Phase-matching condition for enhancement of phase sensitivity in quantum metrology

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We find a phase-matching condition for enhancement of sensitivity in a Mach-Zehnder interferometer illuminated by an arbitrary state in one input port and an odd (even) state in the other port. Under this condition, the Fisher information becomes maximal with respect to the relative phase of two modes, and the phase sensitivity is enhanced. For the case with photon losses, we further find that the phase-matching condition remains unchanged with a coherent state and a coherent superposition state as the input states.

I. INTRODUCTION

With the development of quantum information theory [1–5] and quantum technology [6–8], quantum metrology is becoming more and more practical nowadays. Improving the precision of a parameter is always the basic theme in quantum metrology. Its fundamental procedure is the parametrization process. There are three main methods to perform this procedure: (i) unitary parametrization, (ii) channel parametrization, and (iii) accelerating parametrization. Unitary parametrization is the most useful and well-studied method because it is widely applied in the phase estimation, the main task in quantum metrology. Precision measurement of gravity, temperature, weak magnetic strength, and many other parameters can be classified in the category of phase estimation. Channel parametrization has also been widely studied in frequency estimation, noise estimation [9,10], and other fields for many years. With the help of quantum technology, it may be possible to proceed with the parametrization in an accelerating way [11], in which the relativistic effects cannot be neglected any longer. In this paper, we mainly focus on the category of phase estimation and study how to effectively enhance the phase sensitivity.

In 1981, Caves [12] found out that, for phase estimation, taking a high-intensity coherent state and a low-intensity squeezed vacuum state as the input states of a Mach-Zehnder interferometer, the precision can beat the shot-noise limit(quantum standard limit), i.e., $1/\sqrt{N}$, where N is the total photon number in both modes. Since then, many protocols have been proposed to fulfill a similar job, such as NOON states, entangled coherent states [13], two-mode squeezed states [14], number squeezed states [15], and so on. With high intensity, some of these states can even theoretically achieve or surpass the Heisenberg limit, i.e., $1/N$. In the pioneering work of Caves [12], to enhance the precision, the phases of the two input states need to satisfy a relation. This can be considered a kind of phase-matching condition (PMC). Thus, it is reasonable to study if there is a more general PMC for more general states to enhance the phase sensitivity. This is the major motivation of this paper. To depict the precision of a parameter θ, quantum Fisher information (QFI) is an available useful concept because it describes the lower bound on the variance of the estimator $\hat{\theta}$ due to the Cramér-Rao theorem:

Var$(\hat{\theta}) \geqslant 1/(\nu F)$ [16,17], where Var$(\cdot)$ is the variance, ν is the number of repeated experiments, and F is the QFI.

In this paper, we discuss a general scenario of a Mach-Zehnder interferometer. In this interferometer, one of the input ports is an *arbitrary* state and the other one is an *even* (odd) state. This scenario covers many important cases, including the famous protocol of Caves [12] and a recent one proposed by Pezzé and Smerzi [15]. We give an analytic expression of the QFI and identify a PMC to optimize the parameter precision. In this scenario, the QFI is only determined by the average photon numbers of the two modes and the corresponding expectation values of the square of the annihilation operators. We then give two examples of our scenario. Further, for the case where photon losses occur in both arms with the same transmission coefficients and the input state is a product of a coherent state and a coherent superposition state, the analytic expression of QFI is provided. Based on this expression, we prove that the PMC remains unchanged for any transmission coefficient.

II. MACH-ZEHNDER INTERFEROMETER

The Mach-Zehnder (MZ) interferometer is a well-known optical device in quantum metrology which is constructed with two beam splitters and one or two phase shifts. The interferometer we consider here is constructed with two 50:50 beam splitters and two phase shifts in both arms, as shown in Fig. 1. The well-used 50:50 beam splitter can be described by [18–20] $B_x = \exp(-i\frac{\pi}{2}J_x)$, where $J_x = \frac{1}{2}(a^\dagger b + b^\dagger a)$ is one of the operators in the Schwinger representation of bosons. The others are $J_y = \frac{1}{2i}(a^\dagger b - b^\dagger a)$ and $J_z = \frac{1}{2}(a^\dagger a - b^\dagger b)$. Here a, b are annihilation operators for ports A and B, respectively. Operators J_x, J_y, J_z satisfy the commutation $[J_i, J_j] = i\epsilon_{ijk}J_k$, with ϵ_{ijk} being the so-called Levi-Civita symbol. Usually, a phase shift can be expressed by a unitary transformation $\exp[i\theta \hat{N}_{A(B)}]$, where $\hat{N}_A = a^\dagger a$, $\hat{N}_B = b^\dagger b$, and θ is an unknown parameter. In our scenario, denoting θ as the relative phase between the two arms, one can describe the transformation of the total phase shift by the operator $P_z = \exp(i\theta J_z)$. With the above devices we can construct a well-used MZ interferometer, the transformation of which can be written as $U_{\text{mz}} = B_x P_z B_x^\dagger$. Through some algebra, one can find that this transformation can be simplified as

$$U_{\text{MZ}} = \exp(-i\theta J_y), \tag{1}$$

which is a rotation along the y direction.
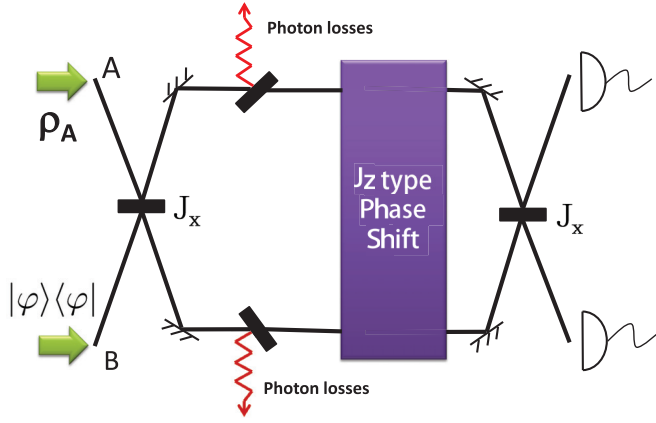
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FIG. 1. (Color online) Sketch of the Mach-Zehnder interferometer. The transformation of the beam splitters is described by $\exp(\pm i\pi J_x/2)$, and that of the phase shift is described by $\exp(i\theta J_z)$. The input states in ports A and B are an arbitrary state and an even (odd) state, respectively.

III. PHASE-MATCHING CONDITION FOR QFI

Quantum Fisher information is a central concept in quantum metrology, and it is defined as [16,17] $F := \mathrm{Tr}(\rho L^2)$, where L is the so-called symmetric logarithmic derivative determined by $\partial_\theta \rho_\theta = (\rho_\theta L + L\rho_\theta)/2$. We consider a separable input state $\rho_{\mathrm{in}} = \rho_A \otimes \rho_B$. Here ρ_A is an arbitrary state, and $\rho_B = |\varphi\rangle\langle\varphi|$ is an even (odd) state. Utilizing the spectral decomposition $\rho_A = \sum_{j=1}^{M} p_j |\psi_j\rangle\langle\psi_j|$, the quantum Fisher information can be written as [16,17,21]

$$F = \sum_{j=1}^{M} 4p_j \langle\phi_j|J_y^2|\phi_j\rangle - \sum_{j,j'=1}^{M} \frac{8p_j p_{j'}}{p_j + p_{j'}} |\langle\phi_j|J_y|\phi_{j'}\rangle|^2, \quad (2)$$

where M is the dimension of the support of ρ_A and $|\phi_j\rangle = |\psi_j\rangle \otimes |\varphi\rangle$ is an eigenstate of ρ_{in} with eigenvalue p_j.

As the input state in port B is an even (odd) state in our scenario, it satisfies $\langle\varphi|b|\varphi\rangle = 0$. Then after some algebra, the QFI can be expressed by

$$F = 2\bar{n}_A \bar{n}_B + \bar{n}_A + \bar{n}_B - 2\mathrm{Re}(\langle a^{\dagger 2}\rangle\langle b^2\rangle), \quad (3)$$

where $\bar{n}_A = \mathrm{Tr}(\rho_A a^\dagger a)$ and $\bar{n}_B = \langle\varphi|b^\dagger b|\varphi\rangle$ are average photon numbers for modes a and b, respectively. To optimize this Fisher information, the input states need to satisfy the following PMC:

$$|\mathrm{Arg}(\langle a^2\rangle) - \mathrm{Arg}(\langle b^2\rangle)| = \pi. \quad (4)$$

Under this condition, the QFI becomes

$$F_m = 2\bar{n}_A \bar{n}_B + \bar{n}_A + \bar{n}_B + 2|\langle a^2\rangle||\langle b^2\rangle|. \quad (5)$$

One key feature of the above result is that the QFI is only determined by the mean photon numbers and the expectations of a^2 and b^2. We emphasize that one input state is arbitrary.

For the case where $|\langle a^2\rangle||\langle b^2\rangle| = 0$, the phases of the input states can be chosen arbitrarily. For example, if ρ_B is a Fock state, i.e., $\rho_B = |n\rangle\langle n|$, then the QFI reduces to $F_m = 2\bar{n}_A \bar{n}_B + \bar{n}_A + \bar{n}_B$, which is the result in Ref. [15] and has been discussed in detail. In the following, we give two examples of the PMC.

Example 1. In this example, we choose ρ_A to be a coherent state $|\beta\rangle\langle\beta|$ and ρ_B to be a coherent superposition state $|\alpha\rangle_+\langle\alpha|$, where $|\alpha\rangle_+ = N_\alpha(|\alpha\rangle + |-\alpha\rangle)$ with $|\alpha\rangle$ also being a coherent state and $N_\alpha^2 = 1/(2 + 2e^{-2|\alpha|^2})$. It is easy to find that $_+\langle\alpha|b|\alpha\rangle_+ = 0$. Here, we denote $\alpha = |\alpha|\exp(i\Phi_\alpha)$, $\beta = |\beta|\exp(i\Phi_\beta)$; then from Eq. (4), the phase-matching condition can be specifically written as

$$|\Phi_\alpha - \Phi_\beta| = \frac{\pi}{2}. \quad (6)$$

Under this condition, the QFI reduces to

$$F_m = 2\bar{n}_A \bar{n}_B + \bar{n}_A + \bar{n}_B + 2\bar{n}_A|\alpha|^2, \quad (7)$$

where $\bar{n}_A = |\beta|^2$ and $\bar{n}_B = |\alpha|^2 \tanh|\alpha|^2$. As $\tanh|\alpha|^2$ is a monotonic function and very close to 1 for $|\alpha| \geqslant 2$, for most values of $|\alpha|$, $\bar{n}_B$ is equal to $|\alpha|^2$, and the QFI reduces to $F_m = 4\bar{n}_A\bar{n}_B + \bar{n}_A + \bar{n}_B$. It is not difficult to obtain that $F_m \leqslant N^2 + N$, where $N = \bar{n}_A + \bar{n}_B$ is the total photon number. The equality can be achieved when $\bar{n}_A = \bar{n}_B$. For the case where the total photon number N is fixed, this bound is the optimal value of the QFI. Considering the PMC, the optimal value can be achieved when $\beta = \pm i\alpha$. From the optimal value of $F_m = N^2 + N \geqslant N^2$, one can find that, with high intensity, the pair of the coherent state $|\pm i\alpha\rangle\langle\pm i\alpha|$ and the coherent superposition state $|\alpha\rangle_+$ can surpass the Heisenberg limit. Reference [13] considered the same input states as in our scenario but with only one phase shift in one arm. In their case, utilizing an analysis similar to that above, it is easy to find that the input states $|\alpha\rangle$ and $|\alpha\rangle_+$ are the optimal choice and can reach the maximum value of QFI.

Example 2. Another well-known even state is the squeezed vacuum state, which is defined as [22] $|\xi\rangle = S(\xi)|0\rangle$, where the squeezing operator reads $S(\xi) = \exp(\frac{1}{2}\xi^* b^2 - \frac{1}{2}\xi b^{\dagger 2})$, with $\xi = |\xi|\exp(i\Phi_\xi)$. For convenience, we still choose the input state in port A to be a coherent state $|\beta\rangle\langle\beta|$. In this example, the PMC is

$$2\Phi_\beta - \Phi_\xi = 0. \quad (8)$$

Under this condition, the QFI can be expressed as

$$F_m = 2\bar{n}_A \bar{n}_B + \bar{n}_A + \bar{n}_B + 2\bar{n}_A\sqrt{\bar{n}_B^2 + \bar{n}_B}, \quad (9)$$

where $\bar{n}_A = |\beta|^2$ and $\bar{n}_B = \sinh^2|\xi|$. This equation is equivalent to the corresponding equation in Ref. [23]. In this expression of QFI, it is only related to the average photon numbers in both ports: $\bar{n}_A$ and $\bar{n}_B$. Figure 2 shows the variation of F_m/N^2 with the change of $\bar{n}_A$, $\bar{n}_B$. From this plot one can find that the optimal value of the quantum Fisher information for a fixed N is obtained near the $n_A = n_B$ line, especially for a large N. This is because when $\bar{n}_B$ is large, for fixed N, $F_m \simeq 4\bar{n}_A\bar{n}_B + \bar{n}_A + \bar{n}_B \leqslant N^2 + N$, and the optimal value can be achieved when $\bar{n}_A = \bar{n}_B$. Adding the PMC, the optimal choice for a large N is that $\beta = \exp(i\Phi_\xi/2)\sinh|\xi|$. Also, from Fig. 2 one can see that with the increase of N, the range that $F_m > N^2$ is increasing, which indicates that the high-intensity input state is good for the enhancement of the phase sensitivity.
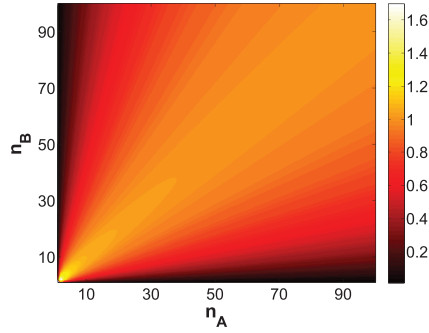
FIG. 2. (Color online) The variation of F_m/N^2 with the average particle numbers $\bar{n}_A$ and $\bar{n}_B$. The input states here are a coherent state $|\beta\rangle\langle\beta|$ in port A and a squeezed vacuum state $|\xi\rangle\langle\xi|$ in port B.

IV. PMC WITH UNBALANCED BEAM SPLITTER

For a more general beam splitter transformation $\exp(i\tau J_x)$ with $\tau \in [0, 2\pi)$, the total setup of the interferometer can be described by

$$U_{MZ} = \exp[i\theta(J_z \cos\tau - J_y \sin\tau)]. \qquad (10)$$

If we restrict the input state ρ_A to a pure state, i.e., $|\psi\rangle\langle\psi|$, and ρ_B is still an even (odd) state, the QFI reads $F = 4\text{Var}(J_z \cos\tau - J_y \sin\tau)$. Due to the property of the even (odd) state, one can find that

$$F = 4\cos^2\tau \text{Var}(J_z) + 4\sin^2\tau \text{Var}(J_y). \qquad (11)$$

It is known that for product states

$$\text{Var}(J_z) = \frac{1}{4}[\text{Var}(a^\dagger a) + \text{Var}(b^\dagger b)] \qquad (12)$$

and $\text{Var}(a^\dagger a)$, $\text{Var}(b^\dagger b)$ are irrelevant to the phase; therefore the phase-matching condition is decided by the second term, $\text{Var}(J_y)$, and keeps the same form as Eq. (4).

V. PMC WITH PHOTON LOSSES

Now we consider the effects of photon losses on the PMC. The scenario we use is shown in Fig. 1. Traditionally, the photon losses can be described by a fictitious beam splitter [24–29]. Here we use $B^T = \exp[i(2\arccos\sqrt{T})J_x]$ to describe this fictitious beam splitter. T is the so-called transmission coefficient, and we also define $R = 1 - T$ as the reflection coefficient. When $T = 1$ ($R = 0$), there are no photon losses in the interferometer, and when $T = 0$ ($R = 1$), all the photons leak out of the interferometer. The whole photon-loss scenario can be mapped into a neat scenario, which includes two steps: first, the input state goes through a particle loss channel, and then the output state imports into a MZ interferometer without losses.

We assume that the leaks in both arms share the same transmission coefficient T and the input state is separable: $\rho_A \otimes \rho_B$. Then after the particle loss channel, the reduced density matrix ρ reads

$$\rho = \text{Tr}_{CD}(\Gamma^D \Gamma^C \rho_A \otimes \rho_B \Gamma^{C\dagger} \Gamma^{D\dagger}), \qquad (13)$$

where the operator of the particle loss channel can be expressed by $\Gamma^C = \exp[i\sqrt{2}\arccos\sqrt{T}(J_x^{AC} + J_y^{BC})]$ and $\Gamma^D =$

$\exp[i\sqrt{2}\arccos\sqrt{T}(J_x^{BD} + J_y^{AD})]$. Here $J_x^{AC} = \frac{1}{2}(a^\dagger c + ac^\dagger)$ and $J_y^{BC} = \frac{1}{2i}(b^\dagger c - bc^\dagger)$, with $c, c^\dagger$ being the annihilation and creation operators of mode C. J_x^{BD} and J_y^{AD} are defined similarly. Next, ρ goes through a regular MZ interferometer, which can be described by $\exp(-\theta J_y)$. Utilizing this representation, this photon-loss scenario can be classified into a normal MZ interferometer scenario with a mixed input state.

Now we choose the initial states to be $\rho_A = |i\alpha e^{i\Phi}\rangle\langle i\alpha e^{i\Phi}|$ and $\rho_B = |\alpha\rangle_+\langle\alpha|$. Here Φ is the relative phase and $\Phi \in [0, \pi)$. Based on the PMC, its optimal value is zero for the no-loss scenario. A similar scenario has been considered with only one phase shift in one arm [13,30,31]. Through some calculations, one can obtain the analytic expression of the QFI:

$$F = 4T|\alpha|^2[N_\alpha^2 + T|\alpha|^2(2N_\alpha^2 - 1)] + 4T^2|\alpha|^4\mathcal{G}\cos^2\Phi$$
$$- 16T^2N_\alpha^4|\alpha|^4(1 - p_r^2)p_t^2\sin^2\Phi, \qquad (14)$$

where $p_r = \exp(-2|\alpha|^2 R)$, $p_t = \exp(-2|\alpha|^2 T)$, and $\mathcal{G} = 1 - 4N_\alpha^4(1 - p_r^2)$. From this equation, one can see that for a fixed α and transmission coefficient T, the maximum value of Eq. (14) can always be obtained at $\Phi = 0$, which indicates that the phase-matching condition remains *unchanged* in this photon-loss case and is not affected by the transmission coefficient T. Under the phase-matching condition, the quantum Fisher information reads

$$F_m = 4TN_\alpha^2|\alpha|^2 + 8T^2N_\alpha^2|\alpha|^4[1 - 2N_\alpha^2(1 - p_r^2)]. \qquad (15)$$

Utilizing the input average photon number $\bar{n}_A$ and the input total photon number N, it can be rewritten as

$$F_m = TN + 2T^2N\bar{n}_A[1 - 2N_\alpha^2(1 - p_r^2)]. \qquad (16)$$

This equation can reduce to Eq. (7) for $T = 1$ ($R = 0$). From this equation, one can find that the photon losses have a negative influence on the QFI. With the decrease of the transmission coefficient T, the QFI reduces. For small losses, namely, R is very small, the QFI reduces to

$$F_m = N + 2N\bar{n}_A - [1 + 4\bar{n}_A(N + 1)]NR. \qquad (17)$$

When $R < R_c$, with $R_c = 2\bar{n}_A/[1 + 4\bar{n}_A(N + 1)]$, the QFI can still be larger than N, which indicates that these input states can still surpass the shot-noise limit. This device is loss tolerant and robust within the region where $R \in [0, R_c]$. For a large N, $R_c \simeq 1/(2N)$.

VI. CONCLUSION

In summary, we have considered a general scenario of a Mach-Zehnder interferometer, where the input state in one port of the interferometer is an arbitrary state and the other one is an even (odd) state. We have provided an analytic expression of the QFI and shown a general PMC under which the phase sensitivity can be enhanced. We give two explicit examples with one port in a coherent state and another in an even coherent state or the squeezed vacuum state. The PMC plays an important role in improving the sensitivity in these cases.

For the unbalanced beam splitters, we find the PMC is unchanged with one port in an arbitrary pure stare and another in an even (odd) state. We also considered the example of a coherent superposition state with photon losses and found

that the PMC remains unchanged and is not affected by the transmission coefficients. In addition, for a small loss, we show that this setup is robust and loss tolerant. The present work sheds light on the problem of how to enhance phase sensitivity in quantum metrology.

[1] V. Giovannetti, S. Lloyd, and L. Maccone, Nat. Photonics **5**, 222 (2011); Phys. Rev. Lett. **96**, 010401 (2006); C. C. Gerry and J. Mimih, Contemp. Phys. **51**, 497 (2010); A. W. Chin, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **109**, 233601 (2012).

[2] M. G. Genoni, S. Olivares, and M. G. A. Paris, Phys. Rev. Lett. **106**, 153603 (2011); L. Pezzé and A. Smerzi, *ibid.* **100**, 073601 (2008); Y. C. Liu, G. R. Jin, and L. You, Phys. Rev. A **82**, 045601 (2010); F. Benatti, R. Floreanini, and U. Marzolino, J. Phys. B **44**, 091001 (2011); C. N. Gagatsos, O. Oreshkov, and N. J. Cerf, Phys. Rev. A **87**, 042307 (2013).

[3] K. P. Seshadreesan, P. M. Anisimov, H. Lee, and J. P. Dowling, New J. Phys. **13**, 083026 (2011); D. W. Berry, B. L. Higgins, S. D. Bartlett, M. W. Mitchell, G. J. Pryde, and H. M. Wiseman, Phys. Rev. A **80**, 052114 (2009); R. Birrittella, J. Mimih, and C. C. Gerry, *ibid.* **86**, 063828 (2012).

[4] C. C. Gerry and A. Benmoussa, Phys. Rev. A **65**, 033822 (2002); C. C. Gerry and R. A. Campos, *ibid.* **64**, 063814 (2001); B. M. Escher, R. L. de MatosFilho, and L. Davidovich, Nat. Phys. **7**, 406 (2011).

[5] X.-M. Lu, S. Luo, and C. H. Oh, Phys. Rev. A **86**, 022342 (2012); X.-M. Lu, Z. Sun, X. Wang, S. Luo, and C. H. Oh, *ibid.* **87**, 050302 (2013); G. Tóth and D. Petz, *ibid.* **87**, 032324 (2013).

[6] J. Estève, C. Gross, A. Weller, S. Giovanazzi, and M. K. Oberthaler, Nature (London) **455**, 1216 (2008); M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, and P. Treutlein, *ibid.* **464**, 1170 (2010).

[7] J. Appel, P. J. Windpassinger, D. Oblak, U. B. Hoff, N. Kjaegaard, and E. S. Polzik, Proc. Natl. Acad. Sci. USA **106**, 10960 (2009).

[8] T. Nagata, R. Okamoto, J. L. O'Brien, K. Sasaki, and S. Takeuchi, Science **316**, 726 (2007).

[9] A. Fujiwara, Phys. Rev. A **63**, 042304 (2001); **65**, 012316 (2001); H. Imai and A. Fujiwara, J. Phys. A **40**, 4391 (2007); A. Fujiwara and H. Imai, *ibid.* **36**, 8093 (2003).

[10] A. Monras and M. G. A. Paris, Phys. Rev. Lett. **98**, 160401 (2007); D. Collins, Phys. Rev. A **87**, 032301 (2013); J. Kahn, *ibid.* **75**, 022326 (2007); M. Hotta and T. Karasawa, *ibid.* **78**, 012332 (2008).

[11] M. Ahmadi, D. E. Bruschi, N. Friis, C. Sabn, G. Adesso, and I. Fuentes, arXiv:1307.7082.

[12] C. M. Caves, Phys. Rev. D **23**, 1693 (1981).

[13] J. Joo, W. J. Munro, and T. P. Spiller, Phys. Rev. Lett. **107**, 083601 (2011).

[14] P. M. Anisimov, G. M. Raterman, A. Chiruvelli, W. N. Plick, S. D. Huver, H. Lee, and J. P. Dowling, Phys. Rev. Lett. **104**, 103602 (2010).

[15] L. Pezzé and A. Smerzi, Phys. Rev. Lett. **110**, 163604 (2013).

[16] C. W. Helstrom, *Quantum Detection and Estimation Theory* (Academic, New York, 1976).

[17] A. S. Holevo, *Probabilistic and Statistical Aspects of Quantum Theory* (North-Holland, Amsterdam, 1982).

[18] B. Yurke, S. L. McCall, and J. R. Klauder, Phys. Rev. A **33**, 4033 (1986).

[19] R. A. Campos, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **40**, 1371 (1989).

[20] B. C. Sanders and G. J. Milburn, Phys. Rev. Lett. **75**, 2944 (1995).

[21] J. Liu, X. Jing, W. Zhong, and X. Wang (unpublished).

[22] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).

[23] M. Jarzyna and R. Demkowicz-Dobrzański, Phys. Rev. A **85**, 011801(R) (2012).

[24] R. Demkowicz-Dobrzanski, U. Dorner, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, and I. A. Walmsley, Phys. Rev. A **80**, 013825 (2009).

[25] S. D. Huver, C. F. Wildfeuer, and J. P. Dowling, Phys. Rev. A **78**, 063828 (2008).

[26] U. Dorner, R. Demkowicz-Dobrzanski, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, and I. A. Walmsley, Phys. Rev. Lett. **102**, 040403 (2009).

[27] T. W. Lee, S. D. Huver, H. Lee, L. Kaplan, S. B. McCracken, C. Min, D. B. Uskov, C. F. Wildfeuer, G. Veronis, and J. P. Dowling, Phys. Rev. A **80**, 063803 (2009).

[28] N. Gkortsilas, J. J. Cooper, and J. A. Dunningham, Phys. Rev. A **85**, 063827 (2012).

[29] X.-X. Zhang, Y.-X. Yang, and X.-B. Wang, Phys. Rev. A **88**, 013838 (2013).

[30] Y. M. Zhang, X. W. Li, W. Yang, and G. R. Jin, arXiv:1307.7353 [Phys. Rev. A (to be published)].

[31] X. Jing, J. Liu, W. Zhong, and X. Wang, arXiv:1307.8009.