Einstein-Podolsky-Rosen paradox and quantum steering in pulsed optomechanics

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We describe how to generate an Einstein-Podolsky-Rosen (EPR) paradox between a mesoscopic mechanical oscillator and an optical pulse. We find two types of paradox, defined by whether it is the oscillator or the pulse that shows the effect Schrödinger called "steering". Only the oscillator paradox addresses the question of mesoscopic local reality for a massive system. In that case, EPR's "elements of reality" are defined for the oscillator, and it is these elements of reality that are falsified (if quantum mechanics is complete). For this sort of paradox, we show that a thermal barrier exists, meaning that a threshold level of pulse-oscillator interaction is required for a given thermal occupation n_0 of the oscillator. We find there is no equivalent thermal barrier for the entanglement of the pulse with the oscillator or for the EPR paradox that addresses the local reality of the optical system. Finally, we examine the possibility of an EPR paradox between two entangled oscillators. Our work highlights the asymmetrical effect of thermal noise on quantum nonlocality.

I. INTRODUCTION

It is an outstanding challenge in fundamental physics to test quantum nonlocality for mesoscopic, massive systems. The Einstein-Podolsky-Rosen (EPR) paradox [1] is one of the most powerful tests of quantum nonlocality. Presented originally as an argument for the completion of quantum mechanics (QM), the EPR paradox has been experimentally realized so far only in optics [2–7].

The observation of an EPR paradox for the position and momentum of mesoscopic mechanical oscillators would represent an important advance, since this would demonstrate the inconsistency of QM with the local reality (LR) of a massive object. First proposed by Giovannetti *et al.* [8], such a realisation would also give an experimental platform to probe the macroscopic reality of an object, along the lines suggested by Schrödinger [9–11]. While mesoscopic superpositions were achieved with ion traps and microwave oscillators, the use of nanomechanical oscillators creates mass distribution superpositions, which tests the effects of gravity.

In light of the exceptional importance of these developments, we examine in this paper the limitations imposed by thermal noise for achieving an EPR paradox in an optomechanical system. There have been numerous proposals and studies, but mainly for the entanglement of optical and/or mechanical modes [12–22]. Relatively little is known about the paradox itself. We expect that an EPR paradox for the positions and momentum of the mechanical oscillator will be strongly masked by thermal motion. Advances in cooling to the ground state of mesoscopic oscillators improve the likelihood of the realization of a massive particle EPR paradox [23].

First, let us recall the important features of the EPR paradox. The original EPR state was an entangled state of two particles (which we label A and B) that have perfectly correlated positions (X_A, X_B) and momenta (P_A, P_B). Measurements on particle A give immediate information about either the position or momentum of particle B. The EPR paradox arises because the assumption that the measurements do not disturb particle B (local realism) would imply a simultaneous and very precise predetermination for *both* of X_B and P_B. No *local quantum* state of the particle B, however, can be consistent with such precise predetermination, for both momentum and position. In this way, an inconsistency between local realism (LR) and the completeness of quantum mechanics is established.

The distinctive feature of the EPR paradox is that, unlike entanglement, it is a form of quantum nonlocality in which the roles of the two systems are *asymmetrical*. In the above example, an inconsistency of QM with LR is established for the local system B. The details about the system A—which acts only to give information about B—are not so important.

The main point of this paper is to understand how to obtain an irrefutable discrepancy between quantum mechanics and the local reality of the *mechanical oscillator* system. We propose to do this by entangling it with an optical pulse. We consider an idealized model, developed by Hofer *et al.* [24], for pulsed optomechanics on fast time scales [25,26]. The model introduces only two parameters: the squeeze parameter r, which is a measure of the pulse-oscillator interaction, and n_0, the thermal occupation number of the mechanical oscillator. Our main conclusion is that thermal noise provides a strong, but not insuperable, barrier to the oscillator EPR paradox. However, the barrier is *directional*, to prevent "steering" of the thermally excited mechanical system, in a sense we explain below.

To detect the EPR paradox one must consider nonideal states, and it is not enough to simply prove entanglement. Suppose we use scaled quadratures, so that we can write the Heisenberg uncertainty relation for particle B as $\Delta X_B \Delta P_B \geqslant 1$. Then the simplest form of an EPR paradox is realized when an inferred uncertainty relation is "violated" under the assumptions of LR, so that [27]

$$E_{B|A} \equiv \Delta_{\inf} X_B \Delta_{\inf} P_B < 1. \qquad (1)$$

Here $\Delta_{\inf} X_B$ and $\Delta_{\inf} P_B$ are the uncertainties associated with the collapsed wave functions created by local measurements

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(made by "Alice") on particle A. These uncertainties allow her to infer either the position or momentum of particle B to a given accuracy, depending on the choice of her measurements. The realization of this EPR criterion (1) poses a more difficult challenge than the realization of entanglement.

There has been a resurgence of interest in this area with new experiments [28–34] motivated by a realization [35,36] that the paradox is also an example of the nonlocality referred to as "steering" [10]. Steering gives a way to quantify how measurements by Alice can collapse the wave packet of B. For a paradox achieved by condition (1), we can conclude that Alice can steer system B [35,37]. The EPR paradox therefore is a stronger test of QM than entanglement.

Our conclusion is that thermal noise n_0 provides a stronger barrier to the EPR paradox than to entanglement. We identify two sorts of EPR paradox: $E_{m|c} < 1$ and $E_{c|m} < 1$, where m and c are the oscillator and cavity field respectively. The most important is $E_{m|c} < 1$. By analyzing the "elements of reality" associated with EPR's argument, we see it is this paradox which enables a test of the mesoscopic nonlocality for the massive system.

Specifically, we find that the thermal noise n_0 of the oscillator induces a threshold for the pulse-oscillator interaction (as measured by r) if one is to observe the paradox $E_{m|c} < 1$. In the limit of large n_0, we require $r > \frac{1}{2}\ln 2$. Consistent with the fact that the field is not thermally excited, we find there is no similar thermal barrier for an EPR paradox $E_{c|m} < 1$, which demonstrates a "steering" of the optical system. The oscillator-pulse system therefore exhibits regimes of "one-way" steering [32,38], where only the steering of the pulse is detectable.

We also find there is no (similar) thermal barrier for the entanglement between the optical pulse and oscillator. In this dissipation-free model, entanglement can exist for any n_0 and $r > 0$. We see, however, that the thermally insensitive entanglement must manifest in an *asymmetric* way, by measurement of the variances of quantities, $X_A - g_x X_B$, $P_A + g_p P_B$, where g_x, g_p are selected real numbers, not equal to 1.

As a final result of the paper, we examine the possibility of an EPR paradox between two mechanical oscillators that are thermally excited. This leads us to distinguish a subclass of "symmetric" entanglement, which can be detected with $g_x = g_p = 1$, and for which a thermal barrier $r > \frac{1}{2}\ln n_0$ *does* exist. We are able to show that this symmetric form of entanglement is relevant to the creation of entanglement between two symmetric thermal oscillators and therefore has its own fundamental significance. The symmetric entanglement is detected by the criterion of Duan *et al.* [39]. By examining two asymmetrically excited oscillators, we conclude that the "EPR steering" of one by the other can be made largely insensitive to the level of thermal excitation of one of the oscillators.

II. THE HWAH MODEL

We consider a mechanical oscillator coupled to an optical mode of a cavity [8]. Hofer, Wieczorek, Aspelmeyer, and Hammerer (HWAH) [24] proposed a scheme (Fig. 1) in which a light pulse is input to an optomechanical cavity mode and interacts with the oscillator mirror mode via
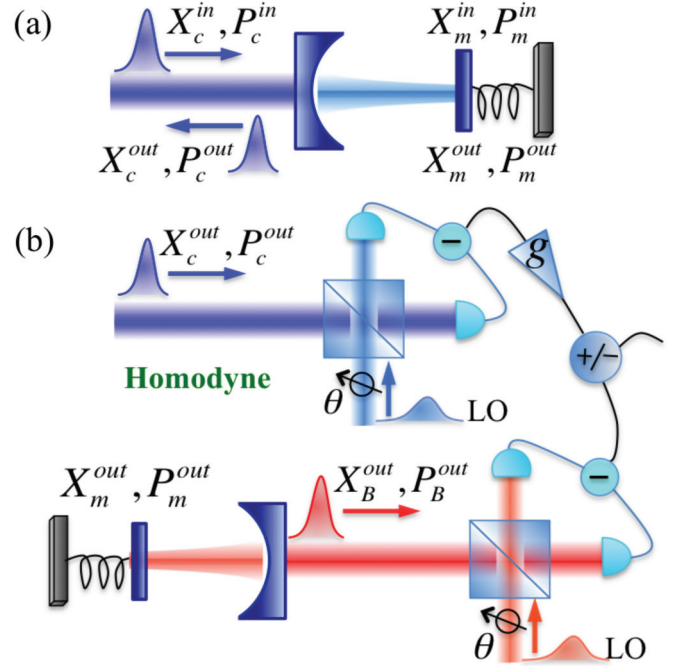

FIG. 1. (Color online) Measurement of the EPR paradox between an oscillator and a pulse: (a) Entangling a pulse with a mechanical oscillator. Following HWAH, an "entangling" blue-detuned pulse interacts with an optomechanical system. The output pulse amplitudes X_c^{out}, P_c^{out} are EPR correlated with the final quadratures X_m^{out}, P_m^{out} of the mechanical oscillator, according to $X_c^{\text{out}} \sim -P_m^{\text{out}}$ and $P_c^{\text{out}} \sim -X_m^{\text{out}}$, in the limit of a large squeezing parameter r. (b) To verify the EPR paradox. The output pulse amplitudes X_c^{out}, P_c^{out} are measured by homodyne detection. The quadratures of the oscillator can be measured by interacting the cavity with a second "verifying" red-detuned pulse.

radiation pressure [40]. The pulse emerges from the cavity with quadratures that are EPR correlated with those of the oscillator. The effective interaction Hamiltonian [40–42] for the cavity-oscillator system in a frame rotating at the laser frequency is

$$H = \omega_m a_m^\dagger a_m + \Delta_c a_c^\dagger a_c + g_R(a_m + a_m^\dagger)(a_c + a_c^\dagger), \quad (2)$$

where $\Delta_c = \omega_c - \omega_1$ is the detuning of the cavity with respect to the laser [24]. The boson creation and destruction operators for the optical cavity and mechanical modes are $a_c^\dagger$, a_c and $a_m^\dagger$, a_m respectively. Quadrature phase amplitudes $X_{c/m}$, $P_{c/m}$ are defined according to $a_c = (X_c + iP_c)/2$ and $a_m = (X_m + iP_m)/2$, where the choice of scaling ensures the normalized EPR inequality $\Delta_{\inf} X_B \Delta_{\inf} P_B < 1$. The term in g_R describes the linearized optomechanical coupling due to the radiation pressure and comprises both a beam-splitter-type coupling (involving $a_m^\dagger a_c + a_c^\dagger a_m$) and a two-mode squeezing interaction term (involving $a_m a_c + a_m^\dagger a_c^\dagger$) of the type known to generate EPR entanglement [27,43].

The physical parameters of the HWAH pulse-oscillator model are the interaction strength g_R, the oscillation frequency ω_m with dissipation rate γ, the optical cavity resonance frequency ω_c with decay rate κ, the pulse carrier frequency ω_1, and duration time τ. The initial occupation number n_0 of the thermal state of the mirror is a vital number which

determines the "quantumness" of the system. HWAH propose the pulse to be either blue-detuned or red-detuned to the cavity resonance [24], to enhance either the two-mode squeezing term (for the purpose of generating entanglement) or the beam-splitter-type term (for the purpose of measurement).

To generate the correlations of the EPR paradox, a blue-detuned pulse interacts with the oscillator cavity. In the case where $g_R \ll \kappa \ll \omega_m$, HWAH derive a set of idealized Langevin equations for the mode operators. To justify neglecting decoherence, they assume the pulse duration and its interaction time with the cavity are short compared to the mechanical decoherence time. The effect of the coupling of the oscillator to an environmental heat bath is ignored.

For the blue-detuned pulse, after making a rotating wave approximation (RWA), with an adiabatic solution for the cavity mode, the simplified Langevin equations lead to solutions for quadratures X_c^{out}, P_c^{out}. The solutions are [24,44]

$$
\begin{aligned}
X_c^{\text{out}} &= -e^r X_c^{\text{in}} - \sqrt{e^{2r}-1}\, P_m^{\text{in}}, \\
P_c^{\text{out}} &= -e^r P_c^{\text{in}} - \sqrt{e^{2r}-1}\, X_m^{\text{in}}, \\
X_m^{\text{out}} &= e^r X_m^{\text{in}} + \sqrt{e^{2r}-1}\, P_c^{\text{in}}, \\
P_m^{\text{out}} &= e^r P_m^{\text{in}} + \sqrt{e^{2r}-1}\, X_c^{\text{in}},
\end{aligned}
\tag{3}
$$

where X_m^{out} and P_m^{out} are the final quadratures of the mechanical oscillator, and $r = g_R^2 \tau / \kappa$ is the "squeezing parameter." The initial quadratures of the oscillator incorporate the effect of the thermal excitation parameter n_0.

The input-output solutions (3) are similar to those of a two-mode squeezed state [45] and will form the basis for modeling the fundamental constraints provided by the thermal noise for an EPR paradox. The solutions (3) in the limit of large r become $X_c^{\text{out}} = -e^r(X_c^{\text{in}} + P_m^{\text{in}})$ and $P_c^{\text{out}} = -e^r(P_c^{\text{in}} + X_m^{\text{in}})$. The EPR nature of the correlations is evident, since

$$
X_m^{\text{out}} = -P_c^{\text{out}}, \quad P_m^{\text{out}} = -X_c^{\text{out}}
\tag{4}
$$

so that a measurement of the quadrature X_c^{out} (or P_c^{out}) of the pulse will immediately give the prediction for the quadrature $-P_m^{\text{out}}$ (or $-X_m^{\text{out}}$) of the oscillator.

The HWAH model is very idealized, and further work is needed to test the validity of the approximations for the pulse and to model the significant decoherence expected for an oscillator interacting with its environment. The model does however capture the main physical effects that generate an EPR correlation and gives a treatment of the thermal noise of the initial state of the oscillator. It can be therefore be used to give a first-order understanding of the asymmetrical effects of thermal noise on the EPR correlation and of the level of thermal cooling that may be necessary, in order to observe an EPR paradox.

III. DETECTING THE ENTANGLEMENT

Often, entanglement is measured as a reduction in two variances, $\{\Delta(X_m^{\text{out}} + P_c^{\text{out}})\}^2$ and $\{\Delta(P_m^{\text{out}} + X_c^{\text{out}})\}^2$, that involve symmetric weightings of oscillator and field quadratures [39]. The symmetric criterion of Duan, Giedke, Cirac, and Zoller (DGCZ) [39] detects entanglement when

$$
\{\Delta(X_m^{\text{out}} + P_c^{\text{out}})\}^2 + \{\Delta(P_m^{\text{out}} + X_c^{\text{out}})\}^2 < 4,
\tag{5}
$$

where we denote the variance using the notation $\{\Delta x\}^2 \equiv \langle x^2 \rangle - \langle x \rangle^2$. This criterion, however, is far from being an optimal signature for entanglement, owing to intrinsic asymmetries.

Here, we examine a less restrictive condition. Entanglement between the oscillator and pulse is proved if one can show [46,47]

$$
\Delta_{g,\text{ent}} = \frac{\{\Delta(X_m^{\text{out}} + g_x P_c^{\text{out}})\}^2 \{\Delta(P_m^{\text{out}} + g_p X_c^{\text{out}})\}^2}{[|g_x g_p| + 1]^2} < 1,
\tag{6}
$$

where g_x and g_p are arbitrary real numbers. The variances in the numerator are directly measurable by the scheme depicted in Fig. 1, where the g_x and g_p are classical gain factors. Here $\Delta_{g,\text{ent}}$ can be minimized to a value Δ_{ent} by choosing the optimal factor $g_x = g_p = g$

$$
g = \frac{\delta + \sqrt{\delta^2 + 4e^{2r}(e^{2r}-1)}}{2e^r \sqrt{e^{2r}-1}},
\tag{7}
$$

where $\delta = \frac{\Delta^2 X_m^{\text{in}} - \Delta^2 P_c^{\text{in}}}{\Delta^2 X_m^{\text{in}} + \Delta^2 P_c^{\text{in}}}$. We assume the initial state of the light field to be the vacuum state and that of the mirror to be a thermal state with mean excitation number n_0. In this case, $\delta = \frac{n_0}{n_0+1}$.

It is shown elsewhere [39,48] that the condition given by Eq. (6) with optimal g is equivalent to the positive partial transpose (PPT) condition developed by Simon [46] for Gaussian states and is therefore necessary and sufficient for Gaussian two-mode entanglement. The entanglement Δ_{ent} can be measured by the arrangement of Fig. 1.

We note that, unlike "steering," entanglement is defined as a property that the two systems share without specification of direction—that is, if A is entangled with B, we know that B is entangled with A. Consistent with this, we see that the criterion (6) is symmetric with respect to interchange of m and c, provided g_x and g_p are interchanged with their reciprocals. Thus, entanglement can be detected using either criterion (6) or the criterion obtained in interchanging m with c, provided the choice of the g_x's and g_p's is kept fully flexible. The prediction of the model (3) for the entanglement measured by $\Delta_{g,\text{ent}}$ is plotted in Fig. 2.

Now we come to the first important result of this paper. Surprisingly, we see from the Fig. 2 that no thermal barrier exists for the presence of entanglement. For any given thermal occupation number n_0, we can always show entanglement for $r > 0$. In other words, once the system is entangled, no amount of thermal noise can completely destroy it. A similar type of robustness of entanglement has been predicted for the steady-state optomechanical entanglement that is generated using continuous wave light fields [12–15]. We note from the figure that the detection of this *thermally insensitive* entanglement is linked to values of the parameter g_{opt} that are very different to 1. As we see below, this result can be understood in terms of the concept of quantum steering.

This result contrasts with that obtained for entanglement detected using the *symmetric* DGCZ condition (5) [39], given by $\Delta_{g,\text{ent}} < 1$, where $g_x = g_p = 1$. We call this sort of entanglement "symmetric entanglement." In that case, a thermal barrier does exist, and entanglement can be detected
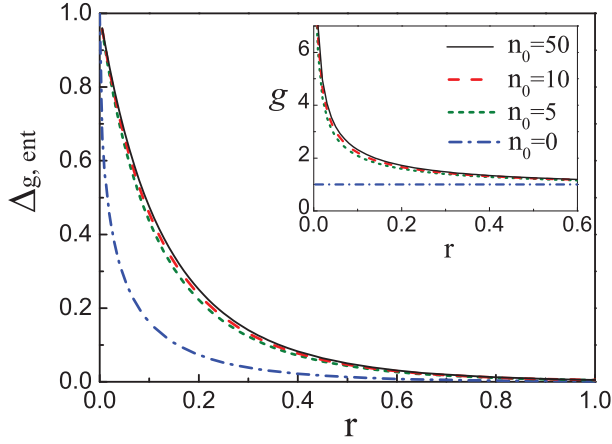
FIG. 2. (Color online) Entanglement $\Delta_{g,\text{ent}}$ plotted vs the squeezing parameter r for $n_0 = 0, 5, 10, 50$, where n_0 is initial occupation number of the oscillator: Entanglement between the oscillator and pulse is observed when $\Delta_{\text{ent}} < 1$. Strong entanglement occurs when $\Delta_{\text{ent}} \to 0$. The optimal g to minimize $\Delta_{g,\text{ent}}$ is shown in the inset. The results indicate presence of entanglement, even for large n_0.

only when the squeezing parameter is sufficiently large [24],

$$ r > \ln \frac{n_0 + 2}{2\sqrt{n_0 + 1}} \overset{n_0 \to \infty}{\sim} \frac{1}{2} \ln n_0. \tag{8} $$

While this thermal barrier becomes relevant to detecting entanglement between two symmetric, thermally excited oscillators, it does not place a limit on the detection of entanglement between the oscillator and a pulse.

IV. DETECTING AN EPR PARADOX AND QUANTUM STEERING

A. Quantum steering of the mechanical oscillator by the pulse

Now we examine how to detect an EPR paradox. An EPR paradox is confirmed if [27]

$$ E_{m|c} = \Delta_{\text{inf}} X_m^{\text{out}} \Delta_{\text{inf}} P_m^{\text{out}} < 1, \tag{9} $$

where $\Delta_{\text{inf}} X_m^{\text{out}}$ is the error in the prediction for the value of the oscillator's position, made by a measurement on the pulse. The $\Delta_{\text{inf}} P_m^{\text{out}}$ is defined similarly. The realization of $E_{m|c} < 1$ is verification of a quantum steering of the mechanical system by measurements made on the pulse [35,37]. For the subclass of quantum systems given by Gaussian states and measurements, as is the case here, this criterion becomes necessary and sufficient to detect steering of the system m by the second system [35]. Walborn *et al.* have derived a more sensitive entropic criterion for "EPR steering" that is useful in other cases [49].

A simple way to determine the conditional uncertainties for Gaussian distributions is to use a linear estimate $g_x P_c^{\text{out}}$, based on the result P_c^{out} for measurement at A [5,27]. We find

$$ \Delta_{\text{inf}} X_m^{\text{out}} = \Delta\left(X_m^{\text{out}} - g_x P_c^{\text{out}}\right), \tag{10} $$

where $g_x = \langle X_m^{\text{out}}, P_c^{\text{out}} \rangle / \{\Delta P_c^{\text{out}}\}^2$ is optimal to minimize $\{\Delta_{\text{inf}} X_m^{\text{out}}\}^2$. Here we use the notation $\langle x, y \rangle \equiv \langle xy \rangle - \langle x \rangle \langle y \rangle$. Similarly, the conditional variance $\{\Delta_{\text{inf}} P_m^{\text{out}}\}^2$ is evaluated by

$$ \{\Delta_{\text{inf}} P_m^{\text{out}}\}^2 = \{\Delta\left(P_m^{\text{out}} + g_p X_c^{\text{out}}\right)\}^2, \tag{11} $$
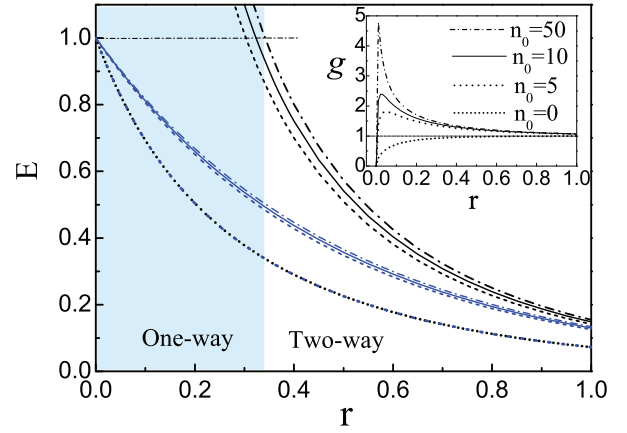

FIG. 3. (Color online) EPR paradox and quantum steering between the oscillator and the pulse: An EPR paradox and quantum steering of the oscillator by the pulse is detected when $E = E_{m|c} < 1$. [The black upper set of lines intercept with the line $E = 1$ at the value for r given by r_{epr} of Eq. (14)]. An EPR paradox and the steering of the pulse by the oscillator is detected when $E = E_{c|m} < 1$ (lower set of lines, blue online). The lowest two superposed curves show $E_{m|c}$ and $E_{c|m}$ with $n_0 = 0$ (dotted). The inset shows the optimal g to minimize $E_{m|c}$.

where $g_p = \langle P_m^{\text{out}}, X_c^{\text{out}} \rangle / \{\Delta X_c^{\text{out}}\}^2$. We note that the values of g_x, g_p that optimize for the EPR paradox are generally different than those that optimize the entanglement given by Eq. (6).

We assume the light to be initially in a vacuum state and the mechanical oscillator to be initially in a thermal state, with mean occupation number n_0. We can then calculate the prediction of the model (3) for the EPR paradox. We find

$$ \Delta_{\text{inf}} X_m^{\text{out}} = \Delta_{\text{inf}} P_m^{\text{out}}, \tag{12} $$

where we take $g_x = g_p = g$ and

$$ g = \frac{2e^r \sqrt{e^{2r} - 1}(n_0 + 1)}{2e^{2r}(n_0 + 1) - (2n_0 + 1)}. \tag{13} $$

The EPR paradox parameter $E_{m|c}$ is given in Fig. 3 vs r, for various values of initial thermal occupation n_0. The EPR paradox is realized when $E_{m|c} < 1$ and is predicted for values of squeezing parameter given by

$$ r > r_{\text{epr}} = \frac{1}{2} \ln \frac{2n_0 + 1}{n_0 + 1} \overset{n_0 \to \infty}{\sim} \frac{1}{2} \ln 2. \tag{14} $$

There is a temperature-dependent minimal squeezing parameter r_{epr} required to observe the paradox. We note, however, that for large thermal excitation n_0, the barrier becomes fixed, at $r_{\text{epr}} = \frac{1}{2} \ln 2$ as $n_0 \to \infty$.

This is the second noteworthy result. If the squeeze parameter r is large enough (that is, if there is enough entanglement between the oscillator and the pulse), the quantum steering of the oscillator by the pulse cannot be destroyed by further increasing the thermal noise value n_0. The quantum steering of the oscillator by the pulse can be achieved when $r > \frac{1}{2} \ln 2$. We expect that this effect arises because the second EPR system, the pulse, is *not* thermally excited. A different effect is expected for the EPR paradox between two oscillators.

B. Quantum steering of the pulse by the oscillator

An EPR paradox can be shown *the other way*, by the criterion

$$E_{c|m} = \Delta_{\inf} X_c^{\mathrm{out}} \Delta_{\inf} P_c^{\mathrm{out}} < 1. \qquad (15)$$

In this case, the optical pulse is "steered" by the measurements made on the mechanical oscillator. Results for the prediction of $E_{c|m}$ based on the model (3) are shown in Fig. 3. Such an EPR paradox is thermally insensitive, being possible for any value of initial thermal noise n_0 and for any squeezing parameter $r > 0$. Since EPR steering requires entanglement [35], this property underpins the thermal insensitivity noted above for entanglement. However, because the thermally insensitive steering is "one way" only, it does not correspond to an entanglement that can be detected symmetrically with respect to the oscillators—that is, with $g = 1$.

We note that there are two regimes for the observation of quantum steering and the EPR paradox. For $r \leqslant \frac{1}{2} \ln 2$, the only EPR paradox possible is $E_{c|m} < 1$ ("one-way steering" [30–32,50]). For $r > \frac{1}{2} \ln 2$, "two-way steering" becomes possible, and both paradoxes $E_{m|c} < 1$ or $E_{c|m} < 1$ can be confirmed.

Now we can understand the reason for the reduced sensitivity of the entanglement to the thermal noise of the oscillator. We have shown that a threshold squeezing parameter r is necessary to enable a steering of the thermal oscillator by the measurements made on the pulse, but there is no threshold for the steering of the pulse by the measurements on the oscillator. Entanglement is a defined as a shared quantity and must exist between the two systems if either form of steering is achieved [35]. Hence, entanglement is detected without the threshold, because this is possible for the quantum steering ($E_{c|m} < 1$) in one direction.

We argue, however, that the symmetric form of entanglement has its own special significance. The DGCZ criterion (5) is defined as that entanglement detected in a symmetric way, with $g_x = g_p = 1$. This distinguishes it from the thermally insensitive entanglement that is detected with g_x, g_p values very different than 1 (Fig. 3). For symmetric systems, such as two equivalently thermally excited oscillators, we conjecture that the limitations for entanglement are determined by the DGCZ criterion.

V. FUNDAMENTAL SIGNIFICANCE OF THE STEERING OF THE OSCILLATOR

To understand the importance of the EPR paradox that demonstrates quantum steering of the oscillator by the pulse, we first review the meaning of an EPR paradox [1]. An EPR paradox arises because the assumption of local realism (LR) would imply a simultaneous, precise predetermination for both of X_m^{out} and P_m^{out}. No quantum state of the oscillator, however, can be consistent with such a predetermination, for both momentum and position. Thus, EPR argue "elements of reality" exist that cannot be described by quantum mechanics, and an inconsistency between LR and the completeness of quantum mechanics is revealed.

We have found there is a thermal barrier for the EPR paradox that corresponds to the "steering" of the oscillator

system. This will make this sort of paradox more difficult to observe in practice. We note, however, that there is a fundamental significance in observing this sort of EPR paradox. On examining the EPR argument, we see that the EPR paradox (in this case) is based on the premise that the action of measuring the pulse cannot change the state of the oscillator. Hence, "elements of reality" are deduced for the oscillator system (not for the pulse system). These "elements of reality" become inconsistent with quantum mechanics when $E_{m|c} = \Delta_{\inf} X_m \Delta_{\inf} P_m < 1$. Hence, if local realism LR is correct, the *oscillator* cannot be described quantum mechanically. Alternatively, with an assumption that quantum mechanics is complete, it is the local reality of the o*scillator* that is disproved. For the second type of paradox, it is the reality of the *optical* state that is addressed. This is less useful for direct insights about quantum effects with matter.

VI. ENTANGLEMENT AND EPR PARADOX BETWEEN TWO MECHANICAL OSCILLATORS

A further challenge is to understand the thermal limits for obtaining an EPR paradox between two thermally excited mechanical oscillators. Bipartite entanglement between two mechanical oscillators can be achieved, in principle, by "swapping" the entanglement between the oscillator $m1$ and the output pulse, to an entanglement between the oscillator $m1$ at A and a second mechanical oscillator $m2$ at location B. Other methods are possible, the simplest being to couple two cavities to two incoming entangled light fields [16–22]. Most previous calculations have been limited to the generation of entanglement in steady-state regimes.

In this paper, we focus for simplicity on the results of calculations based on the first method (Fig. 4). After
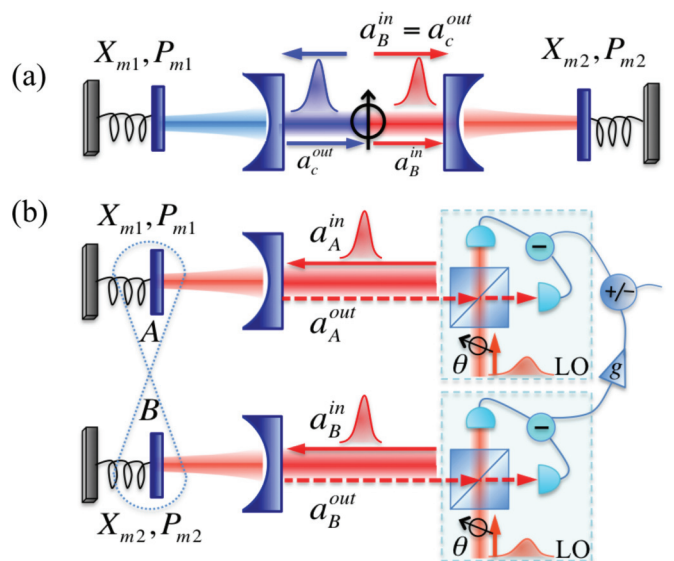

FIG. 4. (Color online) Entangling two oscillators: (a) Generation of the entanglement takes place when the output of the first cavity is injected into the second cavity, as red-detuned. The final states of the two oscillators at A and B will be entangled. (b) The entanglement can be verified, at a later stage, using two red-detuned pulses and the homodyne scheme set-up as depicted, to measure the conditional inference variances $\{\Delta_{\inf} X_{mj}\}^2$ and $\{\Delta_{\inf} P_{mj}\}^2$.

interaction with the first cavity, the entangling pulse "carries" the information about the quadratures of the first oscillator. As $r \to \infty$, we see from Eq. (3) that $X_{m1}^{\text{out}} = -P_c^{\text{out}}$ and $P_{m1}^{\text{out}} = -X_c^{\text{out}}$. Suppose then that after the coupling to the first cavity and oscillator $m1$, the output entangling pulse is then red-detuned relative to a second mechanical oscillator ($m2$) and cavity system. After an interaction with this pulse, the final amplitudes of the second oscillator are [24]

$$
\begin{aligned}
X_{m2}^{\text{out}} &= e^{-r'} X_{m2}^{\text{in}} + \sqrt{1 - e^{-2r'}} P_c^{\text{out}}, \\
P_{m2}^{\text{out}} &= e^{-r'} P_{m2}^{\text{in}} - \sqrt{1 - e^{-2r'}} X_c^{\text{out}},
\end{aligned}
\tag{16}
$$

where r' is the squeezing parameter of the second cavity and P_c^{out}, X_c^{out} are given by (3). As $r' \to \infty$, the relations between the quadratures of the second mechanical oscillator and the pulse are $X_{m2}^{\text{out}} = P_c^{\text{out}}$ and $P_{m2}^{\text{out}} = -X_c^{\text{out}}$, which will "swap" the EPR correlation $\Delta(P_c^{\text{out}} + g X_{m1}^{\text{out}})$, $\Delta(X_c^{\text{out}} - g P_c^{\text{out}})$ into an EPR correlation $\Delta(X_{m2}^{\text{out}} + g X_{m1}^{\text{out}})$, $\Delta(X_{m1}^{\text{out}} - g X_{m2}^{\text{out}})$ between the mechanical oscillators. Thus, an EPR paradox between the pulse and the first mechanical oscillator is directly transformed into an EPR paradox between two mechanical oscillators in the limit of $r' \gg r$ and $r' \to \infty$.

For practical reasons, since the thermal noise on the second oscillator can be significant, it is also informative to consider definite predictions for $r = r'$. The final entanglement and EPR paradox variances are readily calculated. We find

$$
\begin{aligned}
\left\{\Delta_{\inf} X_{m2}^{\text{out}}\right\}^2 &= \left\{\Delta\left(X_{m2}^{\text{out}} + g X_{m1}^{\text{out}}\right)\right\}^2 \\
&= e^{-2r}\left\{\Delta X_{m2}^{\text{in}}\right\}^2 + (g-1)^2(e^{2r}-1)\left\{\Delta P_c^{\text{in}}\right\}^2 \\
&\quad + [(g-1)e^r + e^{-r}]^2\left\{\Delta X_{m1}^{\text{in}}\right\}^2 \\
&= e^{-2r}(2n_{m2}+1) + (g-1)^2(e^{2r}-1) \\
&\quad + [(g-1)e^r + e^{-r}]^2(2n_{m1}+1),
\end{aligned}
\tag{17}
$$

and $\Delta_{\inf} P_{m2}^{\text{out}} = \Delta(P_{m2}^{\text{out}} - g P_{m1}^{\text{out}}) = \Delta_{\inf} X_{m2}^{\text{out}}$. Here, n_{m1}, n_{m2} are the thermal occupation numbers for the two oscillators.

A. Entanglement

Importantly, we note that a thermal barrier exists for the entanglement between two oscillators with equal thermal noise $n_{m1} = n_{m2} = n_0$. We examine the predictions for the entanglement criterion (6) but as applied to the two oscillators $m1$ and $m2$. In this case, the optimal g for the detection of entanglement (given by $\frac{\partial \Delta_{g,\text{ent}}}{\partial g} = 0$) is

$$
\begin{aligned}
g &= \sqrt{1 + (2n_0+1)^2/4e^{4r}(n_0+1)^2} \\
&\quad - (2n_0+1)/2e^{2r}(n_0+1),
\end{aligned}
\tag{18}
$$

which becomes $g \to 1$ in the limit of large n_0 and r. The threshold squeezing parameter for entanglement becomes $r_{\text{ent}} = \frac{1}{2}\ln 2n_0$ in this limit, which has the same large n_0 dependence as for the symmetric entanglement that is detected by the DGCZ entanglement criterion. The similarity is expected for two equivalent oscillators, since any entanglement that can be created between the two oscillators must be symmetric, i.e., unchanged on interchange of $A \longleftrightarrow B$, which will require $g_{\text{opt}} = g_x = g_p = 1$.

B. EPR paradox and steering

We now consider the EPR paradox and quantum steering, in particular how the oscillator $m1$ "steers" the oscillator $m2$. The optimal g for the detection of the EPR paradox $E_{m2|m1}$ is given by $\frac{\partial \Delta_{\inf} X_{m2}^{\text{out}}}{\partial g} = 0$. Solving gives

$$
g = \frac{(e^{2r}-1)(n_{m1}+1)}{e^{2r}(n_{m1}+1) - 1/2}.
\tag{19}
$$

The threshold squeezing parameter for the EPR paradox $E_{m2|m1}$ is then

$$
r_{\text{epr}} = \frac{1}{2}\ln\left(n_{m2}+1+\sqrt{(n_{m2}+1)^2 - \frac{n_{m1}+n_{m2}+1}{n_{m1}+1}}\right).
\tag{20}
$$

The threshold r_{epr} is plotted in Fig. 5(a).

We find that the "steering" of oscillator $m2$ by measurements on oscillator $m1$ is sensitive to the noise n_{m2} on $m2$ and depends logarithmically on n_{m2} in the limit $n_{m2} \to \infty$, but is *insensitive* to the noise n_{m1} on $m1$. For $n_{m2} = 0$, $r_{\text{epr}} = 0$ and no thermal barrier exists, which gives a behavior similar to that of the hybrid paradox $E_{c|m1}$. We note, however, that for $n_{m1} = 0$, a thermal barrier does exist and the threshold is given by $r_{\text{epr}} \to \frac{1}{2}\ln 2n_{m2}$ as $n_{m2} \to \infty$. This gives a different sort of behavior than that of the hybrid paradox $E_{m1|c}$. In that case, the threshold for the steering of the oscillator (by a noiseless pulse) was fixed at $r_{\text{epr}} = \frac{1}{2}\ln 2$ as the thermal noise of the oscillator increased. In this way, we learn that whether a thermally insensitive threshold exists for the steering of a mechanical oscillator will depend on the nature of entanglement preparation.

We can also consider the steering of the oscillator $m1$ by $m2$, which is the EPR paradox obtained when $E_{m1|m2} < 1$. We find

$$
\begin{aligned}
\left\{\Delta\left(X_{m1}^{\text{out}} + g X_{m2}^{\text{out}}\right)\right\}^2 &= [e^r - g(e^r - e^{-r})]^2\left\{\Delta X_{m1}^{\text{in}}\right\}^2 \\
&\quad + (1-g)^2(e^{2r}-1)\left\{\Delta P_c^{\text{in}}\right\}^2 \\
&\quad + g^2 e^{-2r}\left\{\Delta X_{m2}^{\text{in}}\right\}^2,
\end{aligned}
\tag{21}
$$

and $\Delta_{\inf} P_{m1}^{\text{out}} = \Delta(P_{m1}^{\text{out}} - g P_{m2}^{\text{out}}) = \Delta(X_{m1}^{\text{out}} + g X_{m2}^{\text{out}}) = \Delta_{\inf} X_{m1}^{\text{out}}$. The optimal g for the detection of $E_{m1|m2} < 1$ is given by $\frac{\partial \Delta_{\inf} X_{m1}^{\text{out}}}{\partial g} = 0$. Solving gives

$$
\begin{aligned}
g &= (e^{2r}-1)\left(\left\{\Delta X_{m1}^{\text{in}}\right\}^2 + \left\{\Delta P_c^{\text{in}}\right\}^2\right)\Big/\Big\{(e^{2r}+e^{-2r}-2) \\
&\quad \times \left\{\Delta X_{m1}^{\text{in}}\right\}^2 + (e^{2r}-1)\left\{\Delta P_c^{\text{in}}\right\}^2 + e^{-2r}\left\{\Delta X_{m2}^{\text{in}}\right\}^2\Big\}.
\end{aligned}
\tag{22}
$$

In this case, a thermally insensitive barrier to steering does exist; i.e., the threshold for the steering of oscillator $m1$ becomes insensitive to n_{m1}, as $n_{m1} \to \infty$. In fact, as $n_{m1} \to \infty$,

$$
\begin{aligned}
r_{\text{epr}} &= \frac{1}{2}\ln\Big(n_{m2}+2-\frac{1}{2(1+n_{m1})} \\
&\quad + \sqrt{\left[n_{m2}+2-\frac{1}{2(1+n_{m1})}\right]^2 - \frac{2n_{m2}}{1+n_{m1}}-2}\Big) \\
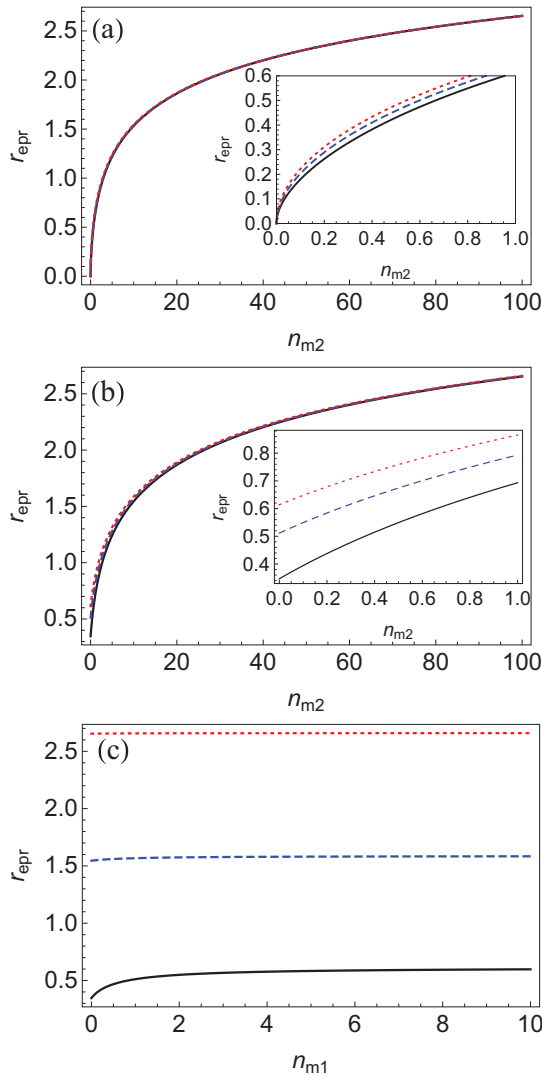&\to \frac{1}{2}\ln 2n_{m2}
\end{aligned}
\tag{23}
$$

FIG. 5. (Color online) Detecting an EPR paradox between two mechanical oscillators prepared using the scheme of Eqs. (16) with $r = r'$: (a) The threshold squeeze parameter r_{epr} for observation of the EPR paradox $E_{m2|m1} < 1$, vs the thermal occupation number of the oscillator $m2$. The curves are for $n_{m1} = 0$ (solid), $n_{m1} = 1$ (dashed), $n_{m1} = 1.5 \times 10^6$ (dotted). (b) The threshold squeeze parameter r_{epr} for observation of the EPR paradox $E_{m1|m2} < 1$, vs the thermal occupation number of the oscillator $m2$. The curves are for $n_{m1} = 0$ (solid), $n_{m1} = 1$ (dashed), $n_{m1} = 1.5 \times 10^6$ (dotted). (c) The threshold squeeze parameter r_{epr} for observation of the EPR paradox $E_{m1|m2} < 1$, vs the thermal occupation number of the oscillator $m1$. The curves are for $n_{m2} = 0$ (solid), $n_{m2} = 10$ (dashed), $n_{m2} = 100$ (dotted).

as evident in the plots of Figs. 5(b) and 5(c). For $n_{m2} = 0$,

$$ r_{epr} = \frac{1}{2}\ln[2 - \frac{1}{2(1 + n_{m1})} + \sqrt{\left[2 - \frac{1}{2(1 + n_{m1})}\right]^2 - 2}], $$

which approaches a fixed value as $n_{m1} \to \infty$, consistent with the result for the threshold r_{epr} for the steering of the oscillator $m1$ by the pulse. However, we note now (different than the result for $E_{m2|m1}$) that there is a sensitivity to the noise n_{m2} of the "steering" system. The limiting value of r_{epr} increases with

n_{m2}. For $n_{m1} = 0$, the optimal g given by Eq. (22) corresponds to a threshold squeezing parameter of $r_{epr} = \frac{1}{2}\ln 2n_{m2}$ in the limit of large n_{m2}.

In short, the steering between the two oscillators will become limited by the thermal noise on them. With a certain method of entanglement preparation, the steering threshold depends logarithmically on the thermal noise of the system being steered. In this case, there is very little dependence on the thermal noise of the steering system. If the entanglement is prepared another way, an oscillator can be steered (in the large thermal limit) independently of its own thermal noise, but then the steering threshold becomes sensitive logarithmically to large levels of thermal noise on the steering system. If we consider equal thermal noise levels n_0 for the two oscillators, the thermal barrier for the quantum steering (of either oscillator) remains sensitive to the thermal noise n_0 in the limit of large r: The threshold becomes $r > \frac{1}{2}\ln 4n_0$. This tells us that enough thermal noise will destroy the possibility of an EPR paradox, for any given squeeze parameter r that creates the entanglement.

The interesting feature noticed for this method of entanglement generation is that both the steering of oscillator $m2$ by $m1$ and the steering of $m1$ by $m2$ show an insensitivity to the thermal excitation level n_1 of oscillator $m1$. Thus, the "steering" of an oscillator M by another oscillator system S can be largely insensitive to the thermal excitation of M, or largely insensitive to the excitation of S, depending on the method of entanglement.

VII. CONCLUSION

In summary, we have examined the effect of an initial thermal excitation of an oscillator on observing an EPR paradox between a mesoscopic mechanical oscillator and a pulse. A thermal barrier exists for an EPR paradox that demonstrates a quantum "steering" of the mechanical oscillator. This is the most interesting paradox, since it tests local reality for a massive, mesoscopic system.

No equivalent thermal barrier exists for the EPR paradox that demonstrates a "steering" of the optical pulse. Similarly, as must be the case given that all types of steering require entanglement, no barrier exists for the entanglement between the oscillator and the pulse. This robust pulse-oscillator entanglement is only detectable using fully sensitive entanglement criteria, such as obtained by the positive partial transpose PPT method.

Importantly, the thermal barrier to the steering of the oscillator by the pulse is not insurmountable: It can be overcome for a large enough squeezing parameter. For temperatures above 100 mK, the condition is $r > 0.4$. This is much more favorable than the conditions $r > 2.4$ at $T \sim 100$ mK, and $r > 7$ at $T = 293$ K, required for observation of the symmetrically measured entanglement (where $g = 1$) [24].

Our results reveal the basic principle that the thermal noise of a mechanical oscillator tends to destroy the manifestation of the EPR paradox for that oscillator. If we consider the paradox between two symmetric oscillators, the thermal barrier increases with the thermal excitation number n_0 of the oscillators. This gives an explanation of the difficulty of observing mesoscopic EPR paradox effects between massive

oscillators at room temperature. We show that a very big advantage is to be gained if we consider the EPR paradox of an oscillator with an optical field, which is not thermally excited, since then (for this simple model) the threshold interaction to demonstrate quantum steering of the oscillator becomes fixed for $n_0 \gg 1$. By analyzing an entanglement swapping scenario that leads to an entanglement of two thermally excited oscillators, we show that this advantage is lost for the EPR paradox between two symmetric oscillators.

We conclude by commenting that a practical prediction for the EPR paradox must fully incorporate the main sources of decoherence. The practical limitation is that larger pulse-cavity interaction times lead to increased mechanical decoherence, due to the coupling to the environmental heat bath at temperature T_{bath}. The results presented here are based on an idealized model which ignores the coupling to the heat bath of the environment. More complete models have been given in Ref. [24]. However, those authors did not analyze the effects of an environmental heat bath on the EPR paradox but restricted their study to the symmetric DGCZ entanglement. Nonetheless, they estimated that the symmetric entanglement is feasible, provided $Qf \gg k_B T_{\text{bath}}/h$, where T_{bath} is the temperature of the environment, f is the frequency of mechanical vibration, and Q is the cavity quality. Based on the results of this paper, more optimistic predictions would be expected, both for entanglement as detected by the PPT condition and also for an EPR paradox.

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