Bragg diffraction of a matter wave driven by a pulsed nonuniform magnetic field

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We have performed a principle-proof experiment of a magneto-optical diffraction (MOD) technique that requires no energy level splitting by a homogeneous magnetic field and a circularly polarized optical lattice, avoiding system errors in an interferometer based on the MOD. The principle for this MOD is that asynchronized switching of the quadrupole trap and the Ioffe trap in a quadrupole-Ioffe-configuration trap can generate a residual magnetic force to drive a Bose-Einstein condensate (BEC) to move. We have observed asymmetric atomic diffraction resulting from the asymmetric distribution of the Bloch eigenstates involved in the diffraction process when the condensate is driven by such a force, and matter-wave self-imaging due to coherent population oscillation of the dominantly occupied Bloch eigenstates. We have classified the mechanisms that lead to symmetric or asymmetric diffraction and found that our experiment presents a magnetic alternative to a moving optical lattice, with a great potential to achieve a very large momentum transfer ($>110\hbar k$) to a BEC using well-developed magnetic trapping techniques.

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Atomic/molecular matter waves [1,2] have played an important role in fundamental research and many practical applications, such as atomic clocks [3,4], gravitational-wave detection [5], gravito-inertial sensors [6,7], atom lithography [8,9], rotation sensing [10,11], detection of tiny effects of general relativity [12,13], measurement of atom surface interactions [14], generation of quantum correlated atom pairs [15], and dispersion manipulation [16]. Among these studies, coherent splitting of atomic beams is a key technique [1,17–19]. Matter-wave Bragg diffraction, analogous to diffraction of an optical beam by a periodic medium, has been intensively investigated for splitting a matter wave into a superposition of momentum states using an optical lattice [20–22] or a magnetic lattice [23–25]. High-momentum transfer splitters are essential for high-precision atom interferometers [26]. So, there is a continuous endeavor [18,27,28] to increase the momentum transfer to the Bose-Einstein condensate (BEC). Recently, momentum transfer of $102\hbar k_L$ from optical fields to a condensate has been successfully demonstrated [28].

A moving optical lattice [29–32], formed by two counterpropagating optical fields with unequal frequencies [33], has shown promise for matter-wave splitting, because the lattice velocity v_L opens a new degree of freedom for controlling the momentum transfer, i.e., the diffracted atoms are prepared to populate in $\pm v_L$ under the resonant Bragg scattering condition [34] in the reference frame where the lattice is stationary, provided that v_L equals to the single-photon recoil velocity $\hbar k_L/m$. More interestingly, a large lattice velocity is capable of realizing large angle beam splitting, as demonstrated by a recent experiment [20,35]. Furthermore, the approach can also realize asymmetric atomic diffraction (the split beams have unequal population) [34], which could be useful for controlling population ratios for the two split beams. In contrast, when an atomic gas at rest is probed by a static optical lattice, no asymmetric atomic diffraction could be induced in general. Recently, however, one experiment [36] shows that using two counterpropagating optical fields with equal frequency but unequal intensities allows asymmetric atomic diffraction for an atomic gas initially at rest. This counterintuitive result is explained later by the local field effect (LFE) [37], i.e., an asymmetric optical lattice, due to asymmetric scattering of the incident optical beams by the condensate, generates the asymmetric diffraction.

Currently, most of matter-wave diffraction experiments apply either an optical lattice or a magnetic lattice. Meanwhile, magneto-optical diffraction of atoms in a magnetic field by a circularly polarized optical standing wave [38] has also been investigated for static atomic clouds to achieve a large momentum transfer. In this approach, Zeeman splitting of energy levels in an external magnetic field is used such that the energy difference of the atoms and the considerably induced wavefront curvature could lead to a systematic phase error in an interferometer based on the splitter [17].

In this paper, we experimentally demonstrate a magneto-optical atomic diffraction, combining a quadrupole-Ioffe-configuration (QUIC) trap and an optical standing wave, which is not circularly polarized as in Ref. [38]. We observe an asymmetric atomic diffraction and matter wave self-imaging, when the quadrupole trap and the Ioffe trap are switched off asynchronously, generating a residual magnetic force and driving the condensate to move. Such an asymmetric atomic diffraction is due to the nonzero initial velocity of the condensate from the magnetic acceleration, rather than LFE. Our experiment shows the potential of a magnetic force in optical atomic diffraction. Magnetic driving of a BEC in this way can be an alternative to a moving optical lattice for achieving a momentum transfer to a BEC.

Our diffraction experiment is performed as follows. A cigar-shaped BEC of $N = 1 \times 10^5$ ^{87}Rb atoms in $|F = 2, m = 2\rangle$ state with a longitudinal Thomas-Fermi radii 40 μm and a transverse radii 4 μm is first prepared in an anisotropic QUIC trap with the axial frequency 20 Hz and the radial frequency

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220 Hz [39–41]. A one-dimensional optical standing wave with a wavelength 852 nm is incident onto the condensate along the long axis direction (x axis). In our experiment, the standing wave is formed with a retroreflected laser beam, well focused on the BEC to a waist of 110 μm. The incident optical field is a square light pulse with an intensity $I_1 = 1.03 \times 10^5$ mW/cm^2. But the reflected light I_2 is controlled by a tunable light attenuator, which is put in front of the reflection mirror. When the QUIC trap is switched off, the optical standing wave is not turned on immediately, but is delayed for a time ΔT for sufficiently exploiting the residual magnetic force. After switching off of the optical fields for 28 ms, an absorption image is taken.

The left panel of Fig. 1 shows the absorption images for different pulse durations and a fixed delay time $\Delta T = 400\mu$s with $I_2 = 0.8 I_1$. Asymmetric diffraction is one prominent feature of our experiment, and self-imaging of the matter wave occurs roughly at 25 μs, 53 μs and 77 μs. These experimental phenomena remind us of a recent experiment [36] dominated by LFE [37], which is also our original motivation to study this Bragg diffraction. However, simulation of our experiment with the LFE fails in fitting our experimental phenomena with an initial wave function $\psi(x,0) = C\exp[-x^2/(2w^2)]$ ($C = (\sqrt{\pi}w)^{-1/2}$) for a BEC released from a harmonic trap, where w is the full width at half maximum; because the huge detuning of about 10^{15} Hz in our experiment is six orders of magnitude larger than that in Refs. [36,37], such that the local refraction index, which is inversely proportional to the detuning, has little influence on the propagation of the optical

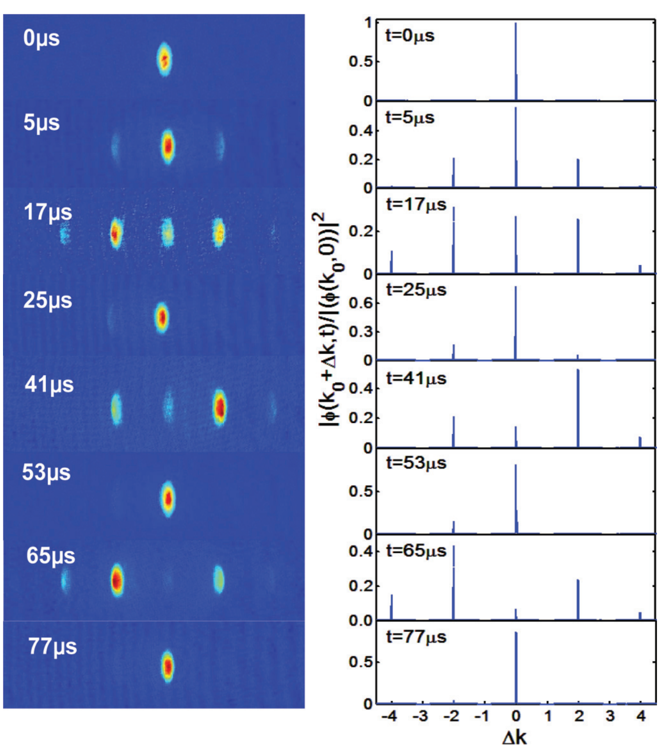

FIG. 1. (Color online) The left panel shows the absorption images of the diffracted BEC for different pulse durations. The right panel shows the related simulation results of the condensate momentum distribution ($\hbar\Delta k$ is the dimensionless net momentum each atom obtains in each diffraction order).

fields. Neglecting the local-field effect, the optical lattice potential is proportional to $\sqrt{I_1 I_2}$. Thus, for fixed $I_1 I_2$, the diffraction processing should be the same for different values of I_1 and I_2.

In our experiment, before the optical field is switched on, the condensate is accelerated during the switching off of the magnetic trap. The QUIC trap generated by the driven currents cannot be switched off instantly but with a relaxation, such that a residual magnetic force is produced to drive the BEC into motion. The force at position X is given by

$$F(X,T) = -\mu\left[i_I\frac{\partial}{\partial X}f_I(X)e^{-T/\tau_I} + i_Q\frac{\partial}{\partial X}f_Q(X)e^{-T/\tau_Q}\right], \quad (1)$$

where f_I (f_Q), $i_I(i_Q)$, and τ_I (τ_Q) are respectively the structure function, driven currents, and relaxation time of the Ioffe-type (quadrupole) trap [42], μ is the atomic magnetic moment. At $T = 0$, $F = 0$. When $\tau_I \neq \tau_Q$, a nonzero magnetic force $F(X,T)$ drives the BEC to move, experimentally confirmed by absorption images (not shown here) of the BEC during releasing process.

We now show the condensate motion is a mechanism beyond LFE for asymmetric diffraction. When the lattice with a large detuning is not distorted by the LFE [37], the matter-wave function $\psi(x,t)$ satisfies the Mathieu equation,

$$i\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\partial^2}{\partial x^2} + 2q\cos(2x)\right]\psi(x,t). \quad (2)$$

Here, dimensionless t, x, and q are respectively related to the real time T, position X, and dipole potential V_0, according to $t = \omega_r T$, $x = k_L X$, $q = V_0/(2\hbar\omega_r)$ with the wave vector of the pump fields k_L and the recoil frequency $\omega_r = \hbar k_L^2/(2m)$. Equation (2) has Bloch eigenfunctions corresponding to eigenenergy ϵ_{2N+s} as [43],

$$\varphi_{N,s} = \exp[i(2N+s)x]\sum_n c_{2n}^{2N+s}\exp(i2nx), \quad (3)$$

in which $-1 \leqslant s \leqslant 1$, and

$$qc_{2n-2}^{2N+s} + (s+2N+2n)^2 c_{2n}^{2N+s} + qc_{2n+2}^{2N+s} = \epsilon_{2N+s}c_{2n}^{2N+s}.$$

The Mathieu function has properties [43]: (i) $\epsilon_{2N+s} = \epsilon_{-2N-s}$; (ii) $c_{2n}^{2N+s} = c_{-2n}^{-2N-s}$.

In the momentum space, the wave function of BEC is $\phi(k,t) = \int_{-\infty}^{\infty}\psi(x,t)\exp(ikx)dx$. For a moving condensate with the initial wave function $\psi(x,0) = C\exp[-x^2/(2w^2)]\exp(-ik_0x)$, using the eigenvalues of the Mathieu equation, we obtain

$$\phi(k,t) = \sum_{\{N,m,n\geqslant -N-\frac{1+k}{2}\}}^{\{n\leqslant -N-\frac{1-k}{2}\}} 2\pi C \times c_{2m}^{-k-2n}c_{2n}^{-k-2n}e^{-\frac{w^2}{2}(k+2n-2m-k_0)^2}e^{-i\varepsilon_{-k-2n}t}.$$

For a large width w, using the steepest descent approximation, the atoms are populated in momentum space around $k = k_0 + 2j$ (j is an integer). Denoting $\phi_j^{k_0} \equiv \phi(k_0+2j,t)$, we have

$$\phi_j^{k_0} \approx \mathcal{N}e^{-i\varepsilon_{-k_0}t}\sum_N\sum_{n\geqslant -N-\frac{1+k}{2}}^{n\leqslant -N-\frac{1-k}{2}} c_{2K}^{-k_0-2K}c_{2K-2j}^{-k_0-2K}e^{-i\Delta\varepsilon_{-2K}t}, \quad (4)$$

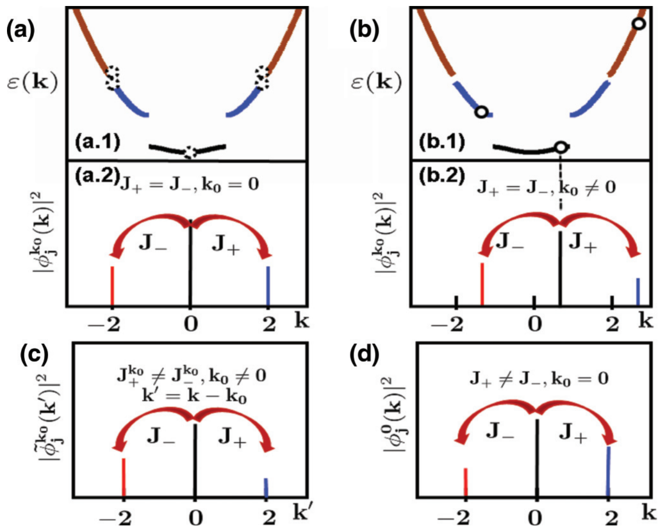

FIG. 2. (Color online) Different mechanisms for symmetric and asymmetric atomic diffraction. (a) and (b) are, respectively, for diffractions of a static and moving BEC in which the involved Bloch eigenstates are shown in circles in (a.1) and (b.1), $\epsilon(k)$ is the Bloch eigenenergy. $\phi_j^{\mathbf{k}_0}$ in (a.2) or (b.2) is the wave function of the diffracted BEC in momentum space. (c) Diffraction of atoms in the frame of the moving condensate, with $\tilde{\phi}_j^{\mathbf{k}_0}$ the atomic wave function in this frame. (d) Atomic diffraction with the local field effect, with ϕ_j^0 the wave function of a stationary BEC. k is dimensionless vector in the reciprocal space.

in which $K = j + n$, $\Delta\varepsilon_{-2K} = \varepsilon_{-k_0-2K} - \varepsilon_{-k_0}$, and $\mathcal{N}$ is a normalization factor (in the followed part, all irrelevant constants are absorbed in this factor).

Equation (4) shows that eigenstates corresponding to eigenenergy ε_{-k_0-2K} have been involved in diffraction process. Figures 2(a.1) and 2(b.1) schematically show the involved eigenstates respectively for $k_0 = 0$ and $k_0 \neq 0$ in an extended band structure of the Mathieu equation. When $k_0 = 0$ [Fig. 2(a.1)], the involved eigenstates shown in dotted circles are symmetrically distributed, thus, we have symmetric diffraction, i.e., $\phi_j^{k_0} = \phi_{-j}^{k_0}$. When $k_0 \neq 0$ [Fig. 2(b.1)], the involved eigenstates shown in solid circles are not symmetrically distributed, such that $\phi_j^{k_0}$ does not equal to $\phi_{-j}^{k_0}$ in general. We numerically calculate the momentum spectrum using the experimental parameters, $k_0 = 0.32$ and $V_0 = 4.45\hbar\omega_r$. k_0 is obtained by measuring the motion of the condensate peak. Actually, assuming that the motion is driven by the residual magnetic force of Eq. (1) with the experimental parameters ($\tau_I = 66$ μs, $\tau_Q = 87$ μs, $i_0 = 21$A), the simulated value of $k_0 \cong 0.3$ is consistent with the measured value. The numerical results are shown at the right panel of Fig. 1. Comparison of the left and right panels of Fig. 1 shows a good agreement of the theory with the experimental results, indicating that the BEC indeed obtains a nonzero velocity due to asynchronized switching off of the quadrupole trap and Ioffe trap.

Finally, we present a classification of all observed symmetric or asymmetric atomic diffraction phenomena. When the initial matter wave has a narrow momentum distribution, no matter whether the local field effect is involved or not, $\phi_j^{k_0}$ satisfies

$$i\frac{\partial}{\partial t}\phi_j^{k_0} = (k_0 + 2j)^2\phi_j^{k_0} + J_-\phi_{j-2}^{k_0} + J_+\phi_{j+2}^{k_0}, \qquad (5)$$

where $J_\pm$ is transition rate from the momentum component $\phi_j^{k_0}$ to $\phi_{j\pm 2m}^{k_0}$. When the local field effect is negligibly small, $J_- = J_+ = q$ is not dependent on k_0. However, the symmetric hopping cannot guarantee symmetric diffraction. When $k_0 = 0$, there is symmetric distribution [Fig. 2(a.2)]. When $k_0 \neq 0$, the jth-order diffraction component is related to the momentum $k_0 + 2j$ [Fig. 2(b.2)], and the matter-wave diffraction is asymmetric.

To see how the asymmetric diffraction happens, we turn to the reference frame moving at the speed k_0, in which the diffraction equation of the wave function $\tilde{\phi}_j^{k_0}$ for the jth-order diffracted atoms is $i\frac{\partial}{\partial t}\tilde{\phi}_j^{k_0} = 4j^2\tilde{\phi}_j^{k_0} + J_-^{k_0}\tilde{\phi}_{j-2}^{k_0} + J_+^{k_0}\tilde{\phi}_{j+2}^{k_0}$, with $J_\pm^{k0} = qe^{\pm i2k_0 t}$. In this moving frame, the jth-order diffraction corresponds to $2j$ momentum as in the static frame; however, $J_+^{k_0} \neq J_-^{k_0}$, leading to asymmetric diffraction [Fig. 2(c)]. This mechanism is different from the mechanism by LFE with the incident and counterpropagating lights of unequal intensities, as shown in Fig. 2(d), where J_- and J_+ are time dependent and are not equal [$J_-(t) \neq J_+(t)$]. In the LFE-dominant case, the condensate leads to unequal scattering of the incident counterpropagating optical fields with unequal intensities, such that the spatial inversion symmetry of the macroscopic wave function for the condensate is induced, and consequently the asymmetric momentum transfer occurs.

We have to emphasize that the asymmetry of the atomic diffraction is induced by the initial acceleration of the BEC. The asymmetric atomic diffraction mechanism in Figs. 2(b)–2(c) also applies to that with a moving optical lattice [44]. Thus, our experiment presents a magnetic alternative to the latter in transferring momentum to the condensate. Driving a static BEC to a very high momentum with a moving optical lattice may require a strong laser intensity [27]. Our experiment shows that we also could exploit the well-established magnetic trap techniques to achieve a very high momentum transfer to a BEC, by reducing the relaxation time of the Ioffe trap and increasing the quadrupole current. We have performed a theoretical calculation of the acceleration got a BEC with QUIC trap parameters $i_Q = i_I = 70$ A, $\tau_Q = 5$ ms, and $\tau_I = 50$ μs. Figure 3(a) displays the spatial distribution of the residue magnetic field at different moments after the QUIC trap currents are switched off. When the Ioffe trap is rapidly switched off ($\tau_I \ll \tau_Q$), the gradient magnetic field forming the quadrupole trap coherently drives the BEC. Figure 3(b) shows the related condensate velocity. With this specially designed QUIC trap, the momentum of the condensate can reach up to $110\hbar k_L$ within 3 ms. With our experimental setup, we have been able to increase the momentum of the BEC to more than $2\hbar k$ by using a current of 24 A in QUIC trap. For further increasing the momentum of the condensate, a better water-cooling system is needed. However, the numerical simulation based on our current experimental configuration indicates that the condensate velocity can be accelerated to more than $100\hbar k$ when a current of 70 A is given. Such big current has already been achieved in some experiments [45]. Our work could pave a way for studying the diffraction

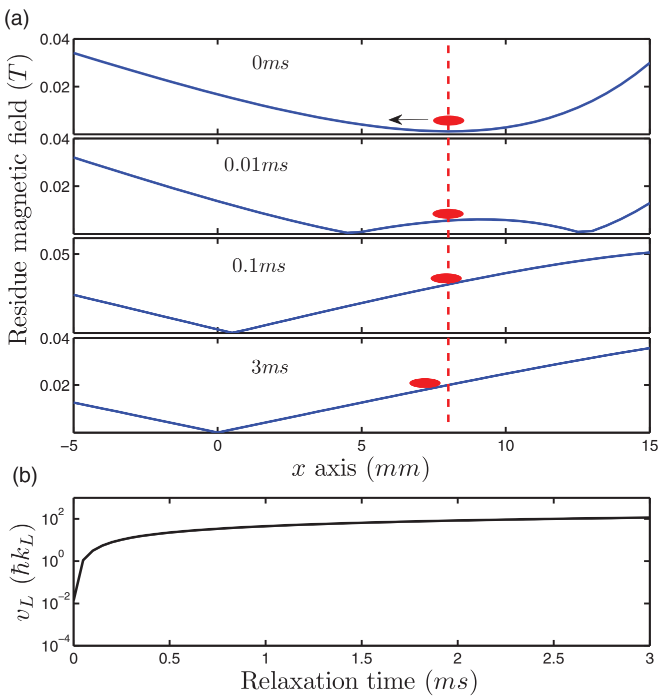

FIG. 3. (Color online) (a) Spatial distribution of the magnetic field within the QUIC trap at the different moments after the QUIC trap is switched off. The condensate is plotted to display the coherent driving. (b) The time evolution of the condensate velocity due to the residue magnetic field after the QUIC trap is switched off.

theory [34] of a fast-moving BEC to achieve a large-momentum transfer atom interferometer.

Our theory also predicts that when the strength of one of the pump field is lowered, i.e., the optical lattice is shallower, the asymmetry of the atomic diffraction would be enhanced. This prediction can be tracked by expanding the light-atom coupling Hamiltonian with the shifted plane-wave modes. In this presentation, the effective Hamiltonian reads

$$H = \sum_i E_i |\phi_{k_0}^i\rangle\langle\phi_{k_0}^i| + J \sum_i \{|\phi_i^{k_0}\rangle\langle\phi_{i+1}^{k_0}| + |\phi_{i+1}^{k_0}\rangle\langle\phi_i^{k_0}|\},$$

where $E_i = \frac{(\hbar k_0 + 2i\hbar k)^2}{2M}$ is the energy level of the free atomic system without lattice. Due to the acceleration of the condensate, the single-particle energy of the quasi mode is asymmetric about the central mode, thus the effective detuning of off diagonal Rabi frequency is asymmetric, resulting an asymmetric atomic Bragg diffraction patterns. However, in the large coupling limit, where the kinetic energy of the atoms is vanishingly small compared to the light-atom coupling, the free part of the single-particle energy can be neglected, rendering the Hamiltonian taking a symmetric form. Therefore, the Rabi oscillations toward the forward and backward directions are nearly identical, leading to more symmetric matter-wave diffractions in the time domain. While in the other limit where the lattice is shallower, the asymmetry of the atomic diffraction would be more prominent.

Equation (4) has also been used to explain the matter wave self-imaging as atomic center-of-mass motion induced interference [36,37,46]. This does not present an analytical result for the self-imaging time. However, Eq. (4) could be used to give a good estimation of the self-imaging time. Not all eigenstates of eigenenergy ε_{-k_0-2K} are essentially involved, so we have a cutoff $N_{\max}$ for N. Thus when $t = T_{\rm si}$, where $T_{\rm si}$ is the least common multiple of the periods of all essentially involved eigenstates, $\phi_0^{k_0}(T_{\rm si}) \approx \phi_0^{k_0}(0)$, i.e., matter-wave self-imaging occurs. For example, in Fig. 1,

$$\left|\phi_{j=0}^{k_0}(t)\right|^2 \approx \mathcal{N}\left|\left(c_0^{k_0}\right)^2 + \left(c_2^{-2-k_0}\right)^2 e^{-i\Delta\varepsilon_{-2}t} + \left(c_2^{-2+k_0}\right)^2 e^{-i\Delta\varepsilon_{-2}t}\right|^2$$

in which $(c_0^{k_0})^2 = 0.588$, $(c_2^{-2-k_0})^2 = 0.335$, $(c_2^{-2+k_0})^2 = 0.052$, $\Delta\varepsilon_2 = 7.337$, and $\Delta\varepsilon_{-2} = 12.419$. Since $(c_2^{-2+k_0})^2 \ll (c_2^{-2-k_0})^2$, $\Delta\varepsilon_{-2}$ is the dominant frequency. The self-imaging time $T_{\rm si}$ is roughly given as $T_{\rm si} \approx 2n\pi/(\Delta\varepsilon_{-2}\omega_r) = 25.5n\ \mu$s ($n = 1,2,3,\ldots$), which agrees well with the experimental values. Thus, the matter-wave self-imaging is a kind of coherent-population oscillation between two Bloch states ϵ_{-k_0} and ϵ_{-k_0-2}.

In summary, we have performed experimental study of diffraction of a BEC released from a QUIC trap by an optical standing wave and observed asymmetric diffraction and matter-wave self-imaging. In contrast with Refs. [36,37], the lattice is not distorted in our experiment. Thus, the experimental phenomena is induced by a mechanism beyond the local field effect. We find that the BEC obtains a velocity due to a residual magnetic force during the asynchronized switching off of the quadrupole trap and the Ioffe trap before the optical lattice is switched on. The initial velocity leads to asymmetric distribution of the involved Bloch eigenstates in momentum space, such that asymmetric diffraction occurs. The matter-wave self-imaging is analytically explained as a coherent-population oscillation between two Bloch eigenstates. Finally, we have presented a clarification of the mechanisms that leads to symmetric or asymmetric diffraction.

Compared to other diffraction schemes using magneto-optical potential [38], our experiment using atoms in the ground state applies no circularly polarized optical lattice, thus the magneto-optical diffraction technique can avoid the phase error in an interferometer due to energy difference of atoms at different energy levels [17]. Moreover, in our approach, the magnetic acceleration and optical diffraction is separated. Therefore, this approach is free from the nonuniformity and fluctuation of a magnetic field and the corresponding energy splitting due to Zeeman effect. In the metrology experiments with lattice-based matter-wave accelerations, the unit of momentum transfer is $2\hbar k$ [28]. This method has been proved to be able to achieve a very large momentum transfer. Our approach is an alternative method based on a gradient magnetic field for accelerating the atoms. Furthermore, in Ref. [28] the wavefront distortions of light pulse broadened the momentum and achieved a contrast of 18% with $102\hbar k$ beam splitters. By using a gradient magnetic field to accelerate atoms, the achieved momentum distribution is as narrow as that of the original condensate, which leads to a high contrast. In our experiment, the current in the magnetic coils can be maintained with stability on the order of 10^{-4}, and the length of ΔT can be controlled with precision on the order of several tens of nanoseconds. Then by fixing the timing sequence of our experiment, the momentum that transferred to condensate can be controlled and distinguished with precision $0.01\hbar k_L$ according to the absorption images. Therefore, our method shows potential in the metrology field.

To achieve a high momentum transfer, it is hard to avoid asymmetric momentum splitting of a condensate through diffraction [47]. This asymmetric diffraction could be used to develop intensity-imbalanced matter-wave interferometers, analogous to intensity-imbalanced optical interferometers which have been used for monitoring the beam size in a particle accelerator [48], or for reducing back action in a two-path interferometer [49]. It is worthy of exploiting the asymmetric diffraction for precision measurement with matter waves in the future. Finally, the asymmetric splitting of matter waves could be used to study symmetry-broken spontaneous four-wave mixing with matter waves [50] to generate directional correlated atom pairs.

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