \leqslant \phi_i < 2\pi$. We obtain that

$$\begin{aligned}\bar{\rho}(t) = e^{-t/\tau_0}\sum_{k=0}^{\infty}\frac{1}{\tau_0^k}\int_\Gamma\cdots\int\prod_{i=1}^{k}dt_i\\ \times\int_\Lambda\cdots\int\prod_{i=1}^{k}dq(\phi_i)\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k).\end{aligned} \tag{A16}$$

Note that the term with $k = 0$ [when ϕ does not change at all in the interval $(0,t)$] will not contain integrals with respect to time and thus is given by where the summation term with $k = 0$ is given by

$$\int dq(\phi_0)\rho(t;\phi_0). \tag{A17}$$

Using the expression (A16),

$$\bar{\rho}(t) = e^{-t/\tau_0}\sum_{k=0}^{\infty}\frac{1}{\tau_0^k}\int_0^{t_k}dt_k\int_0^{t_k}dt_{k-1}\cdots\int_0^{t_2}dt_1\int_0^{2\pi}d\phi_k \times\int_0^{2\pi}d\phi_{k-1}\cdots\int_0^{2\pi}d\phi_0\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k), \tag{A18}$$

and listing explicitly the first terms,

$$\begin{aligned}\bar{\rho}(t)e^{t/\tau_0} &= \int dq(\phi_0)\rho(t;\phi_0) + \frac{1}{\tau_0}\int_0^t dt_1\int dq(\phi_1)\\ &\quad\times\int dq(\phi_0)\rho(t;t_1,\phi_0,\phi_1) + \frac{1}{\tau_0^2}\int_0^t dt_2\int_0^{t_2}dt_1\\ &\quad\times\int dq(\phi_2)\int dq(\phi_1)\\ &\quad\times\int dq(\phi_0)\rho(t;t_1,t_2,\phi_0,\phi_1,\phi_2)+\cdots\\ &= I_0(t)+I_1(t)+I_2(t)+\cdots\end{aligned} \tag{A19}$$

with

$$I_0(t) = \int dq(\phi_0)\rho(t;\phi_0), \tag{A20}$$

$$I_1(t) = \frac{1}{\tau_0}\int_0^t dt_1\int dq(\phi_1)\int dq(\phi_0)\rho(t;t_1,\phi_0,\phi_1), \tag{A21}$$

$$I_2(t) = \frac{1}{\tau_0^2}\int_0^t dt_2\int_0^{t_2}dt_1\int dq(\phi_2) \times\int dq(\phi_1)\int dq(\phi_0)\rho(t;t_1,t_2,\phi_0,\phi_1,\phi_2). \tag{A22}$$

Now using the recurrence relation (A7), we can multiply both sides of Eq. (A20) from the left (right) by $U^1(\phi;\tau,t)$ $[U(\phi;\tau,t)]$, respectively, and also by $dq(\phi)dt/\tau_0$, and then integrate with respect to time from 0 to τ and eliminate the entire series. We can show that

$$\begin{aligned}&\frac{1}{\tau_0}\int_0^\tau\int dq(\phi)U(\phi;\tau,t)I_0(t)U^{-1}(\phi;\tau,t)\\ &\quad=\frac{1}{\tau}\int_0^\tau dt\int dq(\phi)\int dq(\phi_0)U(\phi;\tau,t)\rho(t;\phi_0)\\ &\quad\quad\times U^{-1}(\phi;\tau,t)\\ &\quad=\frac{1}{\tau}\int_0^\tau dt\int dq(\phi)\int dq(\phi_0)\rho(\tau;t,\phi_0,\phi) = I_1(\tau),\end{aligned} \tag{A23}$$

and similarly

$$\begin{aligned}&\frac{1}{\tau_0}\int_0^\tau dt\int dq(\phi)U(\phi;\tau,t)I_k(t)U^{-1}(\phi;\tau,t)\\ &\quad=\frac{1}{\tau_0^{k+1}}\int_0^\tau dt\int_0^t dt_k\int_0^{t_k}dt_{k-1}\cdots\int_0^{t_2}dt_1\\ &\quad\quad\times\int dq(\phi)\int dq(\phi_k)\times\cdots\times\int dq(\phi_0)U(\phi;\tau,t)\\ &\quad\quad\times\rho(t;t_1,\ldots,t_k,\phi_0,\ldots,\phi_k)U^{-1}(\phi;\tau,t)\\ &\quad=\frac{1}{\tau_0^{k+1}}\int_0^\tau dt\int_0^t dt_k\int_0^{t_k}dt_{k-1}\cdots\int_0^{t_2}dt_1\\ &\quad\quad\times\int dq(\phi)\int dq(\phi_k)\times\cdots\times\int dq(\phi_0)\\ &\quad\quad\times\rho(\tau;t_1,\ldots,t_k,t,\phi_0,\ldots,\phi_k,\phi) = I_{k+1}(\tau).\end{aligned} \tag{A24}$$

From the result above,

$$\begin{aligned}&\frac{1}{\tau_0}\int_0^\tau dt\int dq(\phi)U(\phi;\tau,t)\bar{\rho}(t)e^{t/\tau_0}U^{-1}(\phi;\tau,t)\\ &\quad=\frac{1}{\tau_0}\int_0^\tau dt\int dq(\phi)U(\phi;\tau,t)\sum_{k=0}^{\infty}I_k(t)U^{-1}(\phi;\tau,t)\\ &\quad=\sum_{k=0}^{\infty}I_{k+1}(\tau)=\sum_{k=0}^{\infty}I_k(\tau)-\int dq(\phi_{\phi_0})\rho(\tau;\phi_0)\\ &\quad=\bar{\rho}(\tau)e^{\tau/\tau_0}-\int dq(\phi_0)U(\phi_0;\tau,0)\rho(0)U^{-1}(\phi_0;\tau,0),\end{aligned} \tag{A25}$$

and consequently

$$\begin{aligned}\bar{\rho}(\tau)e^{\tau/\tau_0} &= \int dq(\phi_0)U(\phi_0;\tau,0)\rho(0)U^{-1}(\phi_0;\tau,0)\\ &\quad+\frac{1}{\tau_0}\int_0^\tau dte^{t/\tau_0}\int dq(\phi)U(\phi;\tau,t)\bar{\rho}(t)U^{-1}(\phi;\tau,t).\end{aligned} \tag{A26}$$

[1] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zweger, Rev. Mod. Phys. **59**, 1 (1987); A. O. Caldeira and A. J. Leggett, Phys. Rev. A **31**, 1059 (1985).
[2] A. Rivas, A. D. K. Plato, S. F. Huelga, and M. B. Plenio, New J. Phys. **12**, 113032 (2010).
[3] J. Jacobson, S. Pau, H. Cao, G. Björk, and Y. Yamamoto, Phys. Rev. A **51**, 2542 (1995).
[4] S. Pau, G. Björk, J. Jacobson, H. Cao, and Y. Yamamoto, Phys. Rev. B **51**, 14437 (1995).
[5] J. Bernu, S. Deléglise, C. Sayrin, S. Kuhr, I. Dotsenko, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **101**, 180402 (2008).
[6] A. R. Bosco de Magalhães, R. Rossi, and M. C. Nemes, Phys. Lett. A **375**, 1724 (2011).

[7] M. Albiez, R. Gati, J. Folling, S. Hunsmann, M. Cristiani, and M. K. Oberthaler, Phys. Rev. Lett. **95**, 010402 (2005).
[8] G. J. Milburn, J. Corney, E. M. Wright, and D. F. Walls, Phys. Rev. A **55**, 4318 (1997).
[9] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997).
[10] J. Ruostekoski and D. F. Walls, Phys. Rev. A **58**, R50 (1998).
[11] F. Meier and W. Zwerger, Phys. Rev. A **64**, 033610 (2001).
[12] G. Kalosakas and A. R. Bishop, Phys. Rev. A **65**, 043616 (2002).
[13] G. Kalosakas, A. R. Bishop, and V. M. Kenkre, Phys. Rev. A **68**, 023602 (2003).
[14] A. N. Salgueiro *et al.*, Eur. Phys. J. D **44**, 537 (2007).
[15] A. I. Burshtein and Yu S. Oseledchilk, Zh. Eksp. Teor. Fiz. **51**, 1071 (1966) [Sov. Phys. JETP **24**, 716 (1967)].
[16] A. Joshi, J. Mod. Opt. **42**, 2561 (1995).
[17] E. A. Ospina, Master's dissertation, UFMG, Brazil, 2009.
[18] W. H. Zurek, Phys. Rev. D **26**, 1862 (1982).
[19] Y. Yamamoto and R. E. Slusher, Phys. Today **46**, 66 (1993).
[20] C. Weisbuch, M. Nishioka, A. Ishikawa, and Y.Arakawa, Phys. Rev. Lett. **69**, 3314 (1992); R. Houdré, R. P. Stanley, U. Oesterle, M. Ilegems, and C. Weisbuch, Phys. Rev. B **49**, 16761 (1994).
[21] S. Schneider and G. J. Milburn, Phys. Rev. A **57**, 3748 (1998).
[22] S. Dyrting and G. J. Milburn, Quantum Semiclass. Opt. **8**, 541 (1996).
[23] D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **76**, 1796 (1996); **77**, 2346(E) (1996).
[24] S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, and F. Plastina, Phys. Rev. Lett. **100**, 090503 (2008).
[25] J. G. Oliveira, Jr., R. Rossi, and M. C. Nemes, Phys. Rev. A **78**, 044301 (2008).
[26] S. Pascazio and M. Namiki, Phys. Rev. A **50**, 4582 (1994).
[27] Y. Khodorkovsky, G. Kurizki, and A. Vardi, Phys. Rev. Lett. **100**, 220403 (2008).
[28] Y. Khodorkovsky, G. Kurizki, and A. Vardi, Phys. Rev. A **80**, 023609 (2009).