Device-independent certification of the teleportation of a qubit

Melvyn Ho,[1] Jean-Daniel Bancal,[1] and Valerio Scarani[1,2]

[1]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore*

[2]*Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542, Singapore*

(Received 16 August 2013; published 18 November 2013)

We want to certify in a black-box scenario that two parties simulating the teleportation of a qubit are really using quantum resources. If active compensation is part of the simulation, perfect teleportation can be faked by purely classical means. If active compensation is not implemented, a classical simulation is necessarily imperfect: In this case, we provide bounds for the certification of quantumness using only the observed statistics. In particular, if a uniform shrinking of the Bloch vector is observed on the teleported side, an average teleportation fidelity of 85% guarantees the use of quantum resources. In general, the criterion is not simply related to the fidelity; in an extreme case, the assessment of quantumness can be positive even for an average fidelity as low as 50%.

DOI: 10.1103/PhysRevA.88.052318 PACS number(s): 03.67.Hk, 03.65.Wj, 03.67.Mn

I. INTRODUCTION

Shortly after the milestone paper that introduced quantum teleportation [1], the question was asked of which deviation from the ideal case one can tolerate while still claiming that proper quantum effects were being observed. Popescu proved that, if Alice and Bob share no entanglement, the average fidelity for the teleportation of unknown qubit states is bounded by $\bar{F} = \frac{2}{3}$ [2]. This bound has been widely used as a benchmark for experiments [3,4], including the most recent ones [5,6]. The bound at 2/3 holds if one trusts that the quantum systems under study are qubits. By using larger-dimensional alphabets, the model of Gisin [7] reaches up to an average fidelity $\bar{F} \approx 0.87$ with only classical resources. The relation between Bell's theorem and teleportation, which inspired already the authors of [2], has been further studied by Horodecki *et al.* [8], Zukowski [9], and eventually by Clifton and Pope [10]. This work states that $\bar{F} \gtrsim 0.9$ would guarantee that the observed teleportation phenomenon has not been simulated with local variables.

However, with closer scrutiny, even the treatments based on Bell inequalities invoke two-qubit algebra at some point or other to complete the calculation. The fact that it could be critical to resort to qubits at any stage was noticed only several years later, in the context of quantum key distribution [11], where this threatened the security of the existing protocols. This observation in turn lead to the idea of a *device-independent assessment* [12]. The device-independent framework has since been applied to several quantum information tasks (see [13,14] for reviews). It is time to reconsider the assessment of quantum teleportation in this by now well-established framework: This is the goal of the present paper.

II. OPERATIONAL BLACK-BOX DESCRIPTION

A vendor is selling two boxes which allegedly perform quantum teleportation of a qubit state (Fig. 1). The input of each box consists of a unit vector on the surface of sphere $\mathbb{S}^2$ embedded in $\mathbb{R}^3$. Alice's vector $\vec{a}$ represents the state to be teleported as $|\psi\rangle\langle\psi| = \frac{1}{2}(\mathbb{I} + \vec{a}\cdot\vec{\sigma})$.[1] Bob's vector represents his choice of performing the measurement along $\vec{b}\cdot\vec{\sigma}$ on the teleported state. Alice and Bob inform the vendor that they will treat their inputs as defined in the same reference frame. For every input, Alice's box outputs two bits $(c_0,c_1) \in \{0,1\}^2$, while Bob's box outputs a bit $\beta \in \{-1,+1\}$. The two boxes are claimed to be loaded in each run with a maximally entangled two-qubit state, but there is no direct *a priori* evidence for it: In fact, this is what we will aim to infer *a posteriori* by observing the statistical behavior of the boxes. In the ideal teleportation experiment, conditioned on (c_0,c_1), Bob's box contains a qubit in the state $\rho_B = \frac{1}{2}[\mathbb{I} + (\mathbf{R}_{c_0,c_1}\vec{a})\cdot\vec{\sigma}]$, where the four SO(3) matrices are the identity $\mathbf{R}_{00} = I$ and the rotations by π along three orthogonal directions, $\mathbf{R}_{01} = \mathbf{R}(\hat{x},\pi)$, $\mathbf{R}_{10} = \mathbf{R}(\hat{y},\pi)$, and $\mathbf{R}_{11} = \mathbf{R}(\hat{z},\pi)$.

We are going to show that a black-box certification of qubit teleportation is indeed possible and provide explicit bounds for its conclusiveness. As a first step, we point out two immediate consequences of working in a black-box scenario.

A. Consequence 1: Impossibility of active compensation

When teleportation is used as a building block in a larger protocol, one typically wants to recover the input state of Alice deterministically on Bob's side. To this effect, Alice is asked to send (c_0,c_1) over to Bob's location, so that he can apply the unitary transformation corresponding to $\mathbf{R}_{c_0,c_1}$ on the Bloch vector, and ideally recover Alice's state. Experiments that include this active compensation [6] are rightly considered as more advanced than those that do not. If Bob performs a measurement along $\vec{b}$ after the compensation, he expects to find $\langle\beta\rangle = \vec{a}\cdot\vec{b}$. Now, in the black-box scenario, active compensation translates to allowing two more bits of input in Bob's box [Fig. 1(a)], but the way this information is processed within the box may be very different from applying a rotation. Thus, the black-box version of teleportation with active compensation gives more leeway for the vendor to cheat (too much leeway, as it turns out: with those additional two bits of input in Bob's box, the statistics of perfect teleportation can be simulated with only classical resources). This conclusion is an immediate corollary of the Toner-Bacon simulation of the singlet [15]; it supersedes previous, slightly less efficient simulations of teleportation [16]. As we shall see, black-box certification of teleportation becomes possible if one collects

[1]Teleportation should work also for mixed states, i.e., $|\vec{a}| \leqslant 1$. Since the most demanding simulation is that of a pure state, in this paper we focus on $|\vec{a}| = 1$.

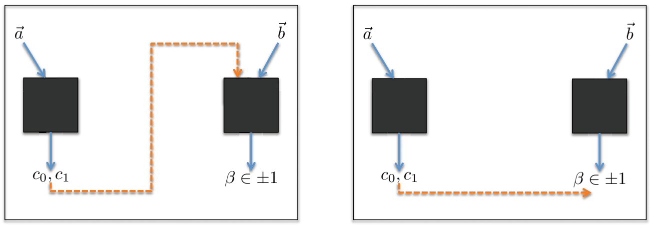

FIG. 1. (Color online) Operational black-box description of the teleportation of a qubit in two different ways Bob can use the two bits of communication. In the first scenario, the two bits are input into Bob's side to perform a compensation depending on (c_0,c_1). In the second scenario, Bob's box cannot benefit from the two bits of information since they are sent only after the outcomes are obtained.

(c_0,c_1) and β separately, then studies the conditional statistics; in other words, if one presents the data of teleportation in analogy with a Bell test [9].

B. Consequence 2: Need to assume that the state to be teleported is known

One of the main features of quantum teleportation is that the protocol works even when the state to be teleported is unknown to the person who sets up the Bell-state measurement. Whether this feature must be requested of simulations has been debated, and ultimately depends on each author's choice of assumptions. For the black-box scenario, the case is clear: The certification must be done against protocols that simulate the teleportation of a *known* state. Indeed, in the two-box scenario presented here, Alice's box simulates both the qubit source and the Bell-state measurement. Since Alice inputs $\vec{a}$ as classical information, this information can be copied and made available at any of the internal steps that happen in the box. It would not help to ask the vendor to build the source as a separate box because this box must send a signal (the alleged qubit) to the box that allegedly performs the Bell-state measurement: That signal could carry the classical description of the state.

III. OPERATIONAL DESCRIPTION OF TELEPORTATION

After these considerations, we focus on the scenario sketched in Fig. 1(b). The available data are a table of values $(c_0,c_1,\beta|\vec{a},\vec{b})$ for each run of the experiment. For each choice of $\vec{a},\vec{b} \in \mathbb{S}^2$, we assume that infinitely many runs of the experiment are performed (that is, we neglect statistical fluctuations). From this table, one can extract the probability distributions $P(c_0,c_1,\beta|\vec{a},\vec{b})$, which will be used to check the violation of a Bell inequality. The ideal case is represented by $P_{\text{ideal}}(c_0,c_1,\beta|\vec{a},\vec{b}) = \frac{1}{8}\,[1+\beta(\mathbf{R}_{c_o,c_1}\vec{a})\cdot\vec{b}]$.

The *average fidelity of teleportation* is the most frequently used measure of quality of a teleportation protocol. To define it in this operational scenario, notice first that $(\mathbf{R}_{c_o,c_1}\vec{V})\cdot\vec{b} = \vec{V}\cdot(\mathbf{R}_{c_o,c_1}\vec{b})$. Thus, Bob will sort the data of the table to reconstruct $P(\beta|c_0,c_1,\vec{a},\mathbf{R}_{c_o,c_1}\vec{b}) = \frac{1}{2}[1+\vec{V}'_{c_o,c_1}(\vec{a})\cdot\vec{b}]$.

On the right-hand side, we have made the assumption (which can be verified *a posteriori*) that the observed $\langle\beta\rangle$ is linear in $\vec{b}$. If this were not the case, it is manifest that Bob is not measuring a qubit. If this linear behavior is observed, Bob can extract a compensated vector $\vec{A}_{c_o,c_1}(\vec{a})$ which should ideally be equal to $\vec{a}$. The average teleportation fidelity can thus be estimated by sampling Alice's inputs at random

$$\bar{F} = \int \frac{d\vec{a}}{4\pi}\sum_{c_0,c_1} P(c_0c_1)\,\frac{1+\vec{A}_{c_o,c_1}(\vec{a})\cdot\vec{a}}{2}\,. \qquad (1)$$

IV. DEVICE-INDEPENDENT CERTIFICATION OF QUANTUM RESOURCES IN TELEPORTATION

Let us move to the constructive description of the certification, based on the scenario. Here we present one approach, not claimed to be optimal that uses, like the authors of [10], the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality. In the protocol, Alice can choose between two inputs $\vec{a}_0$ and $\vec{a}_1$; similarly, Bob can choose between $\vec{b}_0$ and $\vec{b}_1$. Alice's outcome consists of two bits, from which we want to extract one bit $\alpha \in \{-1,+1\}$: we choose the prescription $\alpha \equiv 2c_j - 1$ if Alice's input is $\vec{a}_j$. Now one can evaluate CHSH $= E_{00} + E_{01} + E_{10} - E_{11}$ with

$$\begin{aligned} E_{jk} &\equiv P(\alpha=\beta|j,k) - P(\alpha\neq\beta|j,k) \\ &= P(c_j=0,\beta=-1|j,k) + P(c_j=1,\beta=+1|j,k) \\ &\quad - P(c_j=0,\beta=+1|j,k) - P(c_j=1,\beta=-1|j,k), \end{aligned}$$

where $P(c_j,\beta|j,k) \equiv P(c_j,\beta|\vec{a}_j,\vec{b}_k)$. If CHSH > 2 in a loophole-free assessment, the two boxes must have shared quantum entanglement. This is a very standard device-independent argument by now. The interesting step consists in studying its implications for a teleportation setup.

V. A FIRST APPLICATION

For a first assessment, let us consider a pair of boxes producing the statistics

$$P_{\text{obs}}(c_0,c_1,\beta|\vec{a},\vec{b}) = \tfrac{1}{8}\left[1+\beta\vec{V}_{c_o,c_1}(\vec{a})\cdot\vec{b}\right]. \qquad (2)$$

This model captures in particular Alice's statistics $P(c_0,c_1|\vec{a}) = \frac{1}{4}$, as well as the fact that Bob's statistics are linear in $\vec{b}$ (if this were not the case, Alice and Bob would immediately be suspicious because of a nontrivial departure from the qubit behavior). Then

$$\begin{aligned} \text{CHSH} = &\frac{1}{4}\sum_{c_1}(\vec{b}_0+\vec{b}_1)\cdot\left[\vec{V}_{0,c_1}(\vec{a}_0) - \vec{V}_{1,c_1}(\vec{a}_0)\right] \\ &+ \frac{1}{4}\sum_{c_0}(\vec{b}_0-\vec{b}_1)\cdot\left[\vec{V}_{c_0,0}(\vec{a}_1) - \vec{V}_{c_0,1}(\vec{a}_1)\right]. \end{aligned} \qquad (3)$$

Assume further that

$$\vec{V}_{c_o,c_1}(\vec{a}) = \lambda\mathbf{R}_{c_0,c_1}\vec{a}, \quad \lambda\in[0,1], \qquad (4)$$

that is, in an active-compensation teleportation Bob would always retrieve $\lambda\vec{a}$ if Alice has input $\vec{a}$, independent of c_0 and c_1; again, this is the expected behavior when the resource state is a Werner state, for example, and can be checked on the observed statistics. This assumption also implies that the postprocessing fidelity of each vector is the same as the active compensation case. Then

$$\mathrm{CHSH} = \lambda \left[\sum_{c_1} (\vec{b}_0 + \vec{b}_1)\cdot \vec{a}_{0,x} + \sum_{c_0} (\vec{b}_0 - \vec{b}_1)\cdot \vec{a}_{1,y} \right], \tag{5}$$

where $\vec{a}_{0,x}$ refers to the x component of the vector $\vec{a}_0$, and $\vec{a}_{1,y}$ refers to the y component of the vector $\vec{a}_1$. Here, the vectors $\vec{V}_{c_0,c_1}$ represent the vector that Bob would have had, without compensation to obtain $\vec{A}_{c_0,c_1}$. The fact that the spatial symmetry is broken is just a consequence of our initial choice of α, as alternative choices could be used to pick up any of the following pairs of components from Alice's vector: (a_x,a_y), (a_x,a_z), $(-a_x,a_y)$, and so on. Therefore, if Alice were to choose $\vec{a}_0 = \vec{x}$ and $\vec{a}_1 = \vec{y}$, there are settings for Bob such that the resulting CHSH expression is violated whenever $\lambda \geqslant \frac{1}{\sqrt{2}}$. This translates to a critical average teleportation fidelity $\bar{F} = \frac{1}{2}(1+\lambda) \geqslant \frac{1}{2}(1+\frac{1}{\sqrt{2}}) \approx 0.85$.[2]

From our calculation, one may be tempted to draw the conclusion that $\bar{F} \geqslant 0.85$ is sufficient to certify quantum teleportation in a black-box scenario. However, there is a counterexample to this statement. In 1996, Gisin presented the simulation of the teleportation of a known state, achieving $\bar{F} \approx 0.87$ using only classical resources [7]. We are going to study this model in the next section.

VI. GISIN's CLASSICAL SIMULATION REVISITED

In a classical simulation of teleportation, only the two bits of communication will convey information about $\vec{a}$ to Bob's box. Gisin's protocol uses them to tell in which quarter of the Bloch sphere $\vec{a}$ lies. The Bloch sphere is divided in four equal quarters S_{ij}, each having in its center one of the $\vec{t}_{ij} = \mathbf{R}_{ij}\vec{t}_{00}$ thus defined: $\vec{t}_{00} = \frac{1}{\sqrt{3}}(+1,+1,+1)$, $\vec{t}_{01} = \frac{1}{\sqrt{3}}(+1,-1,-1)$, $\vec{t}_{10} = \frac{1}{\sqrt{3}}(-1,+1,-1)$, and $\vec{t}_{11} = \frac{1}{\sqrt{3}}(-1,-1,+1)$. Bob's box contains $\vec{t}_{00}$ and outputs β distributed according to $\langle\beta\rangle = \vec{b}\cdot\vec{t}_{00}$. Thus the protocol produces

$$P_{\mathrm{Gisin}}(c_0,c_1,\beta|\vec{a},\vec{b}) = \delta(\vec{a}\in S_{c_0,c_1})\,\tfrac{1}{2}\,(1+\beta\vec{t}_{00}\cdot\vec{b}). \tag{6}$$

Notice how Alice and Bob are completely uncorrelated. These statistics lead to CHSH $= 0$. Also, it is easy to see that

$$P_{\mathrm{Gisin}}\big(\beta|c_0,c_1,\vec{a},\mathbf{R}_{c_0,c_1}\vec{b}\big) = \tfrac{1}{2}\left(1+\beta\vec{t}_{c_0,c_1}\cdot\vec{b}\right) \tag{7}$$

for $\vec{t}_{c_0,c_1}$, which is in the same sector as $\vec{a}$. The average teleportation fidelity is therefore

$$\bar{F} = \frac{1}{4}\sum_{c_0,c_1}\frac{d\vec{a}}{\pi}\int_{S_{c_0,c_1}}\frac{1+\vec{t}_{c_0,c_1}\cdot\vec{a}}{2} = \int_{S_{00}}\frac{d\vec{a}}{\pi}\,\frac{1+\vec{t}_{00}\cdot\vec{a}}{2}$$
$$= \frac{1}{2}\left[1+3\frac{1}{\pi}\int_{\pi/3}^{\pi}d\phi\int_0^{u(\phi)}d\theta\cos\theta\sin\theta\right] \approx 0.87$$

with $u \equiv \tan^{-1}[\frac{\sqrt{2}}{\cos(\phi+\frac{\pi}{3})}]$.

[2]This is precisely the fidelity bound that guarantees nonlocality in the Werner state with the qubit assumption [10].

The observable statistics (6) are, however, significantly different from the ones we posited before [Eqs. (2) and (4)]. Notably, Alice's output (c_0,c_1) is deterministic for a given $\vec{a}$. Bob's box is found to contain always the same vector $\vec{t}_{00}$, which does not depend on $\vec{a}$ at all. One could try to modify the protocol to erase these obvious shortcomings. For instance, Alice's output could be randomized by adding two bits of shared randomness (r_0,r_1) to both her and Bob's box. Alice's box would then output $(c_0,c_1) = (c_0'\oplus r_0, c_1'\oplus r_1)$ when $\vec{a}\in S_{c_0',c_1'}$, while Bob's box would contain $\vec{t}_{r_0 r_1}$. Since $\mathbf{R}_{ij}\mathbf{R}_{i'j'} = \mathbf{R}_{i\oplus i',j\oplus j'}$, this hashing leaves Eq. (7) unchanged, thence the fidelity. As for Eq. (6), it is replaced by $P_{\mathrm{Gisin}}(c_0,c_1,\beta|\vec{a},\vec{b}) = \frac{1}{8}$, and gives CHSH $= 0$. So at least Alice does not detect anything obviously wrong locally since $P(c_0,c_1) = \frac{1}{4}$.

One could also randomize Bob's vectors by randomizing the frame of the tetrahedron for each run of the protocol, and for Bob's box to contain $\vec{t}_{00}$ of the that frame for every run. This also gives us $P(c_0,c_1|\vec{a}) = \frac{1}{4}$, and nonzero $\vec{V}_{c_0,c_1}$ such that one obtains a nontrivial CHSH value, but also a low fidelity of 0.5.

The point here is that in excluding local protocols, the local statistics could already indicate if the protocol is truly close to the ideal case. Furthermore, the local statistics may be used as verifiable assumptions, to form bounds specific for a particular experiment. For us, the Gisin model and the variants we just discussed serve to highlight the fact that these assumptions should be verified when concluding if the teleportation protocol utilizes quantum resources, especially in the fidelity region close to 85%. Also, despite the high fidelity of the Gisin model in the active case, these simple modifications do not show a strong relation between the postprocessing fidelity and CHSH value.

VII. LOW FIDELITY, HIGH CHSH

Now we consider whether it is possible to observe a low teleportation fidelity, and yet a high CHSH violation. For this situation, consider a teleportation protocol that maps $\vec{a}$ to the $\vec{V}_{c_0,c_1}(\vec{a})$ that we expect in a perfect teleportation experiment, but only for two vectors to maximize CHSH:

$$\vec{a} \mapsto \vec{V}_{c_0,c_1}(\vec{a}) = \begin{cases} R_{c_0,c_1}\vec{a} & \text{for} \quad \vec{a}\in\{\vec{x},\vec{y}\} \\ 0 & \text{otherwise.} \end{cases}$$

In this case, the average fidelity across the entire sphere is essentially 0.5. However, one can obtain CHSH $= 2\sqrt{2}$ by using the settings that were previously chosen. This maximal violation indicates that a low average fidelity might be weakly related to the performance of the protocol with respect to a finite number of input choices and does not necessarily mean that the protocol is local. While it is not clear that something close to this extreme case can happen in practice, the tools we use here do not allow us to put a tighter bound on the lowest fidelity for which a Bell violation can be observed.

VIII. HIGHEST FIDELITY WITHOUT CHSH VIOLATION

To complete our study, we also describe a possible protocol P_{crit} that has the highest average fidelity without yielding a violation. Here we will not impose the condition in Eq. (4), but

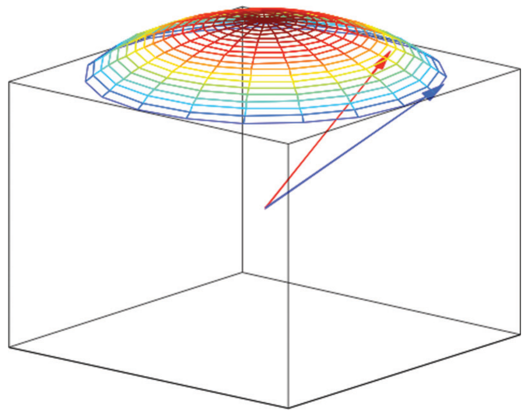

FIG. 2. (Color online) Schematic of the region of the sphere with z component larger than $\frac{1}{\sqrt{2}}$. For each input vector (higher arrow in red), the optimal output vector with the highest fidelity (lower arrow in blue) would be a vector with the same phase ϕ, but with z component limited to prevent any possible violation.

only that the compensated $\vec{V}_{c_0,c_1}$ form a consistent description, i.e., $\forall\,\vec{a}_x, \exists\,\vec{a}_x^B$ s.t. $R_{c_0,c_1}\vec{V}_{c_0,c_1}(\vec{a}_x) = \vec{a}_x^B$; along with Eq. (2).

To construct such a protocol, we recall that the CHSH function we derived earlier (in particular, the specific coarse graining to determine α) picks out a particular component of each of the teleported $\vec{a}_x^B$. This means that to limit CHSH $\leqslant 2$ for all choices of settings, the resultant vectors $\vec{a}_x^B$ must have their individual components limited to $\frac{1}{\sqrt{2}}$.

To see how this limitation works, we note that if the teleported vector $\vec{a}_0^B$ is such that $\vec{a}_{0,z}^B = \pm\frac{1}{\sqrt{2}}$, then $\vec{a}_{1,x}^B$ and $\vec{a}_{1,y}^B$ for any input $\vec{a}_1$ must be limited to $\frac{1}{\sqrt{2}}$ as well. We could also limit the maximal z component of any teleported vector to be an arbitrary W_z, meaning that the x and y components of any other vector should be at most $\sqrt{2} - W_z$ to obtain CHSH $= 2$. However, we checked that the protocol with the highest overall fidelity is for $W_z = \frac{1}{\sqrt{2}}$.

There are two distinct classes of optimal assignments $\vec{a}_x^B$ for every $\vec{a}$. Some vectors could be teleported with perfect fidelity without having any component larger than $\frac{1}{\sqrt{2}}$.[3] For such inputs, we have

$$\vec{a} \mapsto \vec{a}_x^B = \vec{a}.$$

The second class of assignment $\vec{a}_x^B$ is for vectors that would allow for a violation when perfectly teleported. To avoid this, we deterministically assign $\vec{a}_x^B$ in such a way as to maximise the fidelity while keeping the largest component at $\pm\frac{1}{\sqrt{2}}$.

As an example, consider inputs in the upper cap of the Bloch sphere, with the z component larger than $\frac{1}{\sqrt{2}}$ (see Fig. 2). Our protocol teleports these vectors with a reduced fidelity as follows:

$$\text{Upper cap}: \vec{a} = \begin{pmatrix} \sin\theta\ \cos\phi \\ \sin\theta\ \sin\phi \\ \cos\theta \end{pmatrix} \mapsto \vec{a}_x^B = \begin{pmatrix} \frac{1}{\sqrt{2}}\ \cos\phi \\ \frac{1}{\sqrt{2}}\ \sin\phi \\ \frac{1}{\sqrt{2}} \end{pmatrix}.$$

The other five regions are mapped in a similar fashion.

[3]One such example of Alice's input is the vector $\vec{a} = \frac{1}{\sqrt{3}}(+1, +1, +1)$.

Average fidelity over inputs with the z component larger than $\frac{1}{\sqrt{2}}$ are as follows:

$$\bar{F}_{\text{cap}} = \frac{1}{2} + \frac{1}{2}\frac{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/4} \vec{a}_x^B \cdot \vec{a}\ \sin\theta\, d\phi\, d\theta}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/4} \sin\theta\, d\phi\, d\theta} = \frac{1}{2}\left[1 + \frac{\pi}{8(\sqrt{2}-1)}\right]. \tag{8}$$

Assigning a fidelity of 1 to the remaining regions, the fidelity of the protocol P_{crit} is then

$$\bar{F}_{P_{\text{crit}}} = \frac{\left[(12\pi)\left(1-\frac{1}{\sqrt{2}}\right)\right]\bar{F}_{\text{cap}} + \left[4\pi - (12\pi)\left(1-\frac{1}{\sqrt{2}}\right)\right]}{4\pi} \approx 0.97718. \tag{9}$$

IX. CONCLUSION

In the first section, we lay out the framework of quantum teleportation in the device-independent scenario and propose the use of the postprocessing fidelity instead of the active compensation fidelity as an indicator of nonlocality. This, with verifiable assumptions on the local probability distributions, allows us to construct a CHSH-type expression for the outcomes of our teleportation experiment. Here we find that an average fidelity of 85% and 97.7% in the postprocessing scenario is sufficient to quantify nonlocality for different assumptions.

We also explore some local models to see how they perform with respect to our assumptions and the use of the average fidelity. For Gisin's model and its variants, we do not observe any strong relation between fidelity and CHSH. We also give an example with high CHSH and low fidelity to illustrate a possible limitation in using the average fidelity to obtain bounds on the system.

In this work, we have focused on defining the teleportation fidelity in a useful way, and not in finding the most general criterion for certification; in particular, it may be possible to find better bounds using other inequalities than CHSH. For instance, an alternative choice could be to study an inequality whose optimal settings span the three-dimensional nature of the Bloch sphere, instead of only six settings per party in the CHSH case. One such example would be the so-called *elegant inequality* in Ref. [17].

ACKNOWLEDGMENTS

This work was funded by the Singapore Ministry of Education (partly through the Academic Research Fund Tier 3 MOE2012-T3-1-009) and the Singapore National Research Foundation. We acknowledge Rafael Rabelo and Colin Teo for useful discussions during the course of this work. We also thank an anonymous referee for pointing out the alternative choice of a Bell inequality.

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] S. Popescu, Phys. Rev. Lett. **72**, 797 (1994).
[3] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[4] I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, and N. Gisin, Nature (London) **421**, 509 (2003).
[5] J. Yin, J.-G. Ren, H. Lu, Y. Cao, H.-L. Yong, Y.-P. Wu, C. Liu, S.-K. Liao, F. Zhou, Y. Jiang, X.-D. Cai, P. Xu, G.-S. Pan, J.-J. Jia, Y.-M. Huang, H. Yin, J.-Y. Wang, Y.-A. Chen, C.-Z. Peng, and J.-W. Pan, Nature (London) **488**, 185 (2012).
[6] X.-S. Ma, T. Herbst, T. Scheidl, D. Wang, S. Kropatschek, W. Naylor, A. Mech, B. Wittmann, J. Kofler, E. Anisimova, V. Makarov, T. Jennewein, R. Ursin, and A. Zeilinger, Nature (London) **489**, 269 (2012).
[7] N. Gisin, Phys. Lett. A **210**, 157 (1996).
[8] R. Horodecki, M. Horodecki, and P. Horodecki, Phys. Lett. A **222**, 21 (1996).
[9] M. Zukowski, Phys. Rev. A **62**, 032101 (2000).
[10] R. Clifton and D. Pope, Phys. Lett. A **292**, 1 (2001).
[11] A. Acin, N. Gisin, and L. Masanes, Phys. Rev. Lett. **97**, 120405 (2006).
[12] A. Acin, N. Brunner, N. Gisin, S. Massar, S. Pironio, and V. Scarani, Phys. Rev. Lett. **98**, 230501 (2007).
[13] V. Scarani, Acta Physica Slovaca **62**, 347 (2012).
[14] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, arXiv:1303.2849.
[15] B. F. Toner and D. Bacon, Phys. Rev. Lett. **91**, 187904 (2003).
[16] N. J. Cerf, N. Gisin, and S. Massar, Phys. Rev. Lett. **84**, 2521 (2000).
[17] N. Gisin, arXiv:quant-ph/0702021.