Two-level system with broken inversion symmetry coupled to a quantum harmonic oscillator

H. K. Avetissian and G. F. Mkrtchian
Centre of Strong Fields Physics, Yerevan State University, 1 A. Manukian, Yerevan 0025, Armenia
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We study the generalized Jaynes-Cummings model of quantum optics at the inversion-symmetry breaking and in the ultrastrong coupling regime. With the help of a generalized multiphoton rotating-wave approximation, we study the stationary solutions of the Schrödinger equation. It is shown that the problem is reduced to resonant interaction of two position-displaced harmonic oscillators. Explicit expressions for the eigenstates and eigenvalues of generalized Jaynes-Cummings Hamiltonian are presented. We exemplify our physical model with analytical and numerical considerations regarding Rabi oscillations, collapse and revivals of the initial population of a two-level system, and photon distribution function at the direct multiphoton resonant coupling.

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I. INTRODUCTION

The two-level system coupled to a quantum harmonic oscillator (e.g., a single radiation mode) as a simple and tractable model has played a central role in many branches of contemporary physics ranging from quantum optics to condensed-matter physics. In quantum optics it describes a two-level atom resonantly coupled to a single mode electromagnetic radiation [1], the so-called Jaynes-Cummings (JC) model [2]. It accurately describes trapped ion experiments for quantum informatics [3]. In condensed-matter physics we may include here the Holstein model [4], graphene in the magnetic field [5] or in the quantized single mode radiation field [6], quantum dots coupled to photonic cavities [7], and circuit quantum electrodynamics (QED) setups where superconducting qubits are coupled to microwave cavities [8]. Even though the underlying setups of mentioned systems are different, the physics is similar to cavity QED, where first experiments have been done toward the realization of the JC model [9]. Cavity QED can be divided into three coupling regimes: weak, strong, and ultrastrong. For weak coupling, the atom-photon interaction rate is smaller than the atomic and cavity field decay rates. In this case one can manipulate by the spontaneous emission rate compared with its vacuum level by tuning discrete cavity modes [10]. In the strong-coupling regime, when the emitter-photon interaction becomes larger than the combined decay rate, instead of the irreversible spontaneous emission process the coherent periodic energy exchange between the emitter and the photon field in the form of Rabi oscillations takes place [11]. Thanks to recent achievements in solid-state semiconductor [12] or superconductor systems [13] one can achieve the ultrastrong-coupling regime, where the coupling strength is comparable to appreciable fractions of the mode frequency. In this regime new nonlinear phenomena are visible [14]. Besides, in these setups one can enrich the conventional JC model including new interaction terms inaccessible in conventional cavity QED setups. One of the new factors which can be incorporated into the JC model is an inversion-symmetry breaking (ISB). Thus, in the conventional JC model, as well as in the Rabi model with classical radiation field, one assumes that the diagonal matrix elements of the dipole moment operator are zero, that is, the states possess a certain spatial parity, and the levels are not degenerated. Nevertheless, in various systems of interest, there are intrinsic or extrinsic reasons for ISB. The inversion symmetry of a system can be broken either by a system Hamiltonian or the stationary states may not have this symmetry. As has been shown in Refs. [15,16], when the quantum system has permanent dipole moments in the stationary states, or the level is degenerated upon orbital momentum, there are new multiphoton effects in the quantum dynamics of the system subjected to a strong laser field. Furthermore, these systems have an advantage, which allows us to generate radiation with Rabi frequency [17] and moderately high harmonics by optical pulses [18]. For the semiconductor version of the JC model one can achieve ISB by the asymmetric quantum dots [19]. In the circuit QED setups it appears naturally as a consequence of internal asymmetry. For flux qubit the potential landscape is reduced to a double-well potential [13], for a Cooper pair box ISB takes place at a setup far from the charge degeneracy point [20]. Thus, it is of interest to study the consequence of ISB on the quantized version of the Rabi model, where multiphoton effects are expected in the ultrastrong-coupling regime.

In the present work we study the effect of ISB on the quantum dynamics of a two-level system interacting with a quantized harmonic oscillator. Particularly, we consider the consequences of ISB on the eigenstates and eigenenergies of the generalized JC Hamiltonian, and on the dynamics of Rabi oscillations, collapse, and revival. It is shown that ISB substantially alters the dynamics of the system compared with the conventional JC one. Similar to the quasiclassical case [16] it is possible direct multiphoton transitions, and, as a consequence, there are Rabi oscillations with periodic exchange of several photons between the emitter and the radiation (bosonic) field. We consider the ultrastrong-coupling regime. Accordingly, the quantum dynamics of the considered system is investigated using a resonant approximation.

The paper is organized as follows. In Sec. II the model Hamiltonian is presented and diagonalized in the scope of a resonant approximation. In Sec. III we consider temporal quantum dynamics of the considered system and present corresponding numerical simulations. Finally, conclusions are given in Sec. IV.

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II. BASIC MODEL HAMILTONIAN AND DRESSED STATES PICTURE

Assuming here a two-level system with ISB coupled to a quantum harmonic oscillator, the model Hamiltonian can be written as

$$\widehat{H} = \hbar\omega\left(\widehat{a}^{+}\widehat{a} + \frac{1}{2}\right) + \frac{\hbar\omega_0}{2}\widehat{\sigma}_z + \hbar(-\lambda_g\widehat{\sigma}_{\downarrow} + \lambda_e\widehat{\sigma}_{\uparrow} + \lambda_{eg}\widehat{\sigma}_x)(\widehat{a}^{+} + \widehat{a}). \quad (1)$$

The first two terms in Eq. (1) correspond to the free harmonic oscillator of frequency ω and a two-level system with the transition frequency ω_0, respectively. The final term gives the interaction between the oscillator and a two-level system. Creation and annihilation operators, $\widehat{a}^{+}$and $\widehat{a}$, satisfy the bosonic commutation rules, $\widehat{\sigma}_x$, $\widehat{\sigma}_z$ are Pauli operators, $\widehat{\sigma}_{\uparrow} = (\widehat{I} + \widehat{\sigma}_z)/2$ and $\widehat{\sigma}_{\downarrow} = (\widehat{I} - \widehat{\sigma}_z)/2$ are projection operators and are the result of ISB. These terms distinguish the systems being considered from the conventional JC model. At $\lambda_g = \lambda_e = 0$, one will obtain the usual Hamiltonian for the JC model (including also counter-rotating terms) with coupling $\hbar\lambda_{eg}$. In the case of atoms or molecules and quantum dots λ_g and λ_e correspond to mean dipole moments in states of indefinite parity, while λ_{eg} corresponds to the transition dipole moment. In the case of circuit QED see Refs. [13,20]. Without loss of generality we have assumed that ground and excited states have mean dipole moments of opposite signs, and λ_e, $\lambda_g \geqslant 0$.

At first we will diagonalize the Hamiltonian (1), which is straightforward in the dressed states picture. As a JC model, our model does not admit exact analytical solution. One of the most powerful approximations for the solution of a JC model is the resonant or so-called rotating-wave approximation (RWA), which is valid at near resonance $|\omega_0 - \omega| \ll \omega$ and weak coupling between the two systems $|\lambda_{eg}| \ll \omega_0$ [1]. For our model a generalized multiphoton RWA is needed. The first step is to rewrite Hamiltonian (1) in the form

$$\widehat{H} = \widehat{H}_0 + \widehat{V}, \quad (2)$$

where

$$\widehat{H}_0 = \widehat{H}_{\uparrow} \otimes \widehat{\sigma}_{\uparrow} + \widehat{H}_{\downarrow} \otimes \widehat{\sigma}_{\downarrow} \quad (3)$$

represents two noncoupled position-displaced oscillators:

$$\widehat{H}_{\uparrow} = \hbar\omega\left(\widehat{a}^{+}\widehat{a} + \frac{1}{2}\right) + \frac{\hbar\omega_0}{2} + \hbar\lambda_e(\widehat{a}^{+} + \widehat{a}), \quad (4)$$

$$\widehat{H}_{\downarrow} = \hbar\omega\left(\widehat{a}^{+}\widehat{a} + \frac{1}{2}\right) - \frac{\hbar\omega_0}{2} - \hbar\lambda_g(\widehat{a}^{+} + \widehat{a}), \quad (5)$$

and

$$\widehat{V} = \hbar\lambda_{eg}\widehat{\sigma}_x(\widehat{a}^{+} + \widehat{a}) \quad (6)$$

is the interaction part. Hamiltonians (4) and (5) admit exact diagonalization. A schematic illustration of the two position-displaced harmonic oscillators with coupling $\widehat{V}$ is given in Fig. 1. It is easy to see that in each well the eigenstates are

$$|\uparrow, N^{(\lambda_e)}\rangle \equiv |\uparrow\rangle \otimes e^{-\frac{\lambda_e}{\omega}(\hat{a}^{\dagger} - \hat{a})}|N\rangle, \quad (7)$$

$$|\downarrow, N^{(\lambda_g)}\rangle \equiv |\downarrow\rangle \otimes e^{\frac{\lambda_g}{\omega}(\hat{a}^{\dagger} - \hat{a})}|N\rangle, \quad (8)$$

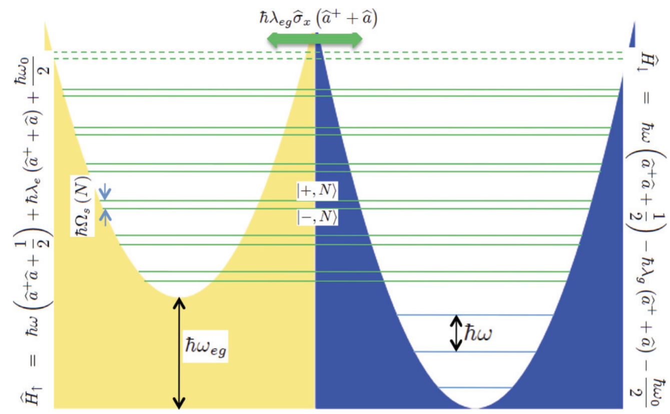

FIG. 1. (Color online) Schematic illustration of the two coupled position-displaced harmonic oscillators. In each well the eigenstates are displaced Fock states. At the resonance, the energy levels starting from the ground state of upper harmonic oscillator are degenerated. The coupling removes this degeneracy, leading to symmetric and asymmetric entangled states. The splitting of levels is defined by the vacuum multiphoton Rabi frequency.

with energies

$$E_{eN}^{(0)} = \frac{\hbar\omega_0}{2} + \hbar\omega\left(N + \frac{1}{2}\right) - \hbar\frac{\lambda_e^2}{\omega}, \quad (9)$$

$$E_{gN}^{(0)} = -\frac{\hbar\omega_0}{2} + \hbar\omega\left(N + \frac{1}{2}\right) - \hbar\frac{\lambda_g^2}{\omega}. \quad (10)$$

Hear $D(\alpha) = e^{\alpha(\hat{a}^{\dagger} - \hat{a})}$ is the displacement operator and quantum number $N = 0,1,\ldots$. The states $|\uparrow\rangle$, $|\downarrow\rangle$ are eigenstates of $\hat{\sigma}_z$ and the states $|N^{(\lambda_e)}\rangle$, $|N^{(\lambda_g)}\rangle$ are position-displaced Fock states:

$$|N^{(\lambda_e)}\rangle = e^{-(\lambda_e/\omega)(\hat{a}^{\dagger} - \hat{a})}|N\rangle = \sum_M I_{N,M}\left(\frac{\lambda_e^2}{\omega^2}\right)|M\rangle,$$

$$|N^{(\lambda_g)}\rangle = e^{(\lambda_g/\omega)(\hat{a}^{\dagger} - \hat{a})}|N\rangle = \sum_M I_{M,N}\left(\frac{\lambda_g^2}{\omega^2}\right)|M\rangle, \quad (11)$$

where $I_{N,M}(\alpha)$ is the Laguerre function and is defined via generalized Laguerre polynomials $L_n^l(\alpha)$ as follows:

$$I_{s,s'}(\alpha) = \sqrt{\frac{s'!}{s!}}e^{-\alpha/2}\alpha^{(s-s')/2}L_{s'}^{s-s'}(\alpha) = (-1)^{s-s'}I_{s',s}(\alpha),$$

$$L_n^l(\alpha) = \frac{1}{n!}e^{\alpha}\alpha^{-l}\frac{d^n}{d\alpha^n}(e^{-\alpha}\alpha^{n+l}). \quad (12)$$

Particularly, $|0^{(\lambda_e)}\rangle$ and $|0^{(\lambda_g)}\rangle$ are the Glauber or coherent states with mean number of photons λ_e^2/ω^2 and λ_g^2/ω^2. Thus, we have two ladders shifted by the energy:

$$\hbar\omega_{eg} = \hbar\left(\omega_0 + \lambda_g^2/\omega - \lambda_e^2/\omega\right). \quad (13)$$

The coupling term (6) $\widehat{V} \sim \widehat{\sigma}_x$ induces transitions between these two manifolds. At the resonance

$$\left|\delta_n^{(0)}\right| \ll \omega, \quad (14)$$

where

$$\delta_n^{(0)} = \omega n - \omega_{eg} \quad (15)$$

is the resonance detuning $(n = 1,2,\ldots)$, the equidistant ladders are crossed: $E_{eN}^{(0)} \simeq E_{gN+n}^{(0)}$, and the energy levels starting from the ground state of upper harmonic oscillator are nearly degenerated. The coupling (6) removes this degeneracy, leading to symmetric and asymmetric entangled states. The splitting of levels is defined by the vacuum multiphoton Rabi frequency. In this case we should apply secular perturbation theory [21], resulting in

$$|\alpha,N\rangle = (C_{\downarrow}^{(\alpha)}|\downarrow,N^{(\lambda_g)}\rangle + C_{\uparrow}^{(\alpha)}|\uparrow,(N-n)^{(\lambda_e)}\rangle), \quad (16)$$

$$E_{\alpha,N}^{(0)} = \frac{E_{gN}^{(0)}+E_{eN-n}^{(0)}}{2} + \alpha\sqrt{\left(\frac{\delta_n^{(0)}}{2}\right)^2 + |V_N(n)|^2}, \quad (17)$$

where $\alpha=\pm$; $C_{\downarrow}^{(\alpha)}$ and $C_{\uparrow}^{(\alpha)}$ are constant with the ratio

$$\frac{C_{\uparrow}^{(\alpha)}}{C_{\downarrow}^{(\alpha)}} = \frac{V_N(n)}{E_{\alpha,N}^{(0)} - E_{eN-n}^{(0)}}, \quad (18)$$

and transition matrix element is

$$V_N(n) = \langle\downarrow,N^{(\lambda_g)}|\widehat{V}|,\uparrow,(N-n)^{(\lambda_e)}\rangle = \hbar\lambda_{eg}\left[\frac{\lambda_g-\lambda_e}{\omega} - \frac{n}{\overline{\lambda}}\right] I_{N-n,N}(\overline{\lambda}^2). \quad (19)$$

Here $\overline{\lambda}=(\lambda_g+\lambda_e)/\omega$ is the effective scaled mean dipole moment. For the exact resonance ($\delta_n^{(0)}=0$), starting from the level $N=n$ we have symmetric and asymmetric entangled states,

$$|\pm,N\rangle = \frac{1}{\sqrt{2}}(|\downarrow,N^{(\lambda_g)}\rangle \pm |\uparrow,(N-n)^{(\lambda_e)}\rangle) \quad (20)$$

with energies $E_{\pm,N}^{(0)} = E_{gN}^{(0)} \pm |V_N(n)|$, while for $N=0,1\ldots n-1$, we have eigenstates $|\downarrow,N^{(\lambda_g)}\rangle$ and energy $E_{gN}^{(0)}$. For the conventional JC model there is a selection rule: $V_N(n)\neq 0$ only for $n=\pm 1$. This also follows from Eq. (19) in the limit $\lambda_e,\lambda_g\to 0$. That is why in that case only one photon Rabi oscillation takes place. In our model due to ISB there are transitions with arbitrary n giving rise to multiphoton coherent transitions. Besides, at $\lambda_g\neq 0$ in the ground state $|\downarrow\rangle\otimes\ |0^{(\lambda_g)}\rangle$ bosonic field is in the coherent state with the mean photon number λ_g^2/ω^2. The solutions (17) are valid at near multiphoton resonance $\omega_{eg}\simeq n\omega$ and weak coupling $|V_N(n)|\ll\hbar\omega$. The latter condition implies that for the multiphoton resonant transitions, systems with large dipole moments ($|\lambda_e+\lambda_g|\gg|\lambda_{eg}|$) are preferable and requires the following restrictions:

$$\frac{\lambda_{eg}}{\omega}\ll 1; \quad \overline{\lambda}^2 \ll 1 \quad (21)$$

on the characteristic parameters of the system considered.

In some cases one should also take into account corrections to the energies (9) and (10) due to nonresonant couplings (nonresonant transitions between two manifolds). These corrections are given by the known formula of perturbation theory [21] in the second order over $\widehat{V}$:

$$E_n^{(2)} = \sum_{m\neq n}\frac{|V_{mn}|^2}{E_n^{(0)}-E_m^{(0)}}. \quad (22)$$

Taking into account Eqs. (22), (19), and condition (21) for the energy corrections we obtain

$$E_{gN}^{(2)} \simeq \frac{\hbar\lambda_{eg}^2}{\omega\overline{\lambda}^2}\sum_{M\neq N-n}\frac{(M-N)^2 I_{M,N}^2(\overline{\lambda}^2)}{N-n-M},$$

$$E_{eN}^{(2)} \simeq \frac{\hbar\lambda_{eg}^2}{\omega\overline{\lambda}^2}\sum_{M\neq N+n}\frac{(M-N)^2 I_{M,N}^2(\overline{\lambda}^2)}{N+n-M}.$$

These corrections may be incorporated into the derived solutions by redefining the energies in Eqs. (16)–(18) as follows:

$$E_{gN} = E_{gN}^{(0)} + E_{gN}^{(2)}, \quad (23)$$

$$E_{eN} = E_{eN}^{(0)} + E_{eN}^{(2)}. \quad (24)$$

As a consequence the resonance detuning in Eq. (17) becomes

$$\delta_n = \delta_n^{(0)} - \frac{E_{eN}^{(2)} - E_{gN+n}^{(2)}}{\hbar}. \quad (25)$$

As will be shown in the next section, the second term in Eq. (25) describes a dynamic Stark shift of levels and for large n it may take the levels off resonance. To compensate this one should take an appropriate detuning

$$\delta_n^{(0)} = \delta_n^{(\mathrm{St})} \simeq \frac{E_{e0}^{(2)} - E_{gn}^{(2)}}{\hbar} = \frac{\lambda_{eg}^2}{\omega\overline{\lambda}^2} \times\left(\sum_{M\neq n}\frac{M^2 I_{M,0}^2(\overline{\lambda}^2)}{n-M} + \sum_{M\neq 0}\frac{(M-n)^2 I_{M,n}^2(\overline{\lambda}^2)}{M}\right). \quad (26)$$

III. MULTIPHOTON RABI OSCILLATIONS

In this section, we consider temporal evolution of the considered system. This is of particular interest for applications in quantum information processing. Here we also present numerical solutions of the time-dependent Schrödinger equation with the full Hamiltonian (1) in the Fock basis considering up to $N_{\max}=200$ excitations. The set of equations for the probability amplitudes has been solved using a standard fourth-order Runge-Kutta algorithm [22].

We first proceed to consider the quantum dynamics of the two-level system and harmonic oscillator starting from an initial state, which is not an eigenstate of the Hamiltonian (1). Assuming arbitrary initial state $|\Psi_0\rangle$ of a system, then the state vector for times $t>0$ is just given by the expansion over the dressed state basis obtained above:

$$|\Psi(t)\rangle = \sum_{N=0}^{n-1}\langle\downarrow,N^{(\lambda_g)}||\Psi_0\rangle e^{-(i/\hbar)E_{gN}t}|\downarrow,N^{(\lambda_g)}\rangle + \sum_{\alpha=\pm}\sum_{N=n}^{\infty}\langle\alpha,N||\Psi_0\rangle e^{-(i/\hbar)E_{\alpha,N}t}|\alpha,N\rangle. \quad (27)$$

For concreteness we will consider two common initial conditions for harmonic oscillator: the Fock state and the coherent state. We will calculate the time dependence of the two-level system population inversion $W_n(t)=\langle\Psi(t)|\widehat{\sigma}_z|\Psi(t)\rangle$ at the

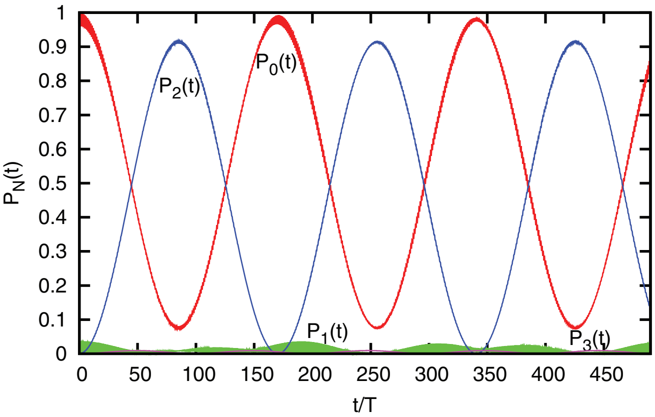

FIG. 2. (Color online) Photon number probability $P_N(t)$ as a function of scaled time at the two-photon resonance ($2\omega = \omega_{eg}$).

exact n-photon resonance $\delta_n = 0$. For the field in the Fock sate and two-level system in the excited state $|\Psi_0\rangle = |\uparrow,0\rangle$, we have

$$W_n(t) = \sum_{N=0}^{\infty} I_{N,0}^2\left(\frac{\lambda_e^2}{\omega^2}\right)\cos\left[\Omega_{N+n}(n)\,t\right], \tag{28}$$

where

$$\Omega_N(n) = \left|2\lambda_{eg}\left(\frac{\lambda_g-\lambda_e}{\omega} - \frac{n}{\overline{\lambda}}\right) I_{N-n,N}(\overline{\lambda}^2)\right| \tag{29}$$

is the multiphoton vacuum Rabi frequency. For $\lambda_e^2 \ll \omega^2$ the main contribution in the sum (28) comes from the first term: $W_n(t) \simeq \cos[\Omega_n(n)\,t]$, which corresponds to Rabi oscillations with periodic exchange of n photons between the two-level system and the radiation (bosonic) field. In this case, taking into account that the Laguerre function is simplified $I_{s,0}(\alpha) = e^{-\alpha/2}\alpha^{s/2}/\sqrt{s!}$, from Eq. (29) for the generalized Rabi frequency we obtain

$$\Omega_R(n) \equiv \Omega_n(n) \simeq \lambda_{eg}\frac{2n}{\sqrt{n!}}e^{-(\overline{\lambda}^2/2)}\overline{\lambda}^{n-1}. \tag{30}$$

In Figs. 2–4 the photon number probability

$$P_N(t) = \langle\uparrow,N||\Psi(t)\rangle\langle\Psi(t)|\uparrow,N\rangle + \langle\downarrow,N||\Psi(t)\rangle\langle\Psi(t)|\downarrow,N\rangle \tag{31}$$

as a function of time is shown for two-, three-, and four-photon resonances. For an initial state we assume a two-level system in the excited state and the field in the vacuum state, $|\uparrow\rangle\otimes|0\rangle$. As is seen from these figures, due to ISB, multiphoton Fock states are excited. For the two-photon resonance, the Schrödinger equation with the full Hamiltonian (1) was numerically solved with the coupling parameters $\lambda_{eg}/\omega = 0.02$, $\lambda_g/\omega = 0$, and $\lambda_e/\omega = 0.1$. For these values, the Rabi frequency and the dynamic Stark shift calculated by Eqs. (30) and (26) are equal to $\Omega_R(2)/\omega = 5.63\times10^{-3}$ and $\delta_n^{(\mathrm{St})}/\omega = 1.6\times10^{-3}$, respectively. Thus, the Rabi frequency is considerably larger than the dynamic Stark shift. Hence, in Fig. 2 we see Rabi oscillations with the time period $t_R/T = \omega/\Omega_R(2) \simeq 178$, where $T = 2\pi/\omega$ is the oscillator period. For three- and four-photon resonances the coupling parameters are taken to be $\lambda_{eg}/\omega = 0.02$, $\lambda_g/\omega = 0.1$, and $\lambda_e/\omega = 0.1$. According

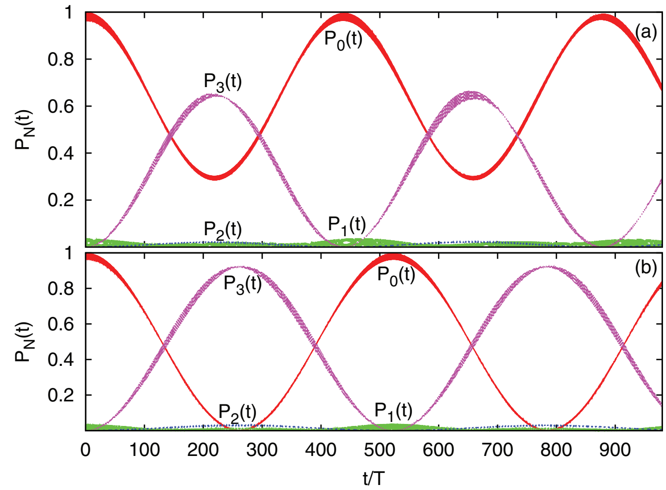

FIG. 3. (Color online) Photon number probability $P_N(t)$ as a function of time at the three-photon resonance ($n = 3$). In (a) the detuning is taken to be $\delta_3^{(0)} = 0$, while in (b) $\delta_3^{(0)} = \delta_3^{(\mathrm{St})}$.

to analytical treatment, for the Rabi frequencies (30) and the dynamic Stark shifts (26) we obtain $\Omega_R(3)/\omega = 1.92\times10^{-3}$, $\delta_3^{(\mathrm{St})}/\omega = 1.24\times10^{-3}$ and $\Omega_R(4)/\omega = 2.56\times10^{-4}$, $\delta_4^{(\mathrm{St})}/\omega = 1.08\times10^{-3}$. In the last case the dynamic Stark shift is larger than the Rabi frequency and should be compensated by an appropriate detuning (26). For zero detuning [Figs. 3(a) and 4(a)], in accordance with the analytical formulas (18), we have a Rabi oscillations with diminished amplitude. Thus, for Rabi oscillations with complete population transfer the dynamic Stark shift should be compensated by the appropriate detuning. The latter is shown in Figs. 3(b) and 4(b).

Finally we turn to the case in which a two-level system begins in the ground state, with an oscillator prepared in a coherent state with a mean excitation (photon) number $\overline{N}$. From Eq. (7) it follows that such state can be represented as $|\Psi_0\rangle = |\downarrow\rangle\otimes|0^{(\lambda_e')}\rangle$, where $\lambda_e' = \sqrt{\overline{N}}\omega$. Taking into account

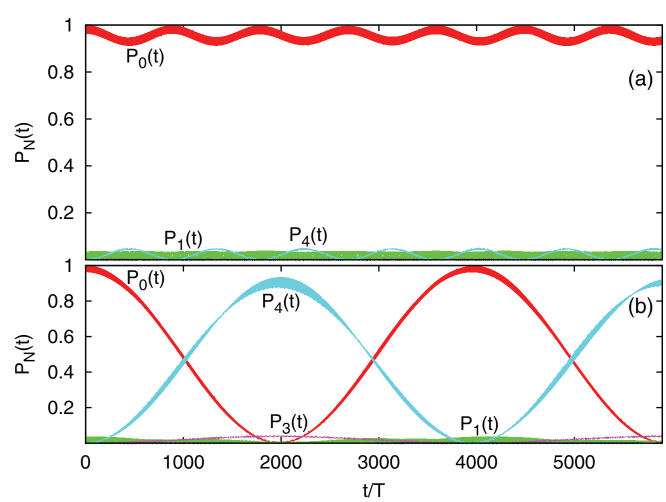

FIG. 4. (Color online) Same as Fig. 3 but for four-photon resonance.

Eqs. (11) and (12), for population inversion we obtain

$$W_n(t) = -1 + 2\sum_{N=n}^{\infty} I_{N,0}^2(\rho)\sin^2\frac{\Omega_N(n)t}{2}, \tag{32}$$

where $\rho = (\sqrt{N} + \lambda_g/\omega)^2$. In this case we have collapse and revival phenomena of the multiphoton Rabi oscillations. First we consider the collapse. In the sum (32) the dominant Rabi frequency is defined via a mean number of photons $\overline{N}$, which for large $\overline{N} \gg n$ can be expressed as

$$\Omega_{\overline{N}}(n) \simeq 2\lambda_{eg}\overline{\lambda}^{n-1}\frac{1}{(n-1)!} \times \left(\overline{N}^{n/2} - \frac{\overline{N}^{n/2+1}}{n+1}\overline{\lambda}^2 + \frac{\overline{N}^{n/2+2}}{2(n+1)(n+2)}\overline{\lambda}^4\right). \tag{33}$$

This formula has been derived from Eq. (29) by expanding the Laguerre function over $\overline{\lambda}$. There are dominant frequencies in Eq. (32) as a result of the spread of probabilities about $\overline{N}$ for a photon numbers in the range $\overline{N} \pm \sqrt{\overline{N}}$. When these terms are oscillating out of phase with each other in sum (32), we expect the cancellation of these terms, i.e., the collapse of Rabi oscillations. Hence, the collapse time may be estimated from the condition

$$t_c^{(n)}(\Omega_{\overline{N}+\sqrt{\overline{N}}}(n) - \Omega_{\overline{N}-\sqrt{\overline{N}}}(n)) \simeq \pi.$$

For large photon numbers $\overline{N} \gg \sqrt{\overline{N}}$ we have

$$t_c^{(n)} \simeq \frac{\pi}{2\sqrt{\overline{N}}}\left(\frac{\partial\Omega_{\overline{N}}(n)}{\partial\overline{N}}\right)^{-1}, \tag{34}$$

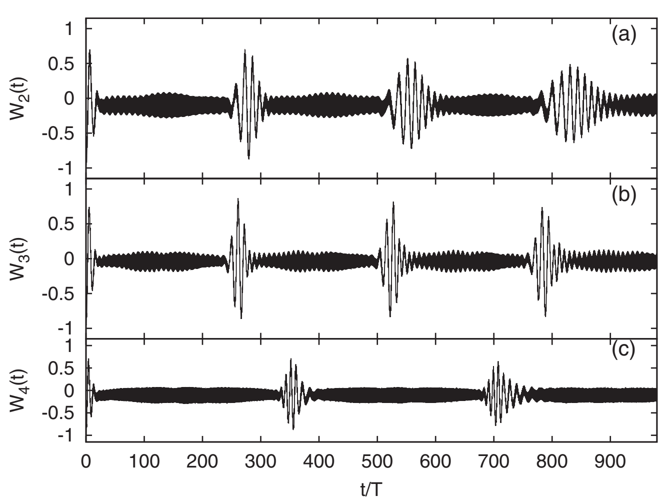

FIG. 5. Collapse and revival of the multiphoton Rabi oscillations. Two-level system population inversion is shown with the field initially in a coherent state. (a) Two-photon resonance with coupling parameters $\lambda_{eg}/\omega = 0.02$, $\lambda_g/\omega = 0$, $\lambda_e/\omega = 0.1$ and mean photon number $\overline{N} = 20$. (b) Three-photon resonance with parameters $\lambda_{eg}/\omega = 0.02$, $\lambda_g/\omega = 0.1$, $\lambda_e/\omega = 0.1$ and mean photon number $\overline{N} = 30$. (c) Same as (b) but for four-photon resonance and $\overline{N} = 60$.

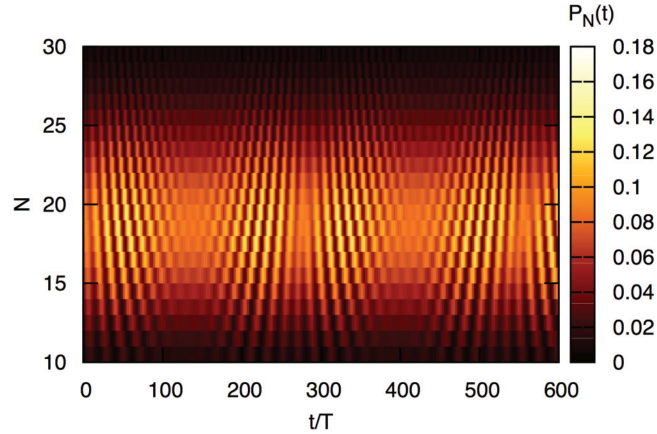

FIG. 6. (Color online) Density plot of photon number probability distribution $P_N(t)$ as a function of photon number and time corresponding to setup of Fig. 5(a).

where

$$\frac{\partial\Omega_{\overline{N}}(n)}{\partial\overline{N}} \simeq \lambda_{eg}\overline{\lambda}^{n-1}\frac{\overline{N}^{n/2-1}}{(n-1)!} \times \left(n - \frac{n+2}{n+1}\overline{N}\overline{\lambda}^2 + \frac{n+4}{2(n+1)(n+2)}\overline{N}^2\overline{\lambda}^4\right). \tag{35}$$

In contrast to the conventional JC model the collapse time (34) strongly depends on the mean photon number.

Now let us consider the revival phenomenon. If the neighboring terms in sum (32) are in phase with each other one can expect a revival of Rabi oscillations. Thus, revivals should occur for times

$$t_{\rm rev}^{(n)}(\Omega_{\overline{N}+1}(n) - \Omega_{\overline{N}}(n)) \simeq 2\pi k; \quad k = 1,2,\ldots.$$

Expanding $\Omega_{\overline{N}+1}(n)$ we arrive at

$$t_{\rm rev}^{(n)} \simeq 2\pi k\left(\frac{\partial\Omega_{\overline{N}}(n)}{\partial\overline{N}}\right)^{-1}. \tag{36}$$

Figures 5 and 6 display collapse and revival of the multiphoton Rabi oscillations. In Fig. 5 the two-level system inversion $W_n(t)$ is shown with the field initially in a coherent state at two-, three-, and four-photon resonances for different mean photon numbers. The consequence of collapse and revival of the multiphoton Rabi oscillations on the photon number probability distribution (31) is shown in Fig. 6. The collapse and revival times $(t_c^{(n)}/T, t_{\rm rev}^{(n)}/T)$ calculated by Eqs. (34) and (36) for two-, three-, and four-photon resonances are estimated as (16, 287), (12, 260), and (11, 330), respectively. As is seen from Fig. 5, the numerical simulations are in agreement with analytical treatment in the multiphoton RWA and confirm the revealed physical picture described above.

IV. CONCLUSION

We have presented a theoretical treatment of the quantum dynamics of a two-level system with ISB interacting with a quantized harmonic oscillator in the ultrastrong coupling regime. With the help of a resonant approach, we have solved the Schrödinger equation and obtained simple analytical expressions for the eigenstates and eigenenergies. The obtained results show that the effect of ISB on the quantum dynamics is considerable. For the n-photon resonance in addition to n nonentangled states we have symmetric and asymmetric entangled states of a two-level system and position-displaced Fock states. The ground state is not entangled, but the bosonic field may be in a coherent state. We have also investigated the temporal quantum dynamics of the considered system and showed that, similar to the one-photon case due to ISB, Rabi oscillations may collapse and the revival of the initial population with periodic multiphoton exchange between the two-level system and the radiation field is possible. The proposed model may have diverse applications in cavity QED experiments, especially in the variant of circuit QED, where the considered parameters are already accessible.

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