Resonant scattering of a photon by an electron in the moderately-strong-pulsed laser field

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Resonant scattering of a photon by an electron in the presence of the moderately strong field of the circularly polarized pulsed laser wave is studied theoretically. The approximation used when a laser pulse duration is significantly greater than the characteristic oscillation time. The probability of such process for the moderately strong field $I \sim 10^{17}$–10^{18} W cm^{-2} is calculated. It is demonstrated that the resonant probability can be significantly greater than the probability of the Compton effect in the absence of the external field. The results may be experimentally verified, for example, in the FAIR project.

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I. INTRODUCTION

High-power pulsed lasers whose field cannot be simulated using a model of the plane monochromatic wave are used in modern experiments on the verification of quantum electrodynamics (QED) effects [1–6]. Thus, theoretical works widely employ the model of the pulsed electromagnetic field that represents the four-potential with an envelope function (see, for example, works studying elementary quantum processes in the presence of the pulsed field [7–27] and tunneling and multiphoton ionization of atoms and ions in the presence of a strong laser field [28–30]):

$$A(\varphi) = g(\phi)\, A_0(\varphi). \tag{1}$$

Here $A_0(\varphi)$ is the four-potential of a plane electromagnetic wave; $g(\phi)$ is the envelope function of the potential that satisfies the conditions $g(0) = 1$ and $g \to 0$ at $|\phi| \gg 1$ ($|\varphi| \gg \varphi_0$); $\varphi_0 = \omega t_{\mathrm{imp}}$, t_{imp} is the pulse duration in the laboratory frame of reference; $\varphi = (kr) = \omega t - \mathbf{k}\cdot\mathbf{r}$ is the wave phase which is responsible for the fast oscillations; $r = (t,\mathbf{r})$ is the four-radius vector; and the variable $\phi = \varphi/\varphi_0$ determines a slow change of the pulse shape.

The following condition is satisfied in the range of the optical frequency and picosecond pulse durations: $\varphi_0 = \omega t_{\mathrm{imp}} \gg 1$. Thus, the spectral density of the four-potential represents a sharp peak with an amplitude in order with φ_0 and a width in order with φ_0^{-1}. Therefore, it is expedient to consider ω as the frequency of the quasimonochromatic field. The relativistic system of units, where $\hbar = c = 1$ will be used throughout this paper.

The intensity of the process is governed by the classical relativistic-invariant parameter [31]

$$\eta = |e|\, a/m, \tag{2}$$

where e and m are the electron charge and mass, $a = F/\omega$, and F and ω are the field strength at the center of a pulse and the wave frequency in the laboratory coordinates. Note that the parameter is introduced when the elementary quantum processes in the presence of the electromagnetic wave field are studied (see, for example, [1–3,7–54]).

The resonant behavior of processes of the second order with respect to the fine-structure constant is one of the fundamental problems of QED in the presence of an external field. The study of this problem was started in the mid-1960s (see, for example, reviews [15,32,33], monographs [14,34–36], and works [37–50]). Oleinik [37,39] first considered resonances in the Compton effect in the field of a plane monochromatic wave but this analysis had rather fragmentary form. In Refs. [44,45] we considered the resonance of the direct and exchange diagrams in the relativistic case for the field of the low-intensity plane monochromatic electromagnetic wave, $\eta^2 \ll 1$. In Ref. [17] we considered the resonance of the direct diagram in the relativistic case for the field of the low-intensity plane pulsed electromagnetic wave:

$$\eta^2 \lesssim \varphi_0^{-1} \ll 1. \tag{3}$$

In this work, we study the scattering of a photon by an electron in the presence of the moderately-strong-pulsed laser field when the following condition is satisfied:

$$\varphi_0^{-1} \lesssim \eta^2 \lesssim 10/\varphi_0, \quad \eta^2 \ll 1. \tag{4}$$

The condition (4) corresponds to the field intensity $I \sim 10^{17}$–10^{18} W cm^{-2} in the range of the optical frequency. In this case the laser pulse duration is significantly greater than the characteristic oscillation time.

II. RESONANT AMPLITUDE

Let us choose the four-potential of a pulsed plane wave (1) propagating along the z axis in the form

$$A_0(\varphi) = a(e_x \cos\varphi + \delta e_y \sin\varphi), \tag{5}$$

where $k = (\omega,\mathbf{k})$ and $e_{x,y} = (0,\mathbf{e}_{x,y})$ are the four-momentum of the external-field photon and four-polarization vectors such that $k^2 = 0$, $e_{x,y}^2 = -1$, and $(e_{x,y}k) = 0$; $\delta = \pm 1$ corresponds to the circularly polarized wave.

For concretization of the theoretical computing, we choose the wave envelope function in the Gaussian form:

$$g(\phi) = \exp(-4\phi^2). \tag{6}$$

The factor 4 in the exponent of formula (6) matches the pulse duration t_{imp}, which corresponds to a decrease of the potential amplitude in e times.

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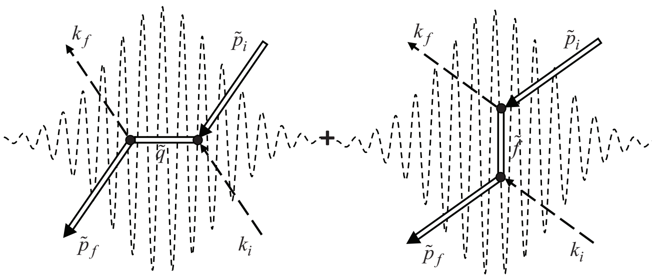

FIG. 1. Feynman diagram for the Compton effect in the field of the pulsed light wave for the direct and exchange parts.

The scattering amplitude of a photon with the four-momentum $k_i = (\omega_i, \mathbf{k}_i)$ by an electron with the four-momentum $p_i = (E_i, \mathbf{p}_i)$ in the external field (5) in the zero approximation with respect to the parameter φ_0^{-1} is given by the following expression (see Fig. 1):

$$S_{fi} = -ie^2 \int d^4r d^4r' \bar{\Psi}_{p_f}(r) \gamma^\mu G(r,r') \gamma^\nu \Psi_{p_i}(r') \times [A^*_\mu(k_f r) A_\nu(k_i r') + A^*_\nu(k_f r') A_\mu(k_i r)], \quad (7)$$

where $p_f = (E_f, \mathbf{p}_f)$ and $k_f = (\omega_f, \mathbf{k}_f)$ are the four-momenta of the final electron and photon, γ^ν ($\nu = 0,1,2,3$) are Dirac matrices, $A_\mu(k_i r') = \sqrt{2\pi/\omega_i}\, e_\mu \exp[-i(k_i r')]$ is the wave function of a photon, e_μ is the four-polarization vector of a photon, $\Psi_{p_i}(r)$ is the wave function of an electron in the rapidly oscillating field [55], and $G(r,r')$ is Green's function of the electron in the field (5) [31].

The amplitude can be represented as the sum of partial components:

$$S_{fi} = \sum_{l,l'=-\infty}^{\infty} S_{fi}^{(l,l')}, \quad (8)$$

where l, l' are integers. Each partial process corresponds to the process of interaction of the initial electron with four-momentum p_i with $|l'|$ photons of the wave ($l' < 0$ corresponds to the absorption from the wave, $l' > 0$ corresponds to the emission to the wave). An electron is transferred to the intermediate state with the four-momentum $q_{l'}$ with absorption of the initial photon for the direct diagram or with the four-momentum $f_{l'}$ with emission of the final photon for the exchange diagram. After interaction of the intermediate electron with $|l + l'|$ wave photons ($l + l' > 0$ corresponds to absorption from the wave, $l + l' < 0$ corresponds to emission to the wave) the electron undergoes a transition to the final state with four-momentum p_f after emission of the final photon for the direct diagram or after absorbing of the initial photon for the exchange diagram.

In the expansion of the amplitude in the small parameter η, up to the terms $\sim\eta^2$, the terms proportional to the zero power of η determine the amplitude of the Compton effect in the absence of the external field [56]. The terms proportional to the first power of the parameter η, i.e., $l' = 0$, $l = 1$, and $l' = 0$, $l = -1$, determine the corrections due to participation of a single wave photon in the process, and the terms proportional to the second power of the parameter η^2 correspond to the involvement of two photons: $l' = 0$, $l = \pm 2$; $l' = 1$, $l = 0$; $l' = 1$, $l = 2$; $l' = -1$, $l = 0$; $l' = -1$, $l = -2$; $l' = 2$, $l = 2$; $l' = -2$, $l = 2$. Note that the expansion of the amplitude in the parameter η was conducted in all terms except for the terms containing the product $\varphi_0 \eta^2$, because they are not small in the considered case (4).

It was shown that the character of partial processes that corresponds to the various numbers of photons absorbed from the wave or emitted to the wave in the pulsed laser field is described by the parameter β [17,48–50]:

$$\beta = \frac{q_{l'}^2 - m^2}{4(kq_{l'})} \varphi_0, \quad (9)$$

where $q_{l'}$ is given by

$$q_{l'} = p_i + k_i - l'k. \quad (10)$$

Processes corresponding to various numbers l, l' were classified as follows:

(i) Resonant processes, if the virtual particle may fall within the mass shell. The following condition is satisfied:

$$\beta(q_{l'}) \lesssim 1. \quad (11)$$

(ii) Nonresonant processes, for which we have

$$\beta(q_{l'}) \gg 1. \quad (12)$$

Previously it was shown that the resonant probability of the Compton effect in the low-intensity light wave can be several orders of magnitude greater than the corresponding probability without the external field [17]. We can compare the contribution of different partial processes for resonant and nonresonant cases to the total probability using an order-of-magnitude estimation:

$$\mathrm{W}^{(l-l',l')} \sim \begin{cases} D\varphi_0^2 \eta^{2(|l-l'|+|l'|)}, & \beta \lesssim 1; \\ D\varphi_0^2 \eta^{2(|l-l'|+|l'|)} \beta^{-2}, & \beta \gg 1; \end{cases} \quad (13)$$

where the function D is rather cumbersome and weakly depends on the parameter η. Note that the resonances are also possible in the next expansion terms. However, according to Eq. (13), the probability of these processes has the additional factor $\eta^2 \ll 1$. Thus, the resonant probability will be much less in the next order of the expansion.

The resonant character of the process can be observed for both direct and exchange amplitudes. The kinematic range that corresponds to the resonance of the exchange diagram is extremely narrow, and the contribution of this resonance to the total probability can be neglected. In this case, the total probability of the Compton effect in the field of the moderately-strong-pulsed wave is determined by the resonance of the direct amplitude only. Hence, the width related to the pulse character of the field dominates and the radiative broadening can be neglected.

The expression for the resonant amplitude of the direct diagram is written as

$$S_{\rm res}^{(d)} = \frac{B}{2(kq_1)} \delta^{(2)}(\mathbf{p}_{i,\perp} + \mathbf{k}_{i,\perp} - \mathbf{p}_{f,\perp} - \mathbf{k}_{f,\perp}) \times \delta(p_{i,-} + k_{i,-} - p_{f,-} - k_{f,-}) I(\beta, a)\, e'^{*\nu} e^\mu \times \bar{u}_{p_f} [M_{-1,\nu}(p_f, q_1)(\hat{q}_1 + m) M_{1,\mu}(q_1, p_i)] u_{p_i}, \quad (14)$$

$$M^{\nu}_{\pm 1}(p_f,q) = \pm\frac{y_0(p_f,q)}{2}e^{\mp ix}\gamma^{\nu} + \frac{m}{2(kq)}[\hat{\varepsilon}^{(\mp)}k^{\nu} - \hat{k}\varepsilon^{(\mp)\nu}] + \frac{m}{4}\left[\frac{1}{(kp_f)} - \frac{1}{(kq)}\right]\hat{\varepsilon}^{(\mp)}\hat{k}\gamma^{\nu}, \quad (15)$$

$$\varepsilon^{(\pm)} = e_x \pm i\delta e_y,$$

$$y_0(p_f,q) = m\eta\sqrt{-g^2(p_f,q)}, \quad \tan x = \frac{\delta(ge_y)}{(ge_x)}, \quad (16)$$

$$g(p_f,q) = \frac{p_f}{(kp_f)} - \frac{q}{(kq)}. \quad (17)$$

Here hats above the notation stand for scalar products ($\hat{\varepsilon} = \varepsilon_\nu\gamma^\nu$, $\hat{k} = k_\nu\gamma^\nu$) and the function $I_{l'}(\beta,a)$ is given by

$$I(\beta,a) = 2(kq_{l'})\left(\frac{4}{\pi^2}\right)\int_{-\infty}^{\infty} f_2(a-\zeta_{l'}) \times \frac{1}{q_{l'}^2 - m^2 + 2(kq_{l'})\zeta_{l'} + i0} f_1(\zeta_{l'})d\zeta_{l'}, \quad (18)$$

$$f_1(\zeta_{l'}) = \int_{-\infty}^{\infty} g(\phi)\exp\left\{i\varphi_0\left[\zeta_{l'}\phi - \frac{\sqrt{2\pi}}{8}\eta^2\frac{u}{\tilde{u}_1}\mathrm{erf}(2\sqrt{2}\phi)\right]\right\}d\phi, \quad (19)$$

$$f_2(a-\zeta_{l'}) = \int_{-\infty}^{\infty} g(\phi)\exp\left\{i\varphi_0\left[(a-\zeta_{l'})\phi + \frac{\sqrt{2\pi}}{8}\eta^2\frac{u'}{\tilde{u}_1}\mathrm{erf}(2\sqrt{2}\phi)\right]\right\}d\phi, \quad (20)$$

where

$$\zeta_{l'} = \frac{q_z - q_{l',z}}{\omega} = -1 - l'_*, \quad a = -l_*, \quad (21)$$

$$u = \frac{(kk_i)}{(kp_i)}, \quad u' = \frac{(kk_f)}{(kp_f)}, \quad \tilde{u}_1 = \frac{2(kq)}{m^2}, \quad (22)$$

$$0 \leqslant u \leqslant \tilde{u}_1, \quad 0 \leqslant u' \leqslant \tilde{u}_1. \quad (23)$$

Here l'_*, l_* are invariant parameters determined from the equations

$$p_i + k_i = q + l'_*k, \quad p_i + k_i + l_*k = p_f + k_f. \quad (24)$$

Under such a condition as Eq. (3), in Ref. [17] the terms proportional to η^2 in formulas (19) and (20) were disregarded. As a result, these formulas can be easily integrated:

$$f_1(\zeta_1) = \frac{1}{2}\sqrt{\pi}\exp\left(-\frac{\varphi_0^2\zeta_1^2}{16}\right), \quad (25)$$

$$f_2(a-\zeta_1) = \frac{1}{2}\sqrt{\pi}\exp\left(-\frac{\varphi_0^2(a-\zeta_1)^2}{16}\right). \quad (26)$$

The aim of this work is to study the behavior of the resonant profile in the moderately strong field (4). The lower end of this range corresponds to the applicability of expressions (25) and (26), and the upper corresponds to the applicability of the stationary phase method when integrals are calculated. In the range (4) the terms $\sim\eta^2\varphi_0$ cannot be neglected in the argument of the exponential function in expressions (19) and (20).

We use the expansion of the exponent in the Fourier series in the interval $-\Delta \leqslant \phi \leqslant \Delta$, where $\Delta = \sqrt{\ln(\varphi_0)/4}$ (the contribution of values $|\phi| \geqslant \Delta \sim \varphi_0^{-1}$ may be neglected), that is,

$$\exp\left(i\varphi_0\frac{\sqrt{2\pi}}{8}\eta^2\frac{u}{\tilde{u}_1}\mathrm{erf}(2\sqrt{2}\phi)\right) = \sum_{n_1=-\infty}^{\infty} a_{n_1}\exp\left(in_1\phi\frac{\pi}{\Delta}\right), \quad (27)$$

where expansion coefficients in the Fourier series are real:

$$a_{n_1} = \frac{1}{\Delta}\int_0^{\Delta}\cos\left[\varphi_0\left(n_1\lambda\phi - \frac{\sqrt{2\pi}}{8}\eta^2\frac{u}{\tilde{u}_1}\mathrm{erf}(2\sqrt{2}\phi)\right)\right]d\phi, \quad (28)$$

$$\lambda = \frac{\pi}{\Delta\varphi_0}. \quad (29)$$

The range of permissible values n_1 is

$$0 \lesssim n_1 \lesssim \eta^2\frac{u}{\lambda\tilde{u}_1}. \quad (30)$$

Similarly,

$$b_{n_2} = \frac{1}{\Delta}\int_0^{\Delta}\cos\left[\varphi_0\left(n_2\lambda\phi + \frac{\sqrt{2\pi}}{8}\eta^2\frac{u'}{\tilde{u}_1}\mathrm{erf}(2\sqrt{2}\phi)\right)\right]d\phi, \quad (31)$$

$$-\eta^2\frac{u'}{\lambda\tilde{u}_1} \lesssim n_2 \lesssim 0. \quad (32)$$

The summands in the expansion of the amplitude for which

$$a_{n_1}^2 \gtrsim \varphi_0^{-1}, \quad b_{n_2}^2 \gtrsim \varphi_0^{-1} \quad (33)$$

should be left only with an accuracy of the precision of calculations. Then functions f_1, f_2 take the form

$$f_1(\zeta_1) = \frac{\sqrt{\pi}}{2}\sum_{n_1=n_{1\min}}^{n_{1\max}} a_{n_1}\exp\left(-\frac{\varphi_0^2(\zeta_1+n_1\lambda)^2}{16}\right), \quad (34)$$

$$f_1(a-\zeta_1) = \frac{\sqrt{\pi}}{2}\sum_{n_2=n_{2\min}}^{n_{2\max}} b_{n_2}\exp\left(-\frac{\varphi_0^2(a-\zeta_1+n_2\lambda)^2}{16}\right). \quad (35)$$

Substitute the expressions for f_1, f_2 into (18) to get

$$I(\beta,a) = \sum_{n_2=n_{2\min}}^{n_{2\max}}\sum_{n_1=n_{1\min}}^{n_{1\max}} b_{n_2}a_{n_1}I^{(n_2,n_1)}(\beta_{n_1},a'), \quad (36)$$

where

$$I^{(n_1,n_2)}(\beta_{n_1},a') = \frac{\pi}{4(kq_{l'})}\exp\left(-\frac{\varphi_0^2a^2 + 8(\beta_{n_1} - \varphi_0a'/4)^2}{16}\right) \times\left[\mathrm{erfi}\left(\frac{\sqrt{2}\,(\beta_{n_1} - a'\varphi_0/4)}{2}\right) + i\right], \quad (37)$$

$$a' = a + (n_1+n_2)\lambda, \quad \beta_{n_1} = \beta - \frac{\varphi_0}{2}n_l\lambda. \quad (38)$$

The characteristic ranges of variation of parameters a and β are

$$0 \lesssim \beta \lesssim \varphi_0\eta^2\frac{u}{\tilde{u}_1}, \quad 2\eta^2\frac{u'}{\tilde{u}_1} \lesssim a \lesssim 2\eta^2\frac{u}{\tilde{u}_1}. \quad (39)$$

The relationship between the resonant parameter β [17] and the frequency of the initial photon ω_i is

$$\frac{\beta}{\varphi_0} = \frac{1}{2}\frac{1-\tilde{u}}{[1+\tilde{u}(\omega_i/\omega_{i,\mathrm{res}}-1)]}\left(\frac{\omega_i}{\omega_{i,\mathrm{res}}}-1\right), \quad (40)$$

$$\omega_i = \omega_{i,\mathrm{res}}\left(1+\frac{\beta}{2\varphi_0}\frac{1}{[1-(1+2\beta/\varphi_0)\,\tilde{u}]}\right). \quad (41)$$

Here the invariant parameter $\tilde{u}$ and frequency $\omega_{i,\mathrm{res}}$ that corresponds to condition $\beta = 0$ are given by

$$\tilde{u} = \frac{(kk_i)}{(p_i k_i)}, \quad 0 \leqslant \tilde{u} \leqslant u_1, \quad u_1 = \frac{2(kp_i)}{m^2}, \quad (42)$$

$$\omega_{i,\mathrm{res}} = \frac{mu_1}{2}\frac{\frac{E_i}{m}+\sqrt{\left(\frac{E_i}{m}\right)^2+u_1-1}\cos\tilde{\theta}_\Pi}{1-u_1+\left[\left(\frac{E_i}{m}\right)^2+u_1-1\right]\sin^2\tilde{\theta}_\Pi}, \quad (43)$$

$$\tilde{\theta}_\Pi = \angle(\mathbf{\Pi},\mathbf{n}_i), \quad \mathbf{\Pi} = \mathbf{p}_i - \mathbf{k}, \quad (44)$$

where $\mathbf{n}_i$ is the unit vector along the propagation direction of the incident photon. Note that the resonance of the amplitude that corresponds to the direct diagram is possible only under the condition $\tilde{u} < 1$. Thus, angles $\tilde{\theta}_\Pi$ are bounded at the values $u_1 > 1$:

$$\alpha_0 < \tilde{\theta}_\Pi < \pi, \quad \alpha_0 = \arccos\frac{E_i - \omega}{|\Pi|}. \quad (45)$$

The dispersion of resonant frequency in accordance with estimation (39) is

$$\Delta\omega_i \approx \omega_{i,\mathrm{res}}\eta^2\frac{u}{2\tilde{u}_1(1-\tilde{u})} \sim \omega_{i,\mathrm{res}}\eta^2 \ll \omega_{i,\mathrm{res}}, \quad (46)$$

where we disregard the scenarios that are close to the condition $\tilde{u} = 1$ ($\tilde{\theta}_\Pi = \alpha_0$), since the corresponding frequency of the resonant photon must be infinitely large, which is practically impossible.

Therefore the area of the resonant frequency increases with the intensity of the pulsed wave, in contrast to the case considered in Ref. [17], where $\Delta\omega_i \sim \varphi_0^{-1}\omega_{i,\mathrm{res}}$. Hence, the condition for the observation of the resonance of the direct diagram is written as

$$\left(\frac{\omega_i}{\omega_{i,\mathrm{res}}}-1\right) \sim \eta^2 \ll 1. \quad (47)$$

Figure 2 shows the resonant range of the initial photon frequency ω_i in the units of the initial electron energy as a function of the parameter $\alpha = (E_i/m)\tilde{\theta}_\Pi$ for the parameters of the E-144 experiment at Stanford Linear Accelerator Center (SLAC) [1]. The chosen geometry of the process is that the momenta of the initial photon and electron and the wave propagation direction belong to the same plane.

Thus, the resonance of the direct diagram is always accompanied by the resonance of the exchange diagram via an electron intermediate state and via a positron state in the range

$$u_1 > 1, \quad 1/u_1 < \tilde{u} < 1 \quad (48)$$

(see Fig. 2). Note that the resonance of the exchange diagram occurs if the final photon is emitted in the strictly determined narrow range of directions (see [17]), and the resonance of the direct diagram occurs for all of the emission angles of the final photon. Thus, the resonance of the only direct diagram can be observed in the experiment if the angles that correspond to the resonance of the exchange diagram are forbidden. Moreover, if the condition for the resonance of the direct diagram is satisfied, the contribution of the exchange diagram to the total probability is proportional to $\sim\varphi_0^{-1} \ll 1$ and, hence, can be neglected.

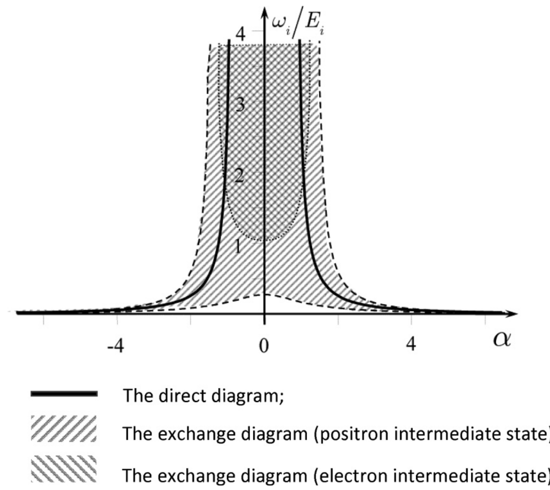

FIG. 2. The resonant frequency range of the initial photon in the units of the initial electron energy.

III. RESONANT PROBABILITY FOR THE DIRECT DIAGRAM

In the resonant approximation, $q^2 = m^2$ is put everywhere except the function $I(\beta,a')$, where q is the four-momentum of the intermediate electron: $q \equiv q_1$ [Eq. (10)]. After averaging over initial and summation over final polarizations of particles, the differential probability is represented as

$$dW_{fi}^{\mathrm{res}} = \frac{e^4 m^4\eta^4}{\pi^3\omega_i E_i V(kq)^2}\frac{\varphi_0^4}{\omega^4}H\,|I_1(\beta,l_*)|^2 \times \delta^{(2)}(\mathbf{p}_{f,\perp} - [\mathbf{p}_{i,\perp} + \mathbf{k}_{i,\perp} - \mathbf{k}_{i,\perp}]) \times \delta(p_{f,-} - [p_{i,-} + k_{i,-} - k_{f,-}])\frac{d^3\mathbf{k}_f d^2\mathbf{p}_{f,\perp}dp_{f,-}}{\omega_f p_{f,-}}. \quad (49)$$

Here, we used the notation

$$H = f(u',\tilde{u}_1)f(u,\tilde{u}_1) - g(u',\tilde{u}_1)g(u,\tilde{u}_1) - \frac{u'\tilde{u}'}{(1+\tilde{u}')} + \left(\frac{u+u'}{\tilde{u}_1} - 2\frac{uu'}{\tilde{u}_1^2}\right)\frac{uu'}{(1+u)(1+u')}, \quad (50)$$

$$f(u,\tilde{u}_1) = 2 + \frac{u^2}{1+u} - 4\frac{u}{\tilde{u}_1}\left(1-\frac{u}{\tilde{u}_1}\right), \quad (51)$$

$$g(u,u_1) = \frac{(2+u)(u_1-2u)u}{2u_1(1+u)}. \quad (52)$$

Under resonant conditions, invariant parameters u, u' [Eq. (22)] and the parameter $\tilde{u}_1$ are given by

$$u = \frac{(kk_i)}{(qk_i)} = \frac{\tilde{u}}{1-\tilde{u}}, \quad u' = \frac{(kk_f)}{(qk_f)}, \quad \tilde{u}_1 = \frac{u_1}{1-\tilde{u}}. \quad (53)$$

Note that $0 \leqslant u \leqslant \tilde{u}_1$ and $0 \leqslant u' \leqslant \tilde{u}_1$, and the parameter $\tilde{u}_1$ takes on the resonant value.

The integration in expression (49) to $d\mathbf{p}'_\perp dp'_-$ is relatively simple owing to the presence of three δ functions:

$$\frac{d^3\mathbf{k}_f d^2\mathbf{p}_{f,\perp} dp_{f,-}}{p_{f,-}\omega_f}\delta^{(2)}(\mathbf{p}_{f,\perp}-[\mathbf{p}_{i,\perp}\mathbf{k}_{i,\perp}-\mathbf{k}_{f,\perp}]) \times\delta(p_{f,-}-[p_{i,-}k_{i,-}-k_{f,-}]) \to \frac{\omega_f}{p_{f,-}}d\omega_f d\Omega', \quad (54)$$

Here, $d\Omega'=\sin\theta' d\theta' d\psi'$ is the element of solid angle in which the final photon is emitted and $\theta'=\angle(\mathbf{k},\mathbf{k}_f)$ and $\psi'=\angle(\mathbf{e}_x,\mathbf{k}_{f,\perp})$ are the azimuthal and polar angles of the scattered photon. Note also that the last two terms in expression (50) are eliminated after the integration of expression (49) with respect to angle ψ':

$$\int_0^{2\pi} H d\psi' = 2\pi[f(u',\tilde{u}_1)f(u,\tilde{u}_1)-g(u',\tilde{u}_1)g(u,\tilde{u}_1)]. \quad (55)$$

We change the integration with respect to the frequency of the resulting photon ω_f by the integration with respect to variable l_*:

$$\omega_f-\omega_f^{(0)} = l_*\frac{(kp_i)+(kk_i)-\left(kk_f^{(0)}\right)}{([p_i+k_i]n_f)-l_*(kn_f)} \approx l_*\frac{(kp_f)}{(pk_i)}\omega_f^{(0)} \Rightarrow d\omega_f \approx \omega_f^{(0)}\frac{(kp_f)}{(pk_i)}dl_*. \quad (56)$$

Thus, the differential probability of the process is represented as

$$W_{fi}^{\rm res} \approx \frac{2e^4\eta^4 m^2}{\pi\omega_i E_i V\tilde{u}_1}\varphi_0^2 t_{\rm imp} \times\left(\int_0^{\tilde{u}_1} P_{\rm res}\frac{f(u',\tilde{u}_1)f(u,\tilde{u}_1)-g(u',\tilde{u}_1)g(u,\tilde{u}_1)}{(1+u')^2}du'\right). \quad (57)$$

Analytic integration in Eq. (57) is not feasible because the function $P_{\rm res}(\beta)$ depends on the parameter u' in contrast to the region (3).

The behavior of the probability in the resonant area (resonant profile) is determined by the function $P_{\rm res}(\beta)$ (see Fig. 3):

$$P_{\rm res}(\beta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}|I(\beta,a)|^2 da. \quad (58)$$

Substituting expression (37) into this expression we obtain

$$P_{\rm res}(\beta)=\sum_{n_1=n_{1\min}}^{n_{1\max}}\sum_{n_2=n_{2\min}}^{n_{2\max}} a_{n_1}^2 b_{n_2}^2\frac{1}{2\pi}\int_{-\infty}^{\infty}|I^{(n_1,n_2)}|^2 da + 2\sum_{n_1=n_{1\min}}^{n_{1\max}}\sum_{n_2=n_{2\min}}^{n_{2\max}}\sum_{n_1'>n_1,n_2'>n_2} a_{n_1}a_{n_1'}b_{n_2}b_{n_2'} \times\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}{\rm Re}(I^{(n_1,n_2)}){\rm Re}(I^{(n_1',n_2')})da + \frac{1}{2\pi}\int_{-\infty}^{\infty}{\rm Im}(I^{(n_1,n_2)}){\rm Im}(I^{(n_1',n_2')}da)\right]. \quad (59)$$

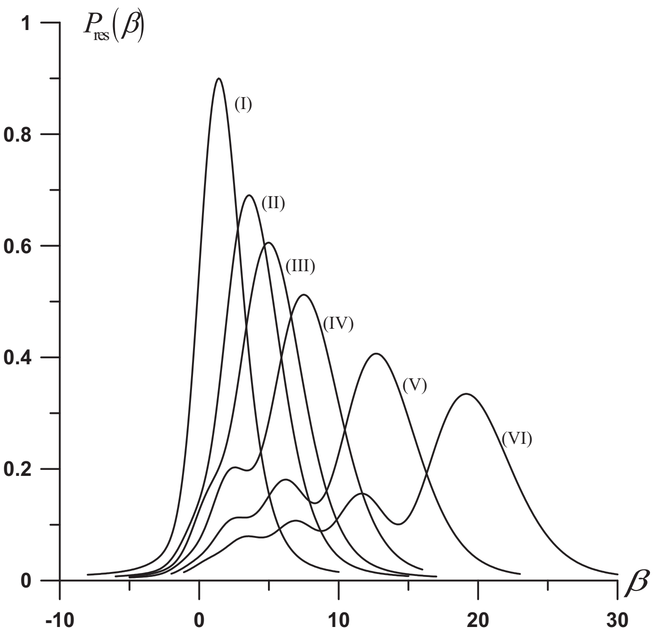

FIG. 3. The dependence of the resonant profile $P_{\rm res}$ on the resonant parameter β for $u/\tilde{u}_1=1/3$ and $u'/\tilde{u}_1=2/3$ in the intense pulsed light field ($\omega=2.35$ eV, $t_{\rm imp}/T=\sqrt{2}/2$). Field intensity in the pulse peak is $I=7\times10^{16}$ (I), $I=1.6\times10^{17}$ (II), $I=2\times10^{17}$ (III), $I=2.8\times10^{17}$ (IV), $I=4.4\times10^{17}$ (V), and $I=6.3\times10^{17}$(VI) W cm^{-2}.

Figure 3 shows that increasing the field intensity involves the appearance of new peaks. This is the result of the interference of the contributions of the front and posterior segments of the pulse. Also the resonant frequency range of the initial photon increases with the intensity increasing.

The ratio of the total resonant probability of the scattering of a photon by an electron via the direct diagram (12) to the total probability of the Compton effect in the absence of the external field [56] is

$$\frac{W_{fi}^{\rm res}}{w_{\rm Compt}} \approx \frac{t_{\rm imp}}{T}R(u,\tilde{u}_1,\beta), \quad (60)$$

$$R(\tilde{u},u_1,\beta)=\frac{2\eta^4\varphi_0^2}{\pi^2}\frac{1}{\tilde{u}_1 F(\tilde{u}_1)}\int_0^{\tilde{u}_1}P_{\rm res}(\beta) \times\frac{f(u',\tilde{u}_1)f(u,\tilde{u}_1)-g(u',\tilde{u}_1)g(u,\tilde{u}_1)}{(1+u')^2}du', \quad (61)$$

where T is the observation time ($T\gtrsim t_{\rm imp}$), which is determined by the experimental conditions, and the function $F(\tilde{u}_1)$ is given by

$$F(\tilde{u}_1)=\left(1-\frac{4}{\tilde{u}_1}-\frac{8}{\tilde{u}_1^2}\right)\ln(1+\tilde{u}_1) + \frac{1}{2}+\frac{8}{\tilde{u}_1}-\frac{1}{2(1+\tilde{u}_1)^2}. \quad (62)$$

Figure 4 demonstrates the ratio of the resonant probability of the scattering of a photon by an electron in the presence of the moderately-strong-pulsed wave field to the probability

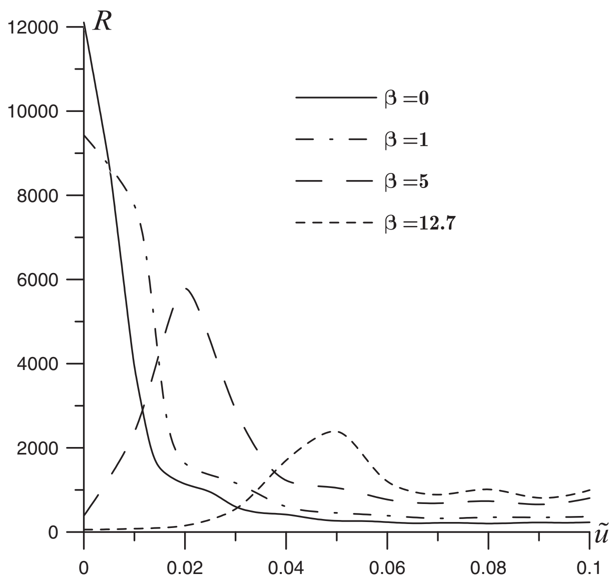

FIG. 4. Ratio R [Eq. (61)] of the resonant probability of scattering of a photon by an electron in the presence of the pulsed light field to the probability of the Compton effect in the absence of the external field vs parameters $\widetilde{u}$ and β at $u_1 = \widetilde{u}$ for the field intensity at the peak $I = 4.4 \times 10^{17}\ \mathrm{W\,cm^{-2}}$.

of the Compton effect as a function of parameters $\widetilde{u}$, β at $t_{\mathrm{imp}}/T = 1$ and $I = 4.4 \times 10^{17}\ \mathrm{W\,cm^{-2}}$. One can see that the resonant probability can be significantly greater than the probability of the Compton effect in the absence of the external field. For example, the probability ratio is $R \sim 10^4$ under the conditions $\eta = 0.25$ and $\widetilde{u} \ll 1$ in the range of the optical frequency $E_i/m \ll m/\omega \sim 10^5$. This effect may be experimentally verified in the framework of the international project at the GSI Facility for Antiproton and Ion Research (FAIR) [57].

IV. CONCLUSIONS

We draw the following conclusions based on the study of scattering of a photon by an electron in the field of a moderately-strong-pulsed optical wave under the conditions in Eq. (4).

(i) The spread of the initial photon frequency for which there is a resonance of the direct diagram in the moderately strong field is wider than in the area given in Eq. (3) and studied in Ref. [17] and increases with the pulsed wave intensity [see Eq. (46)].

(ii) The resonant probability of the Compton effect in the moderately strong field can be greater than the probability in the absence of the external field by several orders of magnitude. For example, the probability ratio is $R \sim 10^4$ for the field intensity at the peak of the pulse $I = 4.4 \times 10^{17}\ \mathrm{W\,cm^{-2}}$ to the case if the photon enters the narrow cone with the direction of propagation of the wave in the range of the optical frequency $E_i/m \ll m/\omega \sim 10^5$.

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