Method for preparing two-atom entangled states in circuit QED and probing it via quantum nondemolition measurements

D. Z. Rossatto* and C. J. Villas-Boas
Departamento de Física, Universidade Federal de São Carlos, CEP 13565-905, São Carlos, SP, Brazil
(Received 7 August 2013; published 17 October 2013)

We propose a probabilistic scheme to prepare a maximally entangled state between a pair of two-level atoms coupled to a leaking cavity mode in circuit QED, without requiring precise time control of the system evolution and initial atomic state. We show that the steady state of this dissipative system is a mixture of two parts, where the atoms are either in their ground state or in a maximally entangled one. Then, by applying a weak probe field to the cavity mode, we are able to discriminate those states without disturbing the atomic system, i.e., to perform a quantum nondemolition measurement via the cavity transmission. In this scheme, one has maximum cavity transmission only when the atomic system is in an entangled state, so that a single click in the detector is enough to be sure that the atoms are in a maximally entangled state. Our scheme relies on an interference effect as it happens in an electromagnetically induced transparency phenomenon so that it works even in the limit of a cavity decay rate much stronger than the atom-field coupling.

DOI: 10.1103/PhysRevA.88.042324 PACS number(s): 03.67.Bg, 42.50.Pq, 03.67.Lx

The preparation and manipulation of entangled states have attracted much interest in recent years, as they have no classical counterpart and are key ingredients in quantum nonlocality tests [1] and play an important role in quantum computation and communication tasks [2], such as quantum cryptography [3], computers [4], and teleportation [5]. Atomic entangled states can be prepared either by coherent control of unitary dynamics [6], as a consequence of measurements [7], or as a result of a dissipative process [8]. Recently, the preparation of quantum systems in an entangled state by dissipative schemes has been actively studied; the noise that is always present in these experiments can itself be used as a resource for entanglement generation, thus avoiding the usual destructive effect on quantum system coherence due to system-environment interaction.

On the other hand, entanglement quantifiers, such as concurrence [9] and negativity [10], are not physical observables, i.e., there are no directly measurable observables, until now, to describe the entanglement of a given arbitrary quantum state. In general, it is necessary to perform quantum state tomography to calculate these entanglement quantifiers, inevitably perturbing the state of the system, although some interesting methods have recently been proposed to construct direct observables related to entanglement in particular systems [11–14]. Whereas the authors in Refs. [11–13] can determine the entanglement when few copies of the quantum system are available, in Ref. [14] the authors do this by introducing a probe atom that performs a quantum nondemolition measurement.

Here we propose a probabilistic scheme to prepare a maximally entangled state between a pair of two-level atoms coupled to a leaking cavity mode in circuit QED, without requiring either precise time control of the system evolution or strong atom-field coupling. The steady state (ρ_{ss}) of this dissipative process is a mixed state with two parts: one describing the possibility of having both atoms in the ground state $|G\rangle$ and the other describing the atoms in a maximally entangled state $|D\rangle$. In both cases, the cavity mode is in the vacuum. In the context of a single experimental run, ρ_{ss} shows us that the atomic system can be either in an uncorrelated state or in a maximally entangled one. Thus, if we are able to distinguish the two states without perturbing the atomic system, we will be able to prepare it in a maximally entangled state. In fact, we can do this by applying a weak probe field to the atom-cavity system. As we show below, when the atomic system is in the uncorrelated state, the cavity transmission goes to zero, as opposed to the maximum transmission which happens only when the atomic system is in the maximally entangled state. Thus, a single click on the detector works as a witness of the entanglement generation of the atomic system. On the other hand, we also show that if we have an unknown atomic mixed state ρ (between the states $|G\rangle$ and $|D\rangle$), then the average transmission of the atom-cavity system is exactly equal to the concurrence of the state ρ, thus providing a direct measure of the degree of entanglement of the atomic system.

Model. Consider a pair of identical two-level atoms ($|g\rangle_j =$ ground state, $|e\rangle_j =$ excited state) coupled resonantly to a cavity mode with coupling strength λ, modeled by the Tavis-Cummings Hamiltonian ($\hbar = 1$) [15],

$$H = \omega a^\dagger a + \frac{\omega}{2} S_z + \lambda(aS_+ + a^\dagger S_-), \tag{1}$$

where the cavity mode and the atomic transition are at frequency ω, with $\omega \gg \lambda$. The operators $S_z \equiv \sum_{j=1}^{2}\sigma_z^j$ and $S_\pm = \sum_{j=1}^{2}\sigma_\pm^j$ are the collective spin operators with $\sigma_\pm^j = (\sigma_x^j \pm i\sigma_y^j)/2$, $\sigma_{x,y,z}^j$ being the Pauli operators for each atom; a ($a^\dagger$) is the annihilation (creation) operator of the cavity field. Assuming a leaking cavity at temperature zero, the dynamics of this system is governed by the master equation [16]

$$\dot{\rho} = -i[H,\rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \tag{2}$$

with κ being the decay rate of the cavity mode. The proposed experimental setup is shown in Fig. 1(a).

*Corresponding author: zini@df.ufscar.br

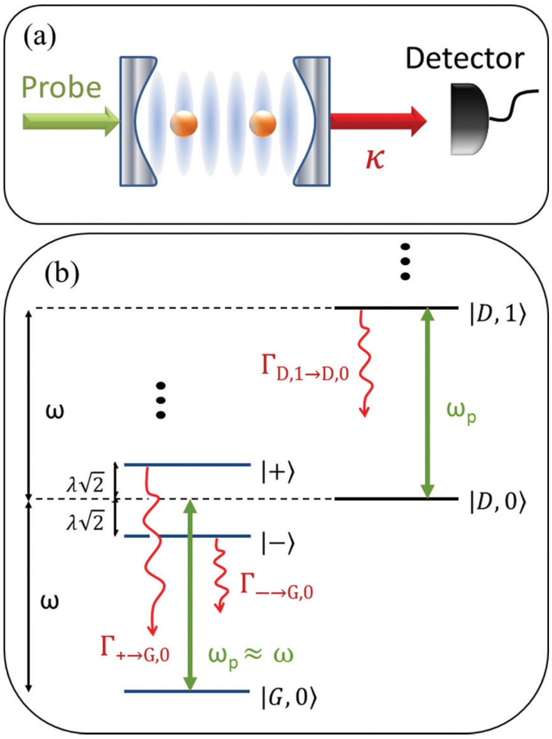

FIG. 1. (Color online) (a) Pictorial experimental setup. A pair of two-level atoms coupled to a leaking cavity mode. Once the system reaches the steady state, the weak probe field is switched on and the cavity transmission is monitored. (b) Energy-level diagram of the whole system with the decay rates and the probe field.

The spectrum of the system, i.e., its allowed states, is given by the dressed states of H,

$$|G,0\rangle = |G\rangle \otimes |0\rangle_c, \tag{3a}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|B\rangle \otimes |0\rangle_c \pm |G\rangle \otimes |1\rangle_c), \tag{3b}$$

$$|D,n\rangle = |D\rangle \otimes |n\rangle_c, \tag{3c}$$

with energies $-\omega$, $\pm\lambda\sqrt{2}$, and $n\omega$, respectively, where $|G\rangle = |g\rangle_1 \otimes |g\rangle_2$, $|B\rangle = (|g\rangle_1 \otimes |e\rangle_2 + |e\rangle_1 \otimes |g\rangle_2)/\sqrt{2}$, $|D\rangle = (|g\rangle_1 \otimes |e\rangle_2 - |e\rangle_1 \otimes |g\rangle_2)/\sqrt{2}$, and $|n\rangle_c$ is the cavity mode state in the Fock basis, with $n = 0,1$. Here we are considering only the lowest eigenstates of our system, since we are interested in its steady state, which is a mixture of those eigenstates. In addition, our scheme requires a weak probe field, which also keeps the cavity field with up to one photon, as we will explain below.

The damping of the cavity mode can promote transitions between the eigenstates of the system, whose rates can be obtained by Fermi's golden rule [17]. As we are considering only the cavity decay, the transition rate from a higher energy state $|i\rangle$ to a lower one $|f\rangle$ is given by $\Gamma_{i\to f} = \kappa|\langle f|a|i\rangle|^2$ [17]. When we take into account the eigenstates of our system, it is easy to see that we have two independent subspaces: $\{|G,0\rangle,|-\rangle,|+\rangle\}$ and $\{|D,n\rangle\}$, in which there are no transitions between states that belong to distinct subspaces. Thus, the nonzero transition rates are $\Gamma_{\pm\to G,0} = \kappa/2$ and $\Gamma_{D,n+1\to D,n} = \kappa$. In Fig. 1(b), the energy-level diagram of the whole system is depicted with the decay rates and the probe field (frequency ω_p) that will be introduced later.

Owing to the existence of two independent subspaces, for any general initial state, the steady state of the system is a mixture of the lowest-energy eigenstates of each subspace, i.e.,

$$\rho_{ss} = (1-P)|G,0\rangle\langle G,0| + P|D,0\rangle\langle D,0|, \tag{4}$$

with $P = \mathrm{Tr}[\rho(0)|D\rangle\langle D|]$ being the projection of the initial state on the dark state $|D\rangle$. This result can be obtained directly from Eq. (2) for $t \to \infty$ ($\dot{\rho} = 0$).

It is important to emphasize that we are not considering independent atomic damping, as it destroys the entanglement in the steady state so that $\rho(t\to\infty) \to |G,0\rangle\langle G,0|$ for any initial state. In other words, the entangled state has a finite lifetime proportional to the atomic one. As a real two-level system always has a spontaneous decay γ, our results are valid in a time window defined by $\gamma t \lesssim 1$ and $\lambda^2 t/\kappa \gg 1$, so that we must have $\lambda^2/\kappa \gg \gamma$ [18]. However, if the atoms are subject to a collective reservoir, as usually occurs in circuit QED [19], then the steady state is still given by Eq. (4), i.e., such reservoir does not destroy the dark state $|D\rangle$, showing that circuit QED is a very good scenario to implement our scheme. We will discuss this point in more detail below.

In the context of a single experimental run, we can see from the ρ_{ss} that the system can be either in the atomic ground state $|G\rangle$ or in the entangled state $|D\rangle$ with probabilities $1-P$ and P, respectively. There is thus a probability of having the atoms in a maximally entangled state. However, a direct measurement of the atoms would destroy such an entangled state. To circumvent this problem, we must be able to measure our atomic system nondestructively. We can do that by probing our system with a weak probe field, which allows us to distinguish the atomic states ($|G\rangle$ or $|D\rangle$) through the cavity transmission without disturbing the atomic system.

To measure the system nondestructively, we must first wait until the system reaches its steady state (ρ_{ss}) and then apply a weak probe field to the cavity, whose Hamiltonian is described by

$$H_p = \varepsilon(a e^{i\omega_p t} + a^\dagger e^{-i\omega_p t}), \tag{5}$$

with $\varepsilon \ll \lambda$. Here, ε and ω_p are the strength and the frequency of the probe field, respectively.

In order to understand how this probe field can provide information about the atomic state, consider first the resonant case, $\omega_p = \omega$. If the system is in the $|D,0\rangle$ state, we can see from Fig. 1(b) that the probe field is able to promote the transition $|D,0\rangle \leftrightarrow |D,1\rangle$. However, as $|D\rangle$ is a dark state, it is decoupled from the cavity mode so that the system behaves as an empty cavity case ($\lambda = 0$). In this case, the asymptotic cavity field state is a coherent field $|\alpha\rangle_c = e^{-|\alpha|^2/2}(|0\rangle_c + \alpha|1\rangle_c + \ldots)$, with $\alpha = -i\varepsilon/\kappa$; then, for a very weak probe field ($\varepsilon \ll \kappa$), the steady state of the atom-field system is given by

$$|\psi\rangle_{ss}^D \approx |D\rangle \otimes \left[\left(1 - \frac{1}{2}\frac{\varepsilon^2}{\kappa^2}\right)|0\rangle_c - i\frac{\varepsilon}{\kappa}|1\rangle_c\right]. \tag{6}$$

On the other hand, if the system is in the $|G,0\rangle$ state, the weak probe field could *a priori* induce two off-resonant transitions: $|G,0\rangle \leftrightarrow |\pm\rangle$, with detuning between the frequencies of the probe and atom-field system given by $\pm\lambda\sqrt{2}$. However, when $\omega_p = \omega$, the probe field does not introduce any photon into the cavity in the stationary regime, whatever the value of the atom-field coupling λ, as explained below. Hence, when the system is in the $|G,0\rangle$ state, the probe field cannot introduce any excitation into the system, so that the steady state of the system is

$$|\psi\rangle_{ss}^{G} \approx |G,0\rangle \,. \tag{7}$$

Therefore, the normalized transmission of the cavity, $T = \langle a^{\dagger}a\rangle/(|\varepsilon|/\kappa)^2$, can be used to provide us with information about the atomic steady state (4): we have $T = 1$ when the atoms are in the maximally entangled state ($|D\rangle$) and $T = 0$ when the atoms are in the separable state ($|G\rangle$). Thus, in the stationary regime, after applying a weak probe field to the system, the transmission works as a nondemolition measurement of the atomic state, allowing us to know whether the system is in a maximally entangled state or not. Moreover, our system does not require a high-efficiency photon detector, since a single click is enough to discriminate between the two atomic states available in the steady state (4).

An entangled state can be prepared simply by monitoring the transmission in the time interval $\kappa/\lambda^2 \ll t < 1/\gamma$ for independent atomic damping: any click in the detector within this time window projects the atomic system in $|D\rangle$. If no click is registered, then we must reset the system and start the experiment again. For collective atomic damping, whose Liouvillian is $\Gamma_c(2S_-\rho S_+ - S_+S_-\rho - \rho S_+S_-)$, with Γ_c being the collective decay rate, the state is prepared in a similar way in the time interval $t \gg \kappa/\lambda^2$. The collective atomic damping does not change the cavity transmission when the atoms are in $|D\rangle$, but it does so when they are in $|G\rangle$ such that, in this case, $T_{|G\rangle} = (\Gamma_c\kappa/\lambda^2)^2/(1+\Gamma_c\kappa/\lambda^2)^2$. Thus, the collective atomic decay decreases the contrast between $T_{|D\rangle}$ and $T_{|G\rangle}$, but it is possible to reach a very reasonable contrast with recently observed experimental parameters in the limit of $\lambda \gg \Gamma_c$ [20]. For instance, for $\Gamma_c \sim \kappa = 0.01g$ [20], $T_{|G\rangle} \sim 10^{-8}$. In this way, the smaller the ratio $\Gamma_c\kappa/\lambda^2$, the better is the efficiency of our method.

The total transmission is expected to be maximum when the atoms are in the dark state $|D\rangle$ because, in this state, the atomic system is decoupled from the cavity mode so that the atom-field system behaves as an empty cavity case ($\lambda = 0$). However, when the system is in the $|G,0\rangle$ state, the transmission is expected to be zero (or close to zero). The reason for this zero transmission could be the detuning between the weak probe field and the atom-field system: the two transitions $|G,0\rangle \leftrightarrow |\pm\rangle$ are coupled by the probe field, but with detuning $\pm\lambda\sqrt{2}$. As both states $|\pm\rangle$ have decay rates $\Gamma_{\pm\to G,0} = \kappa/2$, one can see that for $\lambda\sqrt{2} \gg \kappa/2$, the probe field is very out of resonance with the atom-field system and an absorption close to zero is then expected. If this were the case, one could argue that our system only works in the strong-coupling regime. However, the scheme is also valid for weak atom-field coupling λ, as the real reason for the absence of transmission from the cavity is that our system has two absorption channels, $|G,0\rangle \leftrightarrow |-\rangle$ and $|G,0\rangle \leftrightarrow |+\rangle$, which destructively interfere to produce zero absorption in the resonant case $\omega_p = \omega$, analogously to the phenomenon of electromagnetically induced transparency

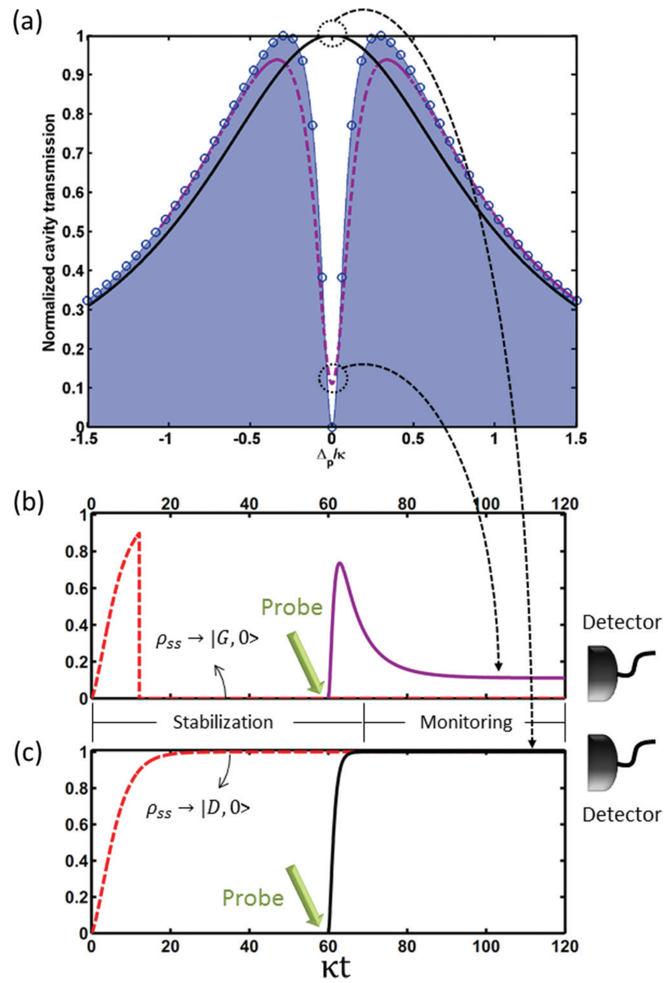

FIG. 2. (Color online) (a) Cavity transmission vs detuning between the probe and the cavity field, setting $\lambda = 0.2\kappa$, $\varepsilon = 0.05\lambda$, and $\Gamma_c = 0.1\lambda$. We observe that even for $\lambda\sqrt{2} \lesssim \kappa/2$ and $\Gamma_c \neq 0$, the cavity transmission is close to zero for $\Delta_p = 0$ when $\rho_{ss} \to |G,0\rangle$ (dashed line), the circles are for $\Gamma_c = 0$, and the solid line represents the cavity transmission when $\rho_{ss} \to |D,0\rangle$ (empty cavitylike). Time evolution of the concurrence C (dashed line) and normalized transmission T (solid line) for a single trajectory when (b) $\rho_{ss} \to |G,0\rangle$ and (c) $\rho_{ss} \to |D,0\rangle$, assuming $|\phi(0)\rangle = |g\rangle_1 \otimes |e\rangle_2 \otimes |0\rangle_c$.

[21,22]. Figure 2(a) shows the cavity transmission as a function of the detuning between the probe field and the cavity mode, $\Delta_p = \omega_p - \omega$, assuming the atom-field coupling $\lambda = 0.2\kappa$, the collective atomic decay rate $\Gamma_c = 0.1\lambda$, and the strength of the probe field $\varepsilon = 0.05\lambda$. This scheme works for any value of λ, but the smaller the value of λ, the longer is the time taken by the system to reach its steady state, since it is proportional to κ/λ^2.

We simulated an experiment with numerical calculations using the quantum jump approach (also called the quantum trajectories method) [23]. Single trajectories are shown in Figs. 2(b) and 2(c), simulating a single run of an experiment for two cases: when $\rho_{ss} \to |G,0\rangle$ [Fig. 2(b)] and when $\rho_{ss} \to |D,0\rangle$ [Fig. 2(c)]. In these simulations, we adopted $\lambda = 0.2\kappa$, $\varepsilon = 0.05\lambda$, $\Gamma_c = 0.1\lambda$, and $\rho(0) = |\phi(0)\rangle\langle\phi(0)|$, with $|\phi(0)\rangle = |g\rangle_1 \otimes |e\rangle_2 \otimes |0\rangle_c$. Here, Wootters' concurrence (C) [9,24] was used as a measure of the degree of entanglement. As we can see in Fig. 2, when the atoms are in the maximally entangled (separable) state, the transmission of the probe field in the monitoring region is maximum (almost zero). This figure also helps us to see the evolution of a single trajectory of the system: at $t = 0$, we have the preparation of the initial state $\rho(0)$, followed by the stabilization of the system; then we switch on the probe field, which requires a second stabilization time; finally, we have the monitoring region where the atomic state is nondestructively measured.

We have mentioned above that the contrast between $T_{|G\rangle}$ and $T_{|D\rangle}$ decreases in the presence of collective atomic damping, even though such damping does not couple the two subspaces $\{|G,0\rangle,|-\rangle,|+\rangle\}$ and $\{|D,n\rangle\}$. The reason for the increasing of $T_{|G\rangle}$ is related to a disturbance in the destructive interference, which produces zero absorption at the resonant case $\omega_p = \omega$. It is possible to show that the collective atomic damping plays the same role of the dephasing in the usual phenomenon of electromagnetically induced transparency in atomic ensemble [21]; that is, such damping (Γ_c) destroys the superposition of the quantum state, $|\phi\rangle = \mathcal{N}(\lambda\sqrt{2}|G\rangle - \varepsilon|B\rangle) \otimes |0\rangle_c$, responsible for the interference effect on the absorption channels of the atom-cavity system, where $\mathcal{N}$ is a normalization factor. Here, $\lambda\sqrt{2}$ (ε) plays the role of the control field (probe field) in the usual phenomenon of electromagnetically induced transparency [21].

As we have seen so far, to be able to generate the maximally entangled state, it is necessary that the initial atomic state $\rho(0)$ has a nonzero projection on the dark state $|D\rangle$. This can be arranged in different ways, for example: (i) if we are able to stimulate the atoms individually, then we can prepare the initial state $|g\rangle_1 \otimes |e\rangle_2 \otimes |0\rangle_c$; (ii) if we are not able to stimulate the atoms individually, then an incoherent field can be applied to both atoms simultaneously, so as to prepare a completely mixed state $\rho(0) = \frac{\mathbf{1}_a}{4} \otimes |0\rangle_c\langle 0|$, with $\mathbf{1}_a$ being the identity atomic matrix ($\mathbf{1}_a = (|g\rangle\langle g| + |e\rangle\langle e|)_1 \otimes (|g\rangle\langle g| + |e\rangle\langle e|)_2$). In the first case, the projection on the dark state is $P = 1/2$, while in the second it is $P = 1/4$, with these being the probabilities of preparing the atoms in a maximally entangled state.

Direct measurement of the concurrence. Besides using our scheme as a source of maximally entangled states, our scheme can also be used as a direct method to measure the concurrence of the atoms. As explained above, for any initial state, the steady state of the atom-field system is given by Eq. (4), which is a mixture of a completely separable state and a maximally entangled one. On applying a weak probe field, when $\Gamma_c\kappa/\lambda^2 \ll 1$ and $\gamma\kappa/\lambda^2 \ll 1$, the steady state turns out to be

$$\rho_{ss} \to \widetilde{\rho}_{ss} \approx (1-P)\,|\psi\rangle_{ss}^{G}\,\langle\psi| + P\,|\psi\rangle_{ss}^{D}\,\langle\psi|\,, \tag{8}$$

where $|\psi\rangle_{ss}^{G}$ and $|\psi\rangle_{ss}^{D}$ belong to distinct subspaces. For this state, the average transmission is $T(\widetilde{\rho}_{ss}) = P$. However, the concurrence of the atomic state is also $C[\mathrm{Tr}_c(\rho_{ss})] = P$, where Tr_c means the trace over the cavity mode variables. Therefore, we see that the transmission of the atom-field system $T(\widetilde{\rho}_{ss})$ is exactly the degree of entanglement (concurrence) between the two atoms. In this way, our scheme works as a direct method to measure the concurrence of the atomic steady state, without requiring any tomographic reconstruction of the atomic density matrix.

In conclusion, we have shown a probabilistic scheme to prepare a maximally entangled state between a pair of two-level atoms coupled to a leaking cavity mode in circuit QED, and how to probe this atomic steady state, without perturbing it, via the cavity transmission. In the case of a single run of the experiment, we have seen that if the atomic system is in an entangled state, the cavity transmission will be maximal. On the other hand, if the system is in an uncorrelated state, the cavity transmission goes to zero. Therefore, a single click in the detector is sufficient to determine that the atoms are in a maximally entangled state. We have also seen that our scheme works as a direct method to measure the concurrence of the atomic steady state, without requiring any tomographic reconstruction of the atomic density matrix.

Acknowledgments. The authors acknowledge financial support from the Brazilian agencies FAPESP, CNPq, and Brazilian National Institute of Science and Technology for Quantum Information (INCT-IQ).

[1] J. S. Bell, Physics **1**, 195 (1964).
[2] C. H. Bennett and D. P. Divincenzo, Nature (London) **404**, 247 (2000).
[3] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[4] D. Gottesman and I. L. Chuang, Nature (London) **402**, 390 (1999).
[5] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[6] A. Rauschenbeutel *et al.*, Science **288**, 2024 (2000); S.-B. Zheng and G.-C. Guo, Phys. Rev. Lett. **85**, 2392 (2000); C. A. Sackett *et al.*, Nature (London) **404**, 256 (2000); L. DiCarlo *et al.*, *ibid.* **460**, 240 (2009); S. Filipp, A. F. van Loo, M. Baur, L. Steffen, and A. Wallraff, Phys. Rev. A **84**, 061805(R) (2011).
[7] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, Phys. Rev. A **59**, 2468 (1999); J. Wang, H. M. Wiseman, and G. J. Milburn, *ibid.* **71**, 042309 (2005); A. R. R. Carvalho, A. J. S. Reid, and J. J. Hope, *ibid.* **78**, 012334 (2008); B. Julsgaard and K. Mø lmer, *ibid.* **85**, 032327 (2012); J. Hong and H.-W. Lee, Phys. Rev. Lett. **89**, 237901 (2002); A. S. Sørensen and K. Mølmer, *ibid.* **91**, 097905 (2003); J. Metz, M. Trupke, and A. Beige, *ibid.* **97**, 040503 (2006).
[8] A. Beige, D. Braun, B. Tregenna, and P. L. Knight, Phys. Rev. Lett. **85**, 1762 (2000); S. G. Clark and A. S. Parkins, *ibid.* **90**, 047905 (2003); M. J. Kastoryano, F. Reiter, and A. S. Sørensen, *ibid.* **106**, 090502 (2011); L. Slodička, G. Hétet, N. Röck, P. Schindler, M. Hennrich, and R. Blatt, *ibid.* **110**, 083603 (2013); S. Diehl *et al.*, Nat. Phys. **4**, 878 (2008); F. Verstraete, M. M. Wolf, and J. I. Cirac, *ibid.* **5**, 633 (2009).
[9] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[10] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
[11] S. P. Walborn, P. H. Souto Ribeiro, L. Davidovich, F. Mintert, and A. Buchleitner, Phys. Rev. A **75**, 032338 (2007).
[12] F. Mintert and A. Buchleitner, Phys. Rev. Lett. **98**, 140505 (2007).

[13] C.-S. Yu and H.-S. Song, Phys. Rev. A **76**, 022324 (2007); C.-S. Yu, C. Li, and H.-S. Song, *ibid*. **77**, 012305 (2008).
[14] Z. N. Li, J. S. Jin, and C. S. Yu, Phys. Rev. A **83**, 012317 (2011).
[15] M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968).
[16] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2007).
[17] P. Xue, Z. Ficek, and B. C. Sanders, Phys. Rev. A **86**, 043826 (2012); C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).
[18] Here, λ^2/κ is the effective collective atomic damping induced by the interaction between the atoms and the leaking cavity. R. K. Bullough, Hyperfine Interact. **37**, 71 (1987); D. Z. Rossatto, T. Werlang, E. I. Duzzioni, and C. J. Villas-Boas, Phys. Rev. Lett. **107**, 153601 (2011).
[19] G. Burkard and F. Brito, Phys. Rev. B **72**, 054528 (2005); F. Nissen, J. M. Fink, J. A. Mlynek, A. Wallraff, and J. Keeling, Phys. Rev. Lett. **110**, 203602 (2013).
[20] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004); A. Blais, J. Gambetta, A. Wallraff, D. I. Schuster, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, *ibid*. **75**, 032329 (2007).
[21] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[22] P. M. Alsing, D. A. Cardimona, and H. J. Carmichael, Phys. Rev. A **45**, 1793 (1992); P. R. Rice and R. J. Brecha, Opt. Commun. **126**, 230 (1996).
[23] H. J. Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics Vol. 18 (Springer, Berlin, 1993); M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998).
[24] $C = \max\{0,\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_1, λ_2, λ_3, and λ_4 are the square roots of the eigenvalues, in decreasing order, of the matrix $R = \rho(\sigma_y^1 \otimes \sigma_y^2)\rho^*(\sigma_y^1 \otimes \sigma_y^2)$. Here, ρ^* denotes the complex conjugate of the matrix ρ in the basis $\{|gg\rangle,|ge\rangle,|eg\rangle, |ee\rangle\}$.