Investigation of atomic radiative recombination processes by the Bohmian-mechanics method

Yang Song, Su-Yu Li, Xue-Shen Liu, Fu-Ming Guo,* and Yu-Jun Yang†
Institute of Atomic and Molecular Physics, Jilin University, Changchun 130012, China
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The light-emission process by the recombination of an ionized electron with its parent ion is investigated by the Bohmian trajectory scheme, and the efficiency of light emission calculated by 20 Bohmian trajectories is qualitatively identical to that obtained by numerically solving the time-dependent Schrödinger equation. It is found that the efficiency of light emission by recombination is determined by the oscillation amplitude of the Bohmian particle's Coulomb acceleration. Along with the decrease of the energy of the incident Bohmian particle, the deepening of the ion's potential well, and the increase of the product factor of the bound-state and continuum-state populations, the oscillation amplitude of the particle's Coulomb acceleration increases.

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I. INTRODUCTION

With the rapid development of the laser technique, the highly intense coherent light source whose duration is several optical cycles and intensity is much higher than one atom unit (3.51×10^{16} W/cm^2) has already been obtained in experiment [1–3]. Using the intense ultrashort laser pulse to interact with the atom, molecule, cluster, and crystal, one can observe the generation of high-order harmonic whose frequency is an integral multiple of that of the incident laser pulse [4–10]. Additionally, we can obtain the small and flexible light source ranging from extreme ultraviolet to soft x ray by utilizing the high-order harmonic generation (HHG), image the molecular structure via the the molecular harmonics [11–17], and generate an isolated attosecond pulse (IAP) which can be used to detect and control the ultrafast motion of an electron by making use of the supercontinuum which exists in the harmonics [18–20].

However, the IAP has not been directly applied to the pump-probe experiment so far, due to the fact that the emission intensity of the IAP is still too low. For this reason, how to improve the emission efficiency of IAP becomes the focus of the research on the strong-field physics. The single atom response and the phase matching between the harmonics generated by different atoms are the main two processes included in the research on the HHG. There are mainly two schemes to improve HHG efficiency: increasing the number of atoms which generate HHG and enhancing the degree of phase matching between the harmonics from different atoms.

Single atom response is the basis of the HHG, and the mechanism of harmonic generation through single atom can be well explained by the three step model [21]: first, the atom is tunnel ionized in the intense laser field; then the ionized electron moves away from the parent ion under the action of the laser pulse, and it is accelerated reversely and driven back to its parent ion when the direction of the laser electric field changes; and finally the ionized electron recombines with its parent ion, emitting high-order harmonics. During this process, the influence of the efficiency of recombination between the ionized electron and its parent ion on the intensity of HHG is of great importance. In order to improve the emission efficiency of HHG, we should first figure out the factors that affect the efficiency of light emission by recombination, then optimize these factors to improve the emission efficiency on the single atom scale, and finally realize the optimization of HHG efficiency in combination with the propagation effect.

In theory, the HHG process is investigated by numerically solving the time-dependent Schrödinger equation (TDSE) [4,22,23]. This scheme, however, is difficult to use to extract the required physical information. In order to increase the transparency of the physics, the Bohmian mechanics (BM) scheme (also called the quantum trajectory method) is therefore adopted to investigate the factors that affect the efficiency of light emission by recombination in this paper. BM is another interpretation of quantum mechanics: the probability density (the squared modulus of the wave function of the system) is viewed as fluid, and its motion can be described by the quantum trajectories which are guided by the wave function. This scheme has been used to investigate the interaction between the intense laser pulses and atoms as well as molecules [24–33], such as the molecular ionization [26], the atomic above threshold ionization [32], the atomic dynamic stabilization [33], and the HHG process [31].

In general, there are two schemes to investigate the interaction between the intense laser and the atoms as well as molecules by BM.

(1) First, numerically solve the TDSE, then calculate the velocity of the classical particles according to the wave function, and finally calculate the positions of the particles at the next moment in combination with the initial positions of the particles [25,26].

(2) First, express the Schrödinger equation that the complex wave function follows as the quantum hydromechanics equation, which is expressed by the modulus and argument of the wave function; effective arithmetic needs to be adopted to solve the derivative function and node problems, directly realizing the simulation for the quantum process [27].

The first scheme is based on the numerical solution of the TDSE, and the corresponding computational quantity increases greatly, but at the same time the accurate motion of the Bohmian particle (BP) can be obtained, and therefore it is usually used to investigate the mechanism of the physical process. As the mechanism that affects the efficiency of light

*guofm@jlu.edu.cn
†yangyj@jlu.edu.cn

emission by recombination concerns us in this paper, we adopt the first scheme.

In this paper, we first numerically solve the TDSE, then calculate the motion of BPs on this basis, and finally obtain the Coulomb acceleration of the particles. According to the information about Coulomb acceleration, we calculate the spectrum of light emission by the recombination of the ionized electron with its parent ion. Adjusting the kinetic energy, the probability of the ionized electron, and the depth of the ion's potential, their influence on the oscillation amplitude of Coulomb acceleration as well as the efficiency of light emission by recombination can be investigated.

II. THEORETICAL METHODS

In order to analyze the factors that affect the efficiency of light emission by recombination, we simplified the physical model and used the wave packet of the free electron in the field free case to collide with the bound-wave packet in the nuclear zone, so as to simulate the light emission by the recombination of the ionized electron with its parent ion under the action of the laser field.

In the absence of an external field, the TDSE is given by (atomic units are used throughout, unless otherwise stated)

$$i\frac{\partial}{\partial t}\psi(x,t)=\left[-\frac{\partial^2}{2\partial x^2}+V_A(x)\right]\psi(x,t), \tag{1}$$

where $V_A(x)$ is the atomic potential function; the soft Coulomb potential $V_A(x)=-q/\sqrt{x^2+\alpha}$ is adopted, whose parameters are $\alpha=0.114$ and $q=0.6338$; and the corresponding energy of the ground state is $E_0=-0.9$, which corresponds to the helium atom. Here, the initial state is chosen as

$$\psi(x,0)=c_0\phi_0(x)+c_1\phi_1(x), \tag{2}$$

where $\phi_0(x)$ refers to the ground-state wave function of the model helium atom in the field free case, and $\phi_1(x)=(2\pi a^2)^{-1/4}e^{-(\frac{x-x_0}{2a})^2}e^{ip_0(x-x_0)}$ is the Gaussian free electron wave function whose width, central position, and central momentum are a, x_0, and p_0, respectively. c_0 and c_1 account for the probability amplitudes of the electron in the $\phi_0(x)$ and $\phi_1(x)$ states, which follow the expression $|c_0|^2+|c_1|^2=1$.

Although we are dealing with the wave function of a single electron, we will speak loosely of multiple BPs, each associated with a different Bohmian trajectory. In the following, the initial spatial positions of a certain amount of BPs are given at random by sampling the electronic wave function at an initial instant through the acceptance-rejection sampling method [34], and the spatial distribution of these particles is in accord with the probability density distribution function of the initial-state electron (i.e., the square of the modulus of the wave function); more BPs are distributed across the position which has the larger probability density. Subsequently, the wave function during the time-dependent evolution follows Eq. (1), and the BPs follow Eqs. (3) and (4) [29–31]:

$$v^k(t)=\mathrm{Im}\left[\frac{1}{\psi(x,t)}\frac{\partial}{\partial x}\psi(x,t)|_{x=x^k(t)}\right]\quad(k=1,2,\ldots,M), \tag{3}$$

$$x^k(t)=x^k(0)+\int_0^t v^k(t')dt'\quad(k=1,2,\ldots,M), \tag{4}$$

where $k=1,\ldots,M$ is the serial number of BP, and $v^k(t)$ and $x^k(t)$ refer to the velocity and position of the kth BP, respectively.

According to BM, the spatial distribution of the BP corresponds to the probability density of the electron, and the trajectory of the BP denotes the motion behavior of the electron. As a result, the relationship between the trajectory of the BP and the wave function of the electron is given by $|\psi(x,t)|^2=\lim_{M\to\infty}\frac{1}{M}\sum_{k=1}^{M}\delta(x-x^k(t))$ [35]. Therefore, the mean value of the electronic Coulomb acceleration is given by

$$\begin{aligned}\bar{a}(t)&=\langle\psi(x,t)|-\frac{\partial V_A(x)}{\partial x}|\psi(x,t)\rangle\\&=\int_{-\infty}^{+\infty}\lim_{M\to\infty}\frac{1}{M}\sum_{k=1}^{M}\delta(x-x^k(t))\left(-\frac{\partial V_A(x)}{\partial x}\right)dx\\&=\lim_{M\to\infty}\sum_{k=1}^{M}\frac{1}{M}\left(-\frac{\partial V_A(x)}{\partial x}\right)\bigg|_{x=x^k(t)},\end{aligned} \tag{5}$$

where $a^k(t)=(-\frac{\partial V_A(x)}{\partial x})|_{x=x^k(t)}$ accounts for the Coulomb acceleration of the kth BP. Carrying out the Fourier transform of the mean value of the BP's Coulomb acceleration, the emission spectrum can be obtained. As more particles are adopted in the calculation, the result of the statistical average will be much closer to that obtained by numerically solving the TDSE [31]. Next, we will briefly explain why we use the Coulomb acceleration at the position of the BP to describe the light emission of each particle.

Once the velocity of a particle at any moment [Eq. (3)] is known, it is easy to figure out its acceleration at that moment:

$$a^k(t)=m\frac{d}{dt}v^k(t)=\left[-\frac{\partial}{\partial x}[V_A(x)+Q(x,t)]\right]_{x=x^k(t)}. \tag{6}$$

Moreover, each particle follows the Bohm-Newton equation [35], and $V_A(x)$ and $Q(x,t)$ on the right side of Eq. (6) are the Coulomb potential and the quantum potential, respectively [35].

Consequently, at some instant, the mean acceleration of all particles is equal to

$$\begin{aligned}\bar{a}'(t)&=\lim_{M\to\infty}\frac{1}{M}\sum_{k=1}^{M}m\frac{d}{dt}v^k(t)\\&=\lim_{M\to\infty}\frac{1}{M}\sum_{k=1}^{M}\left(-\frac{\partial V_A(x)}{\partial x}\bigg|_{x=x^k(t)}\right)\\&\quad+\lim_{M\to\infty}\frac{1}{M}\sum_{k=1}^{M}\left(-\frac{\partial Q(x,t)}{\partial x}\bigg|_{x=x^k(t)}\right).\end{aligned} \tag{7}$$

It is found that the first term on the right side of the above expression is the result of the TDSE, i.e., Eq. (5), and the second term refers to the mean value of quantum force, which is given by

$$\lim_{M\to\infty}\frac{1}{M}\sum_{k=1}^{M}\left(-\frac{\partial Q(x,t)}{\partial x}\bigg|_{x=x^k(t)}\right)$$
$$= -\langle\nabla Q\rangle = \int_{-\infty}^{+\infty} R^2\nabla(\nabla^2 R/R)dx$$
$$= -2\int_{-\infty}^{+\infty}(\nabla^2 R)(\nabla R)dx$$
$$= 2\int_{-\infty}^{+\infty}(\nabla^2 R)(\nabla R)dx = 0, \quad (8)$$

where R is the amplitude of the wave function, which is real.

Through the above calculation, we found that the mean value of the quantum force is zero [36]; that is to say, the results obtained from Eqs. (5) and (7) are the same $[\bar{a}'(t) = \bar{a}(t)]$. For this reason, we can neglect the contribution of quantum force and only consider the Coulomb force.

III. RESULTS AND DISCUSSIONS

In order to facilitate the analysis of the light-emission process by recombination of the electron through the Bohmian trajectory (BT), we randomly select 20 BPs. Additionally, since these BPs are sampled from the probability density function of the superposition state wave function at the initial instant, some of them come from the wave packet inside the nuclear region, and the others come from a wave packet outside the nuclear region. The reason why we select 20 particles to describe the process of recombination is that, along with the increase of the number of BPs, the spectrum obtained from the statistical average of the BPs' Coulomb acceleration [Eq. (5)] is gradually close to that calculated from the TDSE, as is shown in Fig. 1. Where the central kinetic energy of the electronic wave packet is $\frac{p_0^2}{2} = 20$, the width of the wave packet is $\alpha = 30$, and the initial population of the ground state and continuum state is $|c_0|^2 = 0.8$ and $|c_1|^2 = 0.2$, respectively. It can be seen from Fig. 1 that the main parts of the spectrum (around the frequency 20.9) calculated through the 20 BPs

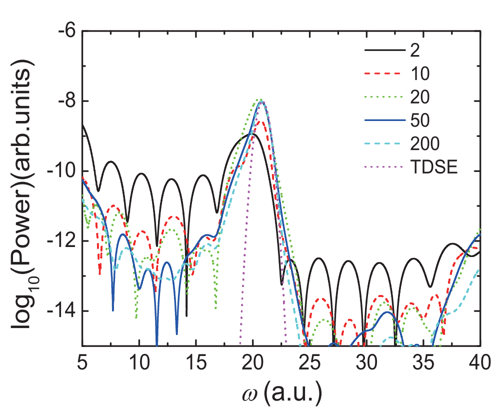

FIG. 1. (Color online) Spectra calculated from the mean Coulomb acceleration of BPs whose number is different and that obtained from the TDSE.

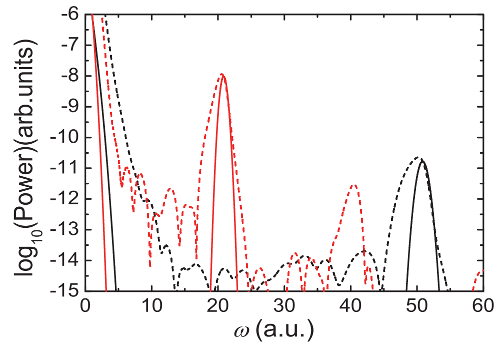

FIG. 2. (Color online) Harmonic spectra calculated from the TDSE (solid) and 20 BTs (dashed) when the kinetic energies of the continuum-state wave packet are 50 a.u (black) and 20 a.u. (red), respectively.

are roughly identical to the result obtained from the TDSE; therefore, it is assumed that the main dynamical behaviors during the light emission by recombination can be basically reflected by the 20 BPs. Moreover, it is convenient to analyze the detailed behavior of each BT, for the number of BPs and computational quantity is small.

By analyzing the dynamical behaviors of the 20 BPs, we find, from the perspective of BM, that the emission spectrum during the recombination of the ionized electron with its parent is determined by high-frequency oscillation of the BP in the nuclear zone of the parent ion, and the oscillation behavior of the BP is affected by three factors: the kinetic energy of the ionized electron, the potential of the parent ion that the electron feels, and the product of the population of the electron in the continuum state and that of the parent ion in the ground state.

In the following, we first discuss the law of the change of the emission spectrum due to the variation of the kinetic energy of the recombined ionized electron. Figure 2 shows the harmonic spectra during the recombination of ionized electrons whose respective central kinetic energies are 50 and 20 a.u. with the parent ion (the population in the ground state and continuum state is $|c_0|^2 = 0.8$ and $|c_1|^2 = 0.2$ in both cases), where the solid black and dashed black curves are the respective results calculated from the TDSE and the 20 BPs when the kinetic energy is 50 a.u., while the solid red and dashed red curves are results calculated from the TDSE and the 20 BPs when the kinetic energy is 20 a.u., respectively. We can clearly see from the figure that the main peaks of light emission are located at 50.9 and 20.9 a.u., which correspond to the sum of the kinetic energy of the recombined electron and ionization potential. It should be noted that the intensity of the spectrum at 20.9 a.u. is much higher than that at 50.9 a.u. Moreover, the spectrum calculated from the mean Coulomb acceleration of 20 particles is roughly identical to the result obtained from the TDSE; therefore, it is assumed that the main dynamical characteristics during the light emission by recombination can be reflected by the 20 particles. To obtain the simpler and more intuitive picture of the mechanism that the kinetic energy of the recombined electron affects the emission spectrum, we show in Fig. 3 the evolution of the Coulomb acceleration of the 20 particles with time. Figures 3(a) and 3(b) show the Coulomb acceleration of the corresponding BPs, where the kinetic energies of the recombined electrons are 50 and 20 a.u.,

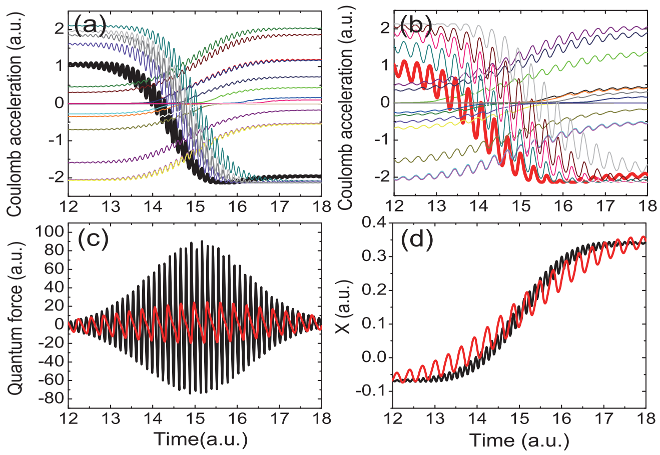

FIG. 3. (Color online) Time evolution of the Coulomb acceleration of the 20 particles as the kinetic energies of the continuum-state wave packet are (a) 50 a.u. and (b) 20 a.u., respectively, where the bold black and red curves are the Coulomb accelerations of the BPs whose initial positions are basically the same. (c) and (d) Time evolution of quantum forces and trajectories of the BPs shown by the bold curves in (a) and (b), where the black and red curves correspond to the cases in which the kinetic energies $\frac{p_0^2}{2}$ of the continuum-state wave packet are 50 and 20 a.u., respectively.

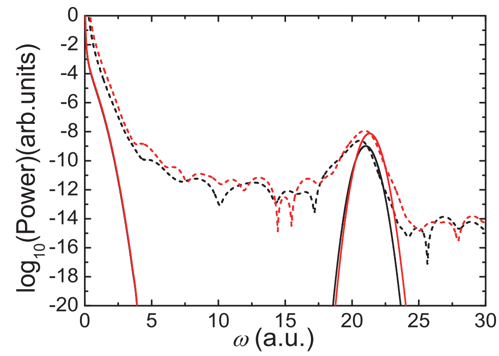

FIG. 4. (Color online) Harmonic spectra during the recombination of the electron with the initial states of different atomic potentials, where the dashed black and dashed red curves refer to the spectra calculated from 20 BPs as the soft Coulomb potentials whose parameters are $a = 0.3$ and 0.17, respectively, and the solid curves are the corresponding results calculated from the TDSE.

respectively. It is found that when the kinetic energy of the continuum state is 50 a.u. the Coulomb acceleration of every trajectory oscillates at high frequency and the corresponding frequency is about 50.9 a.u., while when the kinetic energy of the continuum state is 20 a.u. the oscillation frequency of the Coulomb acceleration of the particles is much lower than the former, which is about 20.9 a.u. In addition, the oscillation amplitudes of the Coulomb acceleration of the former BPs are generally lower than those of the latter ones. Therefore, from the perspective of the BM, we can draw the conclusion that the light emission by the recombination of the ionized electron with its parent ion is caused by the high-frequency oscillation of the BPs in the nuclear zone, and the frequency and intensity of the light emission are determined by the frequency and amplitude of the oscillation of the BPs' Coulomb acceleration, respectively. As the kinetic energy of the recombined electron decreases, the oscillation frequency of the BPs' Coulomb acceleration decreases and the oscillation amplitude of it increases, which induces the change of spectra shown in Fig. 2.

In order to make out the cause of the fact that as the kinetic energy is larger the oscillation amplitude of the BP's Coulomb acceleration will be smaller, and the oscillation frequency of it will be higher, we compare and analyze two trajectories whose initial positions are basically the same (selected from the 20 BTs in the cases in which the kinetic energies of the continuum-state wave packet are 50 and 20 a.u.), and we present the time evolutions of their corresponding quantum forces, which are shown by the dashed black and red curves in Fig. 3(c). Additionally, Fig. 3(d) shows the corresponding trajectories of the two BPs, where the black and red curves refer to the result of the cases in which the kinetic energies of the continuum-state wave packet are 50 and 20 a.u., respectively. It can be seen from Fig. 3(c) that the BP will bear a quantum force which oscillates at high frequency, and this force will make the particle oscillate at the same frequency; as a result, the oscillation of the particle in the space makes the Coulomb acceleration oscillate at the same frequency. It is worth noting that, when the kinetic energy of the wave packet is 50 a.u., though the quantum force the particle feels is larger than that in the case in which the kinetic energy is 20 a.u., the oscillation frequency of the quantum force felt by the former is larger; as a result, under the action of the higher-frequency quantum force, the quiver radius of the BP will be smaller, leading to the lower oscillation amplitude of the BP's Coulomb acceleration. Therefore, the light emission has a higher frequency and lower intensity as the kinetic energy of the wave packet is larger.

Through the above analysis, we know that light emission by the recombination of the ionized electron with its parent ion can be attributed to the high-frequency oscillation of the BP in the vicinity of the nuclear zone. In the following, directly starting from the BT, we investigate the difference between the light emission during the recombination caused by different atomic potentials. We present in Fig. 4 the harmonic spectra during the recombination of the electron in the continuum state whose central kinetic energy is 20 a.u. (the populations in the continuum state and ground state are $|c_1|^2 = 0.2$ and $|c_0|^2 = 0.8$, respectively) with the initial states of different atomic potentials, where the dashed black and red curves refer to the spectra calculated from 20 BPs, as the parameters of the soft Coulomb potentials are $a = 0.3$ and 0.17, respectively, and the solid curves are the corresponding results calculated from the TDSE. It can be found from the figure that the intensity of light emission is higher in the case of the deeper potential well ($a = 0.17$), which can be attributed to the fact that the oscillation amplitude of the BP's Coulomb acceleration is larger when the electron is recombined with the parent ion whose potential well is deeper, as is shown in Figs. 5(a) and 5(b). To make out this phenomenon, we select a pair of trajectories whose initial positions are basically the same and present their quantum forces and trajectories in Figs. 5(c) and 5(d). We can see from the figure that the oscillation amplitude and frequency of the quantum forces and trajectories of the BPs differ only slightly in the two cases, so it can be assumed that the BP's quantum force as well as its position has little influence on the light emission; for this

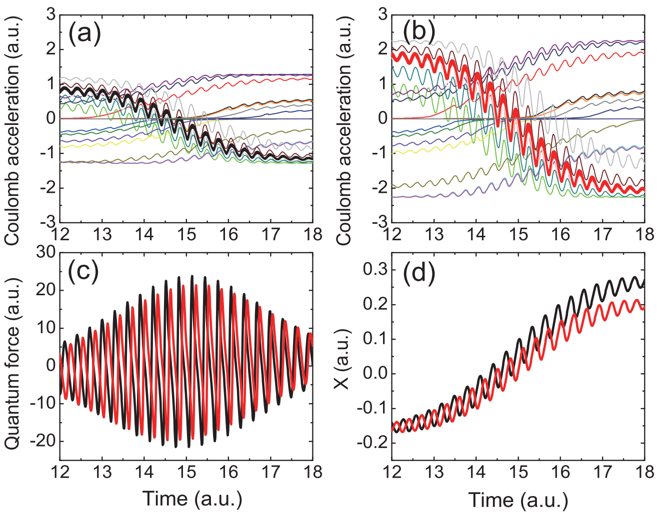

FIG. 5. (Color online) Coulomb accelerations of the 20 BPs in the cases in which the soft Coulomb parameters are (a) $a = 0.3$ and (b) $a = 0.17$, respectively, where the bold black and red curves refer to the Coulomb accelerations of the BPs whose initial positions are basically the same. (c) and (d) Quantum forces and trajectories of the BPs shown by the bold curves in (a) and (b), respectively, where the black and red curves correspond to the cases in which the soft Coulomb parameters are $a = 0.3$ and 0.17, respectively.

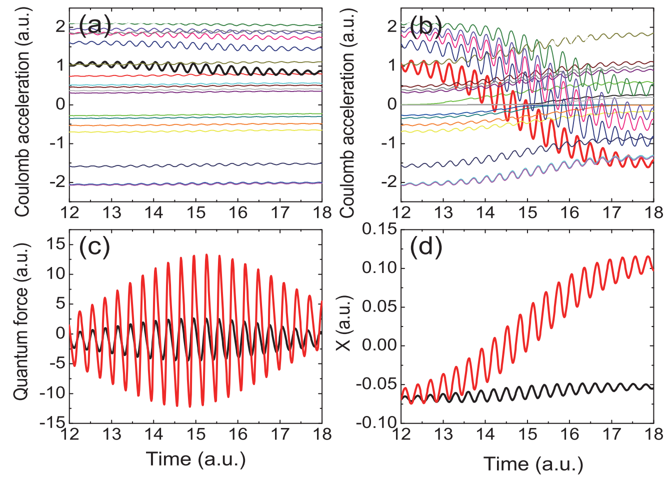

FIG. 7. (Color online) Time evolution of the Coulomb acceleration of the 20 BPs where the populations of the continuum state are (a) $|c_1|^2 = 0.01$ and (b) $|c_1|^2 = 0.1$, respectively, where the bold black and red curves are the Coulomb accelerations of the BPs whose initial positions are basically the same. (c) and (d) Quantum forces and trajectories of the BPs shown by the bold curves in (a) and (b), respectively, where the black and red curves correspond to the cases in which the populations of the continuum state are $|c_1|^2 = 0.01$ and (b) $|c_1|^2 = 0.1$, respectively.

reason, we can draw the following conclusion: the factor that affects the light emission is the Coulomb potential itself; the larger the Coulomb potential well, the larger the oscillation amplitude of Coulomb acceleration that the BP in the vicinity of the nuclear zone bears; this leads to the higher intensity of the light emission.

As is well known, the probability of the recombined electron will affect the intensity of the light emission. To comprehend this from the perspective of the BM, how the probability of the ionized electron affects the BT and the intensity of the light emission by recombination will be discussed in the following. It is found that in the case in which the kinetic energy of the recombined electron and the potential function of the parent ion are fixed, the intensity of the light emission is proportional to the product of the population of the recombined electron in the continuum state and that of the electron in the ground state. As is shown in Fig. 6, the dashed black (red) curve refers to the harmonic spectrum calculated from the 20 BPs in the case in which the populations of the continuum state and ground state are $|c_1|^2 = 0.01(0.1)$ and $|c_0|^2 = 0.99(0.9)$, respectively. Additionally, the solid curves account for the corresponding results calculated from the TDSE. From the figure, it can be seen that as the product of the population of the continuum state and that of ground state $|c_0|^2|c_1|^2$ changes from 0.0099 to 0.09 the intensity of the spectrum increases about one order of magnitude. Starting from the perspective of the BT, we present the change of the BP's Coulomb acceleration with time in the above two cases, as is shown in Figs. 7(a) and 7(b). Figure 7(a) shows the result in the case in which the populations of the continuum state and ground state are $|c_1|^2 = 0.01$ and $|c_0|^2 = 0.99$, and Fig. 7(b) shows the result in the case in which the populations of the continuum state and ground state are $|c_1|^2 = 0.1$ and $|c_0|^2 = 0.9$. We can see that the oscillation frequencies of the BPs' Coulomb accelerations are identical to each other in the two cases, and the particle whose population product is larger has a larger oscillation amplitude. To make out the cause of this phenomenon, we select a pair of particles whose initial positions are basically the same, and we observe the quantum forces they feel and their trajectories, as is shown in Figs. 7(c) and 7(d). It is not hard to find that when the population product is larger ($|c_0|^2|c_1|^2 = 0.09$) the oscillation amplitude of the quantum force the particle feels will be larger; therefore, the oscillation amplitude of the particle's trajectory will be larger, resulting in the increase of the oscillation amplitude of Coulomb acceleration and thus explaining the causes of the fact that the efficiency of light emission will become higher as the population product increases.

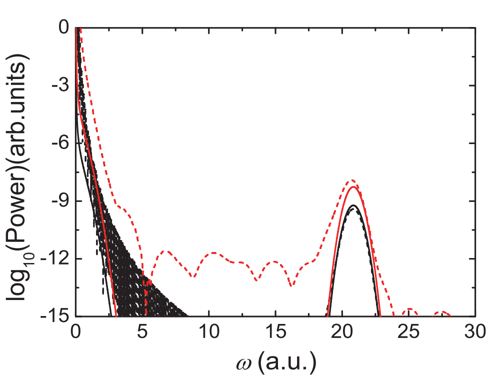

FIG. 6. (Color online) Harmonic spectrum calculated from the TDSE and the 20 BPs, where the solid black (red) refers to the result calculated from the TDSE in the case in which the population of the continuum state is $|c_1|^2 = 0.01(0.1)$, and the dashed black (red) refers to the result in the case in which the population of the continuum state is $|c_1|^2 = 0.01(0.1)$.

In conclusion, we can see from the perspective of the BM that the process of light emission by the recombination of the ionized electron with its parent ion is actually the process in which the BP oscillates at high frequency in the nuclear zone emitting radiation, and the frequency and intensity of the radiation are determined by the oscillation frequency and amplitude of all BPs. The influence of the kinetic energy of the recombined electron, the potential of the nucleus, and the population of the continuum-state and ground-state electrons on the intensity of radiation can be attributed to the influence on the amplitude of the BP's high-frequency oscillation.

IV. CONCLUSIONS

The motions of BPs possess the classical behaviors which follow the Bohm-Newton equation, and the corpuscular property of the electron and all the information about quantum effects are included at the same time. Based on this advantage of BPs, we investigated the light-emission process by the recombination of the ionized electron with its parent ion through the BM scheme, and we obtained the spectrum which is quantitatively consistent with that of the TDSE through the selected 20 BTs. Through the dynamical analysis of these BTs, we present the motion interpretation of the BP during the quantum process, i.e., the light emission by the recombination of the ionized electron. It is found that the light emission by the recombination of the ionized electron is determined by the high-frequency oscillation of the BP in the nuclear zone of the parent ion; the higher the oscillation frequency of the BP's Coulomb acceleration, the higher the frequency of the light, the larger the oscillation amplitude, and the more intense the spectrum will be. Moreover, the motion behaviors of the BP are affected by three factors: the kinetic energy of the recombined electron, the potential function of the nucleus, and the product of the population of the continuum-state wave packet and that of the ground-state wave packet. The larger the kinetic energy of the recombined electron, the smaller the oscillation amplitude of the particle's Coulomb acceleration and the higher the oscillation frequency of it will be; the deeper the potential well of the nucleus, the larger the oscillation amplitude of the particle's Coulomb acceleration and the higher the oscillation frequency of it will be; the larger the product of the population of the continuum-state wave packet and that of the ground-state wave packet, the larger the oscillation amplitude of the particle's Coulomb acceleration will be.

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