Coherent control of tunneling and traversal of ultracold atoms through vacuum-induced potentials

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We study the tunneling and traversal of ultracold Λ-type three-level atoms through vacuum-induced potentials in a high-Q mazer cavity. In particular, we discuss the effects of driving-induced atomic coherence on the passage of ultracold atoms through a high-Q mazer cavity. We consider phase time to study quantum tunneling which exhibits interesting features due to atomic coherence. For example, negative phase time appears for transmission of the atoms in both excited and ground states due to the presence of atomic coherence. Further, for certain values of the driving field, it is found that the phase tunneling time shows an alternate subclassical and superclassical traversal behavior with the increase in atomic momentum.

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I. INTRODUCTION

Quantum coherence and interference [1] lie at the heart of many interesting phenomena such as quantum beat lasers [2], lasing without inversion [3], electromagnetically induced transparency [4], coherent population trapping [5], laser cooling and trapping of atoms [6], and a number of other interesting effects [7]. It also provides some interesting effects in superconducting quantum bits [8], quantum internet [9], entanglement in trapped atomic ions [10], and ultracold atoms in optical lattices [11].

The phenomenon of quantum tunneling has been a problem of considerable interest due to its fundamental nature [12,13]. Phase time is one among various definitions of tunneling time which is studied extensively for a number of physical systems [13]. Recently, this phenomenon was addressed by Arun and Agarwal [14] in the context of tunneling and traversal of ultracold two-level atoms through vacuum-induced potentials [15] in a mazer cavity. The concept of mazer (microwave amplification via z-motion-induced emission of radiation) was introduced by Scully *et al.* [16]. It was shown that the interaction of ultracold atoms with the field inside a high-Q cavity can be treated as a scattering process with many interesting features [17–26]. In our early work, it is shown that the presence of dark states in cascade atomic configuration strongly affect the behavior of phase tunneling time [27]. Similarly, we found that phase tunneling time switches from subclassical to superclassical [28] in the case of off-resonant interactions of two-level atoms with vacuum cavity mode for a particular choice of detuning. In this paper, we study the tunneling and traversal of ultracold atoms in a coherently driven mazer.

II. MODEL

We consider ultracold Λ-type three-level atoms in which the two upper levels are coupled by a coherent driving field Ω [26]. The atoms are prepared initially in their excited states, which interact resonantly with the cavity field in the presence of the driving field. The atomic center-of-mass (c.m.) motion is treated quantum mechanically and the corresponding Hamiltonian of the atom-field system under dipole and rotating wave approximation is $H = H_{\mathrm{AF}} + H_I$, where

$$H_{\mathrm{AF}} = \frac{p_z^2}{2m} + \hbar \sum_{i=a,b,c} \omega_i \,|i\rangle\langle i| + \hbar\omega a^\dagger a, \tag{1}$$

and

$$\begin{aligned} H_I &= \hbar g u(z)(a^\dagger |b\rangle\langle a| + a\,|a\rangle\langle b|) \\ &\quad + \hbar u(z)(\Omega^* |c\rangle\langle a| + \Omega\,|a\rangle\langle c|), \end{aligned} \tag{2}$$

here H_{AF} is the Hamiltonian of the free atom and the free cavity field, and H_I is the interaction Hamiltonian in the presence of a driving field. While p_z is the atomic c.m. momentum along the z axis, m is the mass of the atom, $a(a^\dagger)$ corresponds to annihilation (creation) operators of the single-mode cavity field with frequency ω, g is the atom-field coupling strength, and Ω is the Rabi frequency of the coherent driving field. The operator $|i\rangle\langle i|(i = a,b,c)$ gives the population in level $|i\rangle$ with energy $\hbar\omega_i$ and $|i\rangle\langle j|(i,j = a,b,c; i \neq j)$ describes the transition from level $|j\rangle$ to level $|i\rangle$. For a mesa mode function, we have

$$u(z) = \begin{cases} 1 & \text{for } 0 < z < L, \\ 0 & \text{elsewhere.} \end{cases} \tag{3}$$

For the resonance case when the cavity field frequency ω is assumed to be equal to the transition frequency ω_{ab} (where $\omega_{ab} = \omega_a - \omega_b$) corresponding to levels $|a\rangle$ and $|b\rangle$ of the atom, the eigenstates and eigenvalues of the Hamiltonian can easily be obtained as discussed in Ref. [26]:

$$V_n^0 = 0, \quad V_n^{\pm} = \pm\hbar\sqrt{G_n^2 + |\Omega|^2}, \tag{4}$$

$$|\phi_n^0\rangle = \frac{G_n}{\sqrt{G_n^2 + |\Omega|^2}}\,|c,n\rangle - \frac{\Omega}{\sqrt{G_n^2 + |\Omega|^2}}\,|b,n+1\rangle\,, \tag{5}$$

$$|\phi_n^{\pm}\rangle = \frac{\sqrt{2}}{2}\left(|a,n\rangle \pm \frac{\Omega^*}{\sqrt{G_n^2 + |\Omega|^2}}\,|c,n\rangle \pm \frac{G_n}{\sqrt{G_n^2 + |\Omega|^2}}\,|b,n+1\rangle\right), \tag{6}$$

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where $G_n = g\sqrt{n+1}$; and n is the photon number of the cavity field.

We consider the initial atom-field state $|a,n\rangle$, and the initial wave packet of a moving free atom can be written as $\psi(z,t) = \exp(-ip_z^2 t/2m\hbar)\int dk A(k)e^{ikz}$. Here it is assumed that the Fourier amplitude $A(k)$ determining the position and extent of the wave packet is such that at $z=0$ (entry of the cavity) it has its peak at $t=0$. Therefore, the initial wave packet of the atom-field system comes to be $|\Psi(z,0)\rangle = \psi(z,0)|a,0\rangle$. The wave function of the atom-field system after the interaction may be obtained by expanding $|a,0\rangle$ in the dressed state basis $|\phi_n^{\pm}\rangle$. The component corresponding to $|\phi_n^{+}\rangle$ observes a square potential barrier of height V_n^+ and the component corresponding to $|\phi_n^{-}\rangle$ observes a square potential well with depth V_n^-. These components ($|\phi_n^{\pm}\rangle$) suffer both reflection and transmission. While the component associated with $|\phi_n^0\rangle$ passes through the cavity freely as it sees zero potential. The transmitted wave function after the interaction can be written as

$$|\Psi_T(z,t)\rangle = \int dk A(k) e^{-i\frac{\hbar k^2}{2m}t}[T_{a,n}|a,n\rangle + T_{b,n+1}|b,n+1\rangle + T_{c,n}|c,n\rangle]e^{ikz}, \quad (7)$$

where k is the c.m. momentum, t is the interaction time, and

$$T_{a,n} = \frac{1}{2}(\tau_n^+ + \tau_n^-), \quad (8)$$

$$T_{b,n+1} = \frac{G_n}{2\sqrt{G_n^2+|\Omega|^2}}(\tau_n^+ - \tau_n^-), \quad (9)$$

and

$$T_{c,n} = \frac{\Omega^*}{2\sqrt{G_n^2+|\Omega|^2}}(\tau_n^+ - \tau_n^-) \quad (10)$$

are the transmission amplitudes for atoms finally transmitted in states $|a,n\rangle$, $|b,n+1\rangle$, and $|c,n\rangle$, respectively. Here

$$\tau_n^{\pm}(k) = e^{-ikL}[\cos(k_n^{\pm}L) - i\Sigma_n^{\pm}(k)\sin(k_n^{\pm}L)]^{-1}, \quad (11)$$

with

$$k_n^{\pm} = \sqrt{k^2 \mp k_n^2}, \quad (12)$$

$$k_n^2 = 2m\sqrt{G_n^2+|\Omega|^2}, \quad (13)$$

$$\Sigma_n^{\pm}(k) = \frac{1}{2}\left(\frac{k_n^{\pm}}{k} + \frac{k}{k_n^{\pm}}\right). \quad (14)$$

In the absence of a coherent driving field, i.e., $\Omega = 0$, Eqs. (8)–(10) reduce to the results of Ref. [16], which concern one-photon mazer action with a two-level atom without atomic coherence.

To explain the atom-field interaction, the dressed-state picture of the Λ-type three-level atomic system is considered, which provides a clear insight of the interaction. The bare states of the system describe transitions between $|a\rangle \longrightarrow |b\rangle$ and $|a\rangle \longrightarrow |c\rangle$, governed by the Rabi frequency g and driving field Ω, respectively. In terms of the dressed states Eqs. (5) and (6), we have

$$|a,0\rangle = \frac{1}{\sqrt{2}}(|\phi_0^+\rangle + |\phi_0^-\rangle), \quad (15)$$

$$|b,1\rangle = \frac{1}{\sqrt{g^2+|\Omega|^2}}\left[\frac{g}{\sqrt{2}}(|\phi_0^+\rangle - |\phi_0^-\rangle) - \Omega^*|\phi_0^0\rangle\right], \quad (16)$$

$$|c,0\rangle = \frac{1}{\sqrt{g^2+|\Omega|^2}}\left[\frac{\Omega}{\sqrt{2}}(|\phi_0^+\rangle - |\phi_0^-\rangle) + g\,|\phi_0^0\rangle\right]. \quad (17)$$

Here the atomic transition between level $|a\rangle$ and $|b\rangle$ corresponds to the Rabi frequencies $\pm\Omega$ and $\pm(2g+\Omega)$, whereas the transition between $|a\rangle$ and $|c\rangle$ follows the $\pm g$ and $\pm(2\Omega + g)$ vacuum Rabi frequencies. It is evident that the atomic transitions $|a\rangle$ to $|b\rangle$ and $|a\rangle$ to $|c\rangle$ do not follow the vacuum Rabi frequency g and driving frequency Ω, respectively, as in the case of bare atomic state transitions. As a result, atoms initially in their lower states $|b\rangle$ or $|c\rangle$ experience a dark state.

III. PHASE TUNNELING TIME

In this section, we study the tunneling time of the atoms initially prepared in their excited states passing through a high-Q mazer cavity initially in a vacuum state. For this purpose, we consider the transmission amplitude say in the excited state $T_{a,0} \equiv |T_{a,0}|e^{i\phi(k)}$ which incorporates the effects of driving-induced atomic coherence see Eq. (8), and depends on the vacuum coupling energy $\hbar g \equiv \hbar^2 k_0^2/2m$. Here k_0 represents the amount of momentum for which the corresponding kinetic energies of the incoming atoms reach to the height of the potential barrier V_n^+. We consider a Gaussian wave packet with amplitude $A(k) = \exp[-(k-\bar{k})^2/\sigma^2]$, associated with the atoms with $\bar{k}$ and σ as its mean momentum and width, respectively. The normalized transmitted wave function for the atoms in the excited state $|a\rangle$ with $z \geqslant L$ is given by

$$|\Psi_T(z,t)\rangle = \frac{1}{(2\pi)^{3/4}}\sqrt{\frac{2}{\sigma}}\int_{-\infty}^{\infty} dk\ \exp[-(k-\bar{k})^2/\sigma^2] \times e^{-i(\hbar k^2/2m)t}|T_{a,0}|e^{i\phi(k)}e^{ikz}|a,0\rangle. \quad (18)$$

The integrand in Eq. (18) survives for small width σ of the wave packet only in a small range of wave numbers k centered about the mean momentum $\bar{k}$. In an earlier study, Arun and Agarwal obtained an approximate solution of the integral in Eq. (18) by considering terms up to second order in the Taylor expansion and assuming $\sigma \ll \bar{k}$ to approximate $|T_{a,0}(k)| \approx |T_{a,0}(\bar{k})|$ such that the transmitted wave function at $z = L$ is given by [14]

$$|\Psi_T(z,t)\rangle|_{z=L} \approx \frac{1}{(2\pi)^{3/4}}\sqrt{\frac{2}{\sigma}}\exp[i(\bar{k}L + \phi(\bar{k}) - \bar{E}t/\hbar)] \times |T_{a,0}(\bar{k})|\sqrt{\frac{2\pi}{\left(\frac{2}{\sigma^2}+i\alpha\right)}}\exp\left(\frac{-\bar{E}(t-t_{\rm ph})^2}{m\left(\frac{2}{\sigma^2}+i\alpha\right)}\right)|a,0\rangle. \quad (19)$$

Here, $\bar{E} = (\hbar\bar{k})^2/2m$ is the average energy of the incident atom, whereas $\alpha = \hbar t/m - \partial^2\phi/\partial k^2|_{k=\bar{k}}$ represents the spreading of the wave packet as it propagates. It is clear that phase time has no significance when Taylor expansion of the phase does not converge or terms beyond the second order are important in the expansion.

The envelop of the transmitted wave packet $|\langle a,0|\Psi_T(z,t)\rangle|^2$ reaches its peak value when the total phase $\Theta(k)$ of the integrand exhibits extremum at the wave number $k = \bar{k}$. The time for the appearance of the peak of the wave packet at the exit of the cavity can be obtained using the stationary phase condition as in [14]

$$\left.\frac{\partial\Theta(k)}{\partial k}\right|_{k=\bar{k}} = \left.\frac{\partial}{\partial k}[kL + \phi(k) - (\hbar k^2/2m)t]\right|_{k=\bar{k}} = 0. \tag{20}$$

This leads to the phase tunneling time

$$t_{\mathrm{ph}} = \frac{m}{\hbar k}\left.\left(\frac{\partial\phi}{\partial k} + L\right)\right|_{k=\bar{k}}. \tag{21}$$

In the absence of cavity there will be zero reflection, thus the transmission probability gets to its maximum value, i.e., $|T_{a,0}| = 1$, with invariant phase $\frac{\partial\phi}{\partial k} = 0$. Consequently, in free space, as clear from Eq. (21), the phase time converges to the classical time $t_{\mathrm{cl}} \equiv mL/\hbar\bar{k}$.

IV. RESULTS AND DISCUSSION

Here we present the results of our numerical simulations using Eq. (21). In Fig. 1, we show the plots of phase time (solid line) for atoms transmitted in excited state $|a,0\rangle$ as a function of mean momentum $\bar{k}/k_0$ for cavity length $k_0L = 10\pi$. We take, $\Omega = 400$ kHz [29] and by taking coupling constant g as 100–10 kHz, temperature of the atoms will be in the range 10^{-7}–10^{-8} K for $\bar{k}/k_0 = 0.1$ [14]. In the absence of any driving field [see Fig. 1(a)], the result discussed in [14] is obtained, which is quite obvious from the fact that our system reduces to a two-level atom. Here it is clear that the phase time is smaller than the classical time, i.e., the system behaves superclassically. Next we take the driving field of strength $\tilde{\Omega} = \Omega/g = 26$, while keeping the rest of the parameters unchanged. There is a clear change in the behavior of phase time [see Fig. 1(b)] as compared to the case with zero driving field. Here we get negative phase time values for ultracold atoms ($1 \gg \bar{k}/k_0$). Negative phase time implies that even before entering into the interaction region, the peak of the wave packet emerges from the exit of the cavity. It is a quite interesting situation and is probably due to the interference between the incoming wave and the wave which is reflected from the inner walls of the cavity. Quantitatively a negative value of phase time appears when the derivative of the phase of transmission amplitude has negative value with magnitude greater than L [see Eq. (21)]. Negative phase time is quite similar to the concept of negative group velocities for electromagnetic pulse propagation through dielectric media [30]. It is clear that phase time switches to negative values in the presence of a driving field. It is also noted that the phase time follows the resonant behavior of the transmission probability.

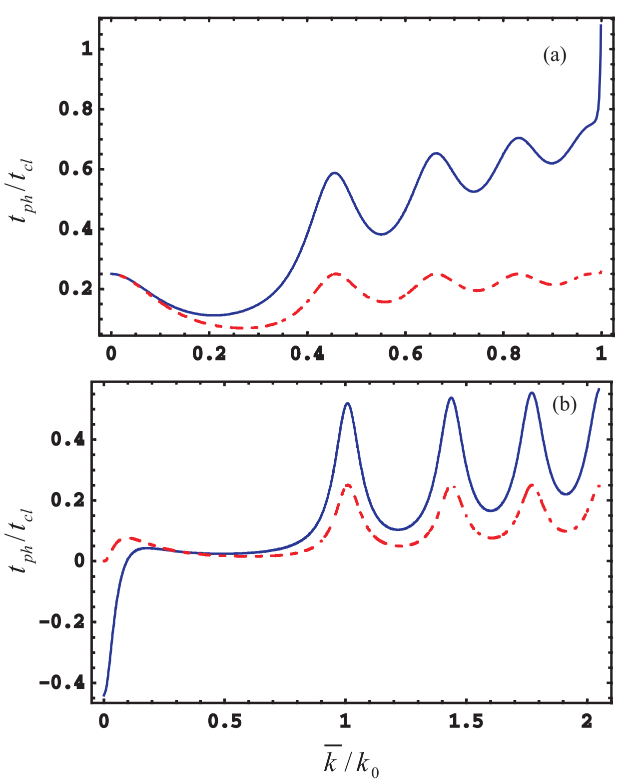

FIG. 1. (Color online) Dimensionless phase time (solid curve) and transmission probability $|T_{a,0}|^2$ (dashed curve) of a three-level atom in excited state vs the mean momentum. Here, $k_0L = 10\pi$ (a) $\tilde{\Omega} = 0$ and (b) $\tilde{\Omega} = 26$.

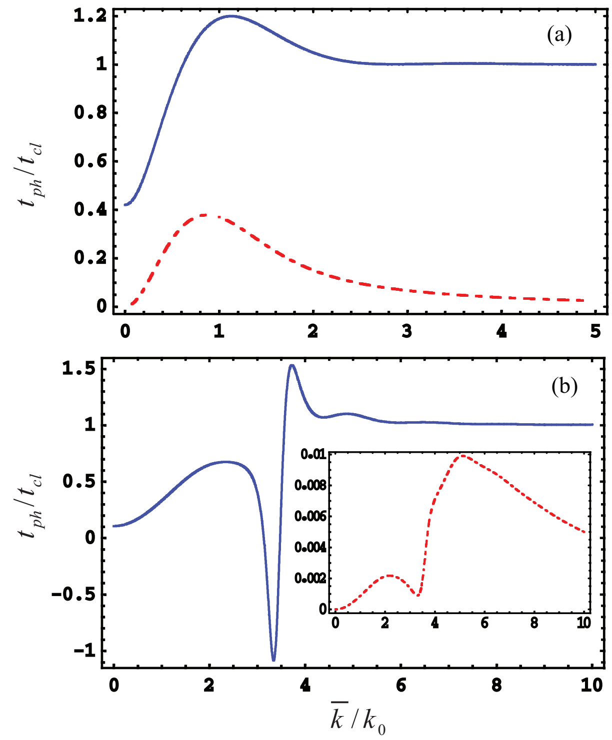

FIG. 2. (Color online) Dimensionless phase time (solid curve) for transmission of a three-level atom in state $|b,1\rangle$ vs the mean momentum. The transmission probability $|T_{b,1}|^2$ is shown with a dashed curve in (a) and in the inset of (b). Here, $k_0L = \pi/2$ and (a) $\tilde{\Omega} = 0$ and (b) $\tilde{\Omega} = 10$.

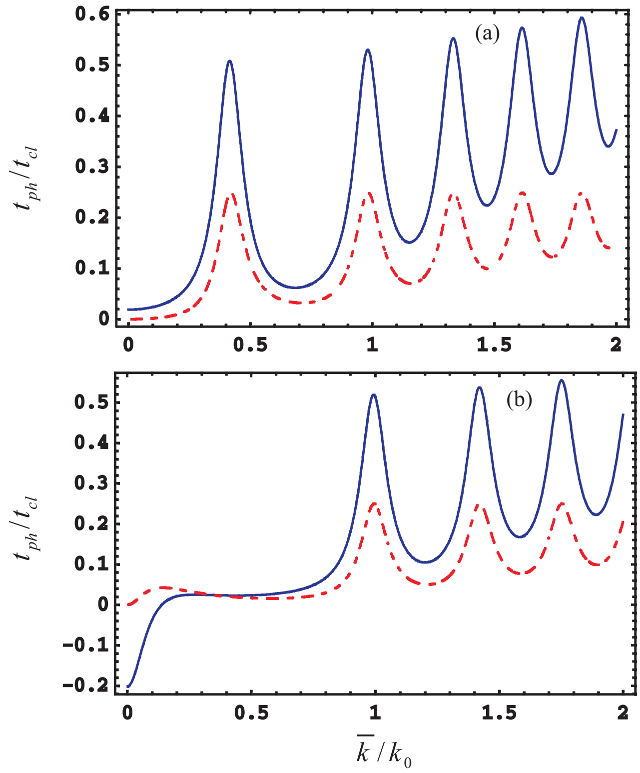

FIG. 3. (Color online) Dimensionless phase time (solid curve) and transmission probability (dashed curve) of a three-level atom in state $|c,0\rangle$ vs the mean momentum. Here, $k_0L = 10\pi$ and (a) $\tilde{\Omega} = 15$ and (b) $\tilde{\Omega} = 25$.

Next, we consider the behavior of the phase tunneling time for transmission of the atoms in their lower states, $|b,1\rangle$ and $|c,0\rangle$. In Fig. 2(a) for zero driving field, phase time is positive and shows both superclassical and subclassical behaviors corresponding to various values of $\bar{k}/k_0$. Now by introducing a driving field $\tilde{\Omega} = 10$, phase time takes negative values for some particular momentum and approaches the classical limit for fast atoms as shown in Fig. 2(b). The switching of phase time from positive to negative values by adjusting $\tilde{\Omega}$ appears for transmission of the atoms in state $|b,1\rangle$. For transmission of the atoms in state $|c,0\rangle$ a similar flipping of phase tunneling time by tuning $\tilde{\Omega}$ is shown in Fig. 3.

To confirm the early arrival of the transmitted peak, we perform a numerical simulation of the Gaussian wave-packet propagation. In Fig. 4, we plot the normalized probability density $P \equiv |\langle b,1|\Psi_T(z,t)\rangle|^2/\sigma$ at the exit of the cavity, i.e., $z = L$. It can be seen that the peak of the transmitted wave packet appears at $t/t_{\rm cl} \approx -1.1$ which matches the phase

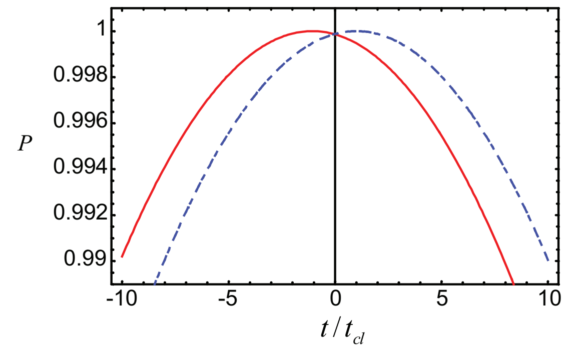

FIG. 4. (Color online) Normalized probability density $P \equiv |\langle b,1|\Psi_T(z,t)\rangle|^2/\sigma$ at $z = L$ as a function of dimensionless time $t/t_{\rm cl}$. The solid (dashed) curve represents P after transmission through the cavity (in free space). The parameters used are $k_0L = \pi/2$, $\tilde{\Omega} = 10$, $\sigma/k_0 = 0.01$, $\bar{k}/k_0 = 3.33$. Both solid and dashed curves are normalized to unity.

time in Fig. 2(b) at $\bar{k}/k_0 = 3.33$. For comparison, we also plot the envelope of the wave packet that travels through the same distance L in free space. In this case, peak of the free wave packet occurs at the expected time, i.e., $t/t_{\rm cl} = 1$.

V. CONCLUSIONS

In conclusion, we have considered the tunneling and traversal of coherently driven ultracold Λ-type three-level atoms through a high-Q mazer cavity. Our results show that the behavior of phase tunneling time is remarkably modified in the presence of driving-induced atomic coherence. In particular, we obtained negative phase time values for all three states with different values of the driving field. More interestingly, we find that phase time can be switched between superclassical and subclassical traversal behavior by controlling the applied driving field Ω. It is also noted that phase time shows an alternate subclassical and superclassical traversal behavior with the increase in energies of the atoms.

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