Testing genuine tripartite quantum nonlocality with three two-level atoms in a driven cavity

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It is known that the violation of Svetlichny's inequality (SI), rather than the usual Mermin's inequality (MI), is a robust criterion to confirm the existence of genuine multipartite quantum nonlocality. In this paper, we propose a feasible approach to test SI with three two-level atoms (TLAs) dispersively coupled to a driven cavity. The proposal is based on the joint measurements of the states of three TLAs by probing the steady-state transmission spectra of the driven cavity: each peak marks one of the computational basis states and its relative height corresponds to the probability superposed in the detected three-TLA state. With these kinds of joint measurements, the correlation functions in SI can be directly calculated, and thus the SI can be efficiently tested for typical tripartite entanglement, i.e., genuine tripartite entanglement [e.g., Greenberger-Horne-Zeilinger (GHZ) and W states] and biseparable three-qubit entangled states (e.g., $|\chi\rangle_{12}|\xi\rangle_3$). Our numerical experiments show that the SI is violated only by three-qubit GHZ and W states, not by biseparable three-qubit entangled state $|\chi\rangle_{12}|\xi\rangle_3$, while the MI can still be violated by biseparable three-qubit entangled states. Thus the violation of SI can be regarded as a robust criterion for the existence of genuine tripartite entanglement.

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I. INTRODUCTION

Quantum entanglement [1] and quantum nonlocality [2] are two fundamental features of quantum systems. They are in fact two different concepts in quantum theory, even if they are indeed intimately related. Mathematically, quantum entanglement means that the state of a quantum system cannot be expressed as the form of product state or the convex sum of product states. Quantum nonlocality refers to the correlations, among the results of measurements on all the constituent subsystems, which cannot be reproduced by any local hidden-variable model. This nonlocal character of quantum system can be revealed through the violation of certain appropriate Bell-type inequalities. As a consequence, the magnitude of the quantum-mechanical violation of such an inequality could be regarded as a measure of the quantum nonlocality. However, for certain entangled quantum systems, Bell's inequality can still be satisfied. For example, two-qubit Werner states, typical entangled quantum states, cannot violate the Bell-type inequality for certain parameters [3]. Therefore, quantum entanglement does not necessarily imply quantum nonlocality. Actually, it is just a necessary condition for a state to be nonlocal.

Based on the Einstein, Podolsky, and Rosen (EPR) paradox [4], Bell proposed a quantitative inequality [5] (i.e., Bell's inequality) to test the contraction between the bipartite quantum nonlocality and the local hidden variable (LHV) models. Physically, if the Bell's inequality is violated, then the quantum nonlocal correlation (i.e., entanglement) is sustained. During the past decades, many experiments had been made to test the Clauser-Horner-Shimony-Holt (CHSH) version [6] of Bell inequality with various bipartite entanglement, e.g., photons [7], trapped ions [8], an atom and a photon [9], superconducting qubits [10], and neutrons [11], etc. Note that all these experiments agree well with quantum-mechanical predictions and thus rule out the LHV theories.

For testing tripartite quantum nonlocality, two kinds of Bell-type inequalities are proposed. One is Mermin's inequality (MI) [12]

$$M \leqslant 2, \tag{1}$$

satisfied by all *LHV theories*, and another is Svetlichny's inequality (SI) [13]

$$S \leqslant 4, \tag{2}$$

satisfied by all *hybrid local-nonlocal hidden-variable theories*. Above, $M = |E(\theta_1,\theta_2,\theta_3') + E(\theta_1,\theta_2',\theta_3) + E(\theta_1',\theta_2,\theta_3) - E(\theta_1',\theta_2',\theta_3')|$ and $S = |E(\theta_1,\theta_2,\theta_3) + E(\theta_1,\theta_2,\theta_3') + E(\theta_1,\theta_2',\theta_3) - E(\theta_1,\theta_2',\theta_3') + E(\theta_1',\theta_2,\theta_3) - E(\theta_1',\theta_2,\theta_3') - E(\theta_1',\theta_2',\theta_3) - E(\theta_1',\theta_2',\theta_3')|$ are the Mermin (M) and Svetlichny (S) functions, respectively. Certainly, θ_j and θ_j' are the controllable local variables of three independent qubits and $E(\theta_1,\theta_2,\theta_3)$ is the correlation function of these three qubits regarding the local variables θ_1, θ_2, and θ_3. Presently, the maximal violation of MI has been experimentally demonstrated with tripartite GHZ states [14] shared by photons [15,16] and superconducting qubits [17,18], respectively. However, it has been shown that all the biseparable three-qubit entangled states (i.e., two of the qubits are separable from the third, e.g.,$|\chi\rangle_{12}|\xi\rangle_3$) can still violate MI [19,20]. This is because the MI for any of these states is reduced to the relevant CHSH-Bell inequality [6]. This indicates that *the violation of MI cannot unambiguously identify genuine tripartite quantum nonlocality*. Instead, it has been shown theoretically that the SI can be violated only by genuine tripartite entanglement, not by biseparable states [13,19–21]. Recently, the violation of SI has been

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experimentally demonstrated only with the photonic GHZ states (genuine tripartite entanglement) [22,23].

As another possible demonstration to experimentally verify the genuine tripartite quantum nonlocality in cavity-QED systems, in this paper we discuss how to test SI with three two-level atoms (TLAs) in a driven cavity. The basic idea is the tripartite correlations required for testing SI are calculated by jointly probing the qubits in the computational basis via the spectra of the driven cavity. Indeed [24,25], beyond the usual mean-field approximation each of the detected transmission peaks marks one of the computational basis states and their superposed probabilities in the detected state. As a consequence, the required correlation functions can be directly calculated by the spectral information. With such a spectral technique we demonstrate the tests of the SI for typical tripartite entanglement, i.e., genuine tripartite entanglement (e.g., GHZ and W states [26]) and biseparable three-qubit entangled states (e.g., $|\chi\rangle_{12}|\xi\rangle_3$). We can find that the SI could be violated only by three-qubit GHZ and W states, not by biseparable three-qubit entangled state $|\chi\rangle_{12}|\xi\rangle_3$. This also confirms the existence of the genuine multipartite nonlocal correlations. Compared with the previous schemes for testing SI with photons [22,23], a potential advantage of our proposal is that the required tripartite correlations can be obtained by relatively-simple joint spectral measurements, rather than the photonic coincidence measurements used previously.

The remainder of this paper is organized as follows. In Sec. II, we briefly review how to realize joint measurements of three TLAs in a driven cavity. Then, with this kind of joint measurement, the tests of the SI for tripartite entanglement (GHZ states, W states, and biseparable three-qubit entangled state $|\chi\rangle_{12}|\xi\rangle_3$) are demonstrated in Sec. III. Finally, discussions and conclusions are given in Sec. IV.

II. JOINT MEASUREMENTS OF THREE TLAs IN A DRIVEN CAVITY

We consider a driven cavity-TLAs system shown in Fig. 1, wherein three separated-spatially TLAs couple to a common cavity. Under the standard rotating-wave approximation the system can be described by a Tavis-Cummings Hamiltonian

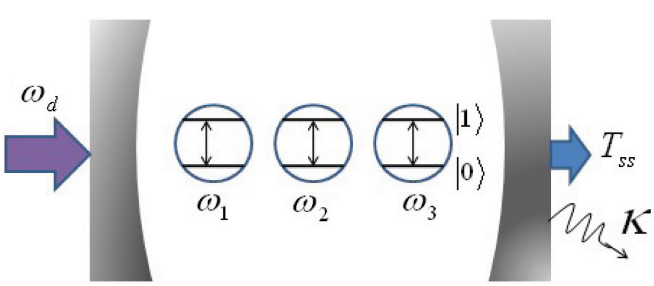

FIG. 1. (Color online) Schematics of three TLAs (with transition frequency ω_1, ω_2, ω_3, respectively) dispersively coupled to a cavity. An external driving with the frequency ω_d is applied to the cavity. κ denotes the cavity decay rate. The joint measurements of three TLAs can be realized by probing the steady-state transmission spectra T_{ss} of the driven cavity.

($\hbar = 1$; hereafter the same) [27]

$$H = \omega_r a^\dagger a + \sum_{j=1,2,3}\left[\frac{\omega_j}{2}\sigma_{zj} + g_j(a^\dagger\sigma_{-j} + a\sigma_{+j})\right]. \quad (3)$$

Here, ω_r is the cavity frequency, $a^\dagger(a)$ its creation (annihilation) operator of the cavity photon; ω_j is the transition frequency of jth TLA with the Pauli operators: $\sigma_{+j} = |1\rangle_j\langle 0|$, $\sigma_{-j} = |0\rangle_j\langle 1|$, and $\sigma_{zj} = |0\rangle_j\langle 0| - |1\rangle_j\langle 1|$. And g_j is the coupling strength between the jth TLA and the cavity. For the reliable readout of qubits by probing the transmission of the cavity, we assume that the dispersive condition,

$$0 < \left|\frac{g_j}{\Delta_j}, \frac{g_j g_{j'}}{\Delta_j\Delta_{jj'}}, \frac{g_j g_{j'}}{\Delta_{j'}\Delta_{jj'}}\right| \ll 1, \quad j \neq j' = 1,2,3, \quad (4)$$

is satisfied to eliminate the influence of the effective interatom interactions. Here, $\Delta_j = \omega_j - \omega_r$ is the detuning between the jth TLA and the cavity, and $\Delta_{jj'} = \omega_j - \omega_{j'}$ the detuning between the jth TLA and the j'th TLA.

In a frame rotating at the drive field (applying to the cavity), the driven cavity-TLAs system can be described by the following effective Hamiltonian:

$$\widetilde{H} = \left(-\Delta_r + \sum_{j=1,2,3}\Gamma_j\sigma_{zj}\right)a^\dagger a + \sum_{j=1,2,3}\frac{\widetilde{\omega}_j}{2}\sigma_{zj} + \epsilon(a^\dagger + a). \quad (5)$$

Here, the last term describes the driving of the cavity [28], with ϵ being the time-independent real amplitude and ω_d the frequency of the driving field. Above, $\Delta_r = \omega_d - \omega_r$ is the detuning between the external driving frequency and the cavity frequency, $\Gamma_j = g_j^2/\Delta_j$ and $\widetilde{\omega}_j = \omega_j + \Gamma_j$. From Eq. (5), one can see that the interaction between the cavity and three TLAs takes the form of $\sum_{j=1}^{3}\Gamma_j\sigma_{zj}a^\dagger a$. This indicates that these three TLAs *jointly* cause the atomic state-dependent frequency shift of the cavity. To be specific, if the register of three TLAs is prepared at the computational basis state $|111\rangle$ ($|110\rangle$, $|101\rangle$, $|100\rangle$, $|011\rangle$, $|010\rangle$, $|001\rangle$, or $|000\rangle$), then the frequency of the cavity is shifted as $-\Gamma_1 - \Gamma_2 - \Gamma_3$ ($-\Gamma_1 - \Gamma_2 + \Gamma_3$, $-\Gamma_1 + \Gamma_2 - \Gamma_3$, $-\Gamma_1 + \Gamma_2 + \Gamma_3$, $\Gamma_1 - \Gamma_2 - \Gamma_3$, $\Gamma_1 - \Gamma_2 + \Gamma_3$, $\Gamma_1 + \Gamma_2 - \Gamma_3$, or $\Gamma_1 + \Gamma_2 + \Gamma_3$). Thus the frequency shifts of the cavity can be used to mark all the possible computational basis states of these TLAs. This argument can be verified by probing the transmitted spectra of the driven cavity.

Neglecting the relatively small decays of the TLAs and under the Born-Markov approximation, the dynamics of the above driven system can be described by the following master equation [29]:

$$\frac{d\rho}{dt} = -i[\widetilde{H},\rho] + \kappa(a\rho a^\dagger - a^\dagger a\rho/2 - \rho a^\dagger a/2), \quad (6)$$

with ρ being the density matrix of the system and κ the photon decay rate of the cavity. The steady-state transmission spectra of the driven cavity is defined as the driving-frequency-dependent expectable value

$$T_{ss} = \frac{\langle a^\dagger a\rangle_{ss}}{\epsilon^2} \quad (7)$$

of the intercavity photons. Specifically, this expectable value is determined by the following differential equation:

$$\frac{d\langle a^\dagger a\rangle}{dt} = -\kappa\langle a^\dagger a\rangle - 2\epsilon\,\mathrm{Im}\langle a\rangle, \tag{8}$$

with

$$\frac{d\langle a\rangle}{dt} = \left(i\Delta_r - \frac{\kappa}{2}\right)\langle a\rangle - i\epsilon - i\Gamma_1\langle a\sigma_{z1}\rangle - i\Gamma_2\langle a\sigma_{z2}\rangle - i\Gamma_3\langle a\sigma_{z3}\rangle. \tag{9}$$

Consequently, the differential equations for the higher-order correlations (e.g., $\langle a\sigma_{zj}\rangle$) can also be obtained, and the equation chains are naturally cut up to the four-partite correlations. Physically, only the incident photon whose frequency is equivalent to one of the state-dependent frequencies of the cavity can transmit the cavity and then be detected. This means that the detected probability is the superposed probability of the computational basis state of the TLAs. Therefore, the relative height of the peak corresponds to the superposed probability of the computational basis state in the detected state. Experimentally, the steady-state transmission spectra can be statistically observed by performing the measurement on an ensemble of the identically detected state. In this way, the joint measurement of the TLAs can be realized by probing steady-state transmission spectra of the driven cavity.

III. SI TESTS BY JOINT MEASUREMENTS

With the above joint measurements of three TLAs, we now discuss how to test the SI with typical tripartite entanglement, i.e., genuine tripartite entangled states (e.g., well-known GHZ and W states) and biseparable three-qubit entangled states (e.g., $|\chi\rangle_{12}|\xi\rangle_3$), in a relatively simple way.

A. SI test with tripartite GHZ states

First, we assume that the TLAs are initially prepared in one of the GHZ states [14],

$$|\psi\rangle_{123} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{123}. \tag{10}$$

Then the desirable local variables $\theta_j (j = 1,2,3)$ can be encoded into the above GHZ state by performing the single-qubit operations,

$$U_j(\theta_j) = R_z(-\theta_j/2)R_y(\pi/4)R_z(\theta_j/2) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & e^{-i\theta_j} \\ -e^{i\theta_j} & 1\end{pmatrix}, \tag{11}$$

on each TLA. It has been shown that the above single-qubit operations, $R_y(\pi/4) = e^{i\sigma_y\pi/4}$ and $R_z(\theta/2) = e^{i\sigma_z\theta/2}$, can be easily realized [30]. After the encoding operations, the GHZ state $|\psi\rangle_{123}$ is changed into

$$|\Psi\rangle_{123} = U_1(\theta_1)U_2(\theta_2)U_3(\theta_3)|\psi\rangle_{123}. \tag{12}$$

Now, let us perform the joint measurements to determine the required correlation functions for various combinations of the local variables. Theoretically, the correlation function in the SI can be calculated as

$$E(\theta_1,\theta_2,\theta_3) = \langle\Psi|P|\Psi\rangle = -\cos(\theta_1+\theta_2+\theta_3), \tag{13}$$

with the projective operator $P = \sigma_{z1}\otimes\sigma_{z2}\otimes\sigma_{z3}$. For one set of local variables $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\}$, the S function in Eq. (2) can be calculated as

$$\begin{aligned} S_1 = |&\cos(\theta_1+\theta_2+\theta_3) + \cos(\theta_1+\theta_2+\theta_3') \\ &+ \cos(\theta_1+\theta_2'+\theta_3) - \cos(\theta_1+\theta_2'+\theta_3') \\ &+ \cos(\theta_1'+\theta_2+\theta_3) - \cos(\theta_1'+\theta_2+\theta_3') \\ &- \cos(\theta_1'+\theta_2'+\theta_3) - \cos(\theta_1'+\theta_2'+\theta_3')|. \end{aligned} \tag{14}$$

For the typical set of local variables, $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\} = \{\frac{\pi}{4},\frac{\pi}{2},0,\frac{3\pi}{4},\pi,\frac{\pi}{2}\}$, we have

$$S_1 = 4\sqrt{2} > 4. \tag{15}$$

This means that SI [13], namely $S \leqslant 4$, is maximally violated. Experimentally, the above encoding and measurement can be repeated many times for the same local variables. Consequently, the correlation function $E(\theta_1,\theta_2,\theta_3)$ can be determined by

$$E(\theta_1,\theta_2,\theta_3) = P_{000} + P_{011} + P_{101} + P_{110} - P_{001} - P_{010} - P_{100} - P_{111}. \tag{16}$$

Here $\sum_{i,j,k=0,1} P_{ijk} = 1$ with P_{ijk} being the superposed probability of the computational basis state $|ijk\rangle$ in the encoded state $|\Psi\rangle$, while all the required superposed probability P_{ijk} required for calculating the correlation function in Eq. (16) can be determined by the joint measurements of the TLAs. Specifically, for one set of local variables chosen as $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\} = \{\frac{\pi}{4},\frac{\pi}{2},0,\frac{3\pi}{4},\pi,\frac{\pi}{2}\}$, Fig. 2 shows the relevant steady-state transmission spectra T_{ss} of the driven cavity as a function of the detuning $\Delta_r = \omega_d - \omega_r$ for the encoded state $|\Psi\rangle$. Here the steady-state transmission spectra T_{ss} of the driven cavity is calculated with Eq. (7), and the parameters are chosen as $(\Gamma_1,\Gamma_2,\Gamma_3,\kappa) = 2\pi\times(20,12,5,1)$MHz. Panels (a)–(h) correspond to the local variables $\{\theta_1,\theta_2,\theta_3\}$, $\{\theta_1,\theta_2,\theta_3'\}$, $\{\theta_1,\theta_2',\theta_3\}$, $\{\theta_1,\theta_2',\theta_3'\}$, $\{\theta_1',\theta_2,\theta_3\}$, $\{\theta_1',\theta_2,\theta_3'\}$, $\{\theta_1',\theta_2',\theta_3\}$, and $\{\theta_1',\theta_2',\theta_3'\}$. From each subfigure in Fig. 2, we can read out all the correlation functions in SI. For example, in Fig. 2(a), it can be seen that eight peaks mark the computational basis states $|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$, and $|111\rangle$, respectively. And, the relative heights of these peaks read $P_{000} = P_{011} = P_{101} = P_{110} = 0.213$ and $P_{001} = P_{010} = P_{100} = P_{111} = 0.037$. According to Eq. (16), the correlation function $E(\theta_1,\theta_2,\theta_3)$ is calculated as

$$E(\theta_1,\theta_2,\theta_3) = 0.704. \tag{17}$$

Similarly, from Figs. 2(b)–2(h) and according to Eq. (16), we can also get the correlation functions

$$\begin{aligned} &E(\theta_1,\theta_2,\theta_3') = 0.704, \\ &E(\theta_1,\theta_2',\theta_3) = 0.704, \\ &E(\theta_1,\theta_2',\theta_3') = -0.704, \\ &E(\theta_1',\theta_2,\theta_3) = 0.704, \\ &E(\theta_1',\theta_2,\theta_3') = -0.704, \\ &E(\theta_1',\theta_2',\theta_3) = -0.704, \\ &E(\theta_1',\theta_2',\theta_3') = -0.704. \end{aligned} \tag{18}$$

Substituting these correlation functions in (17) and (18) into Eq. (2), we get

$$S_1' = 5.632 > 4. \tag{19}$$

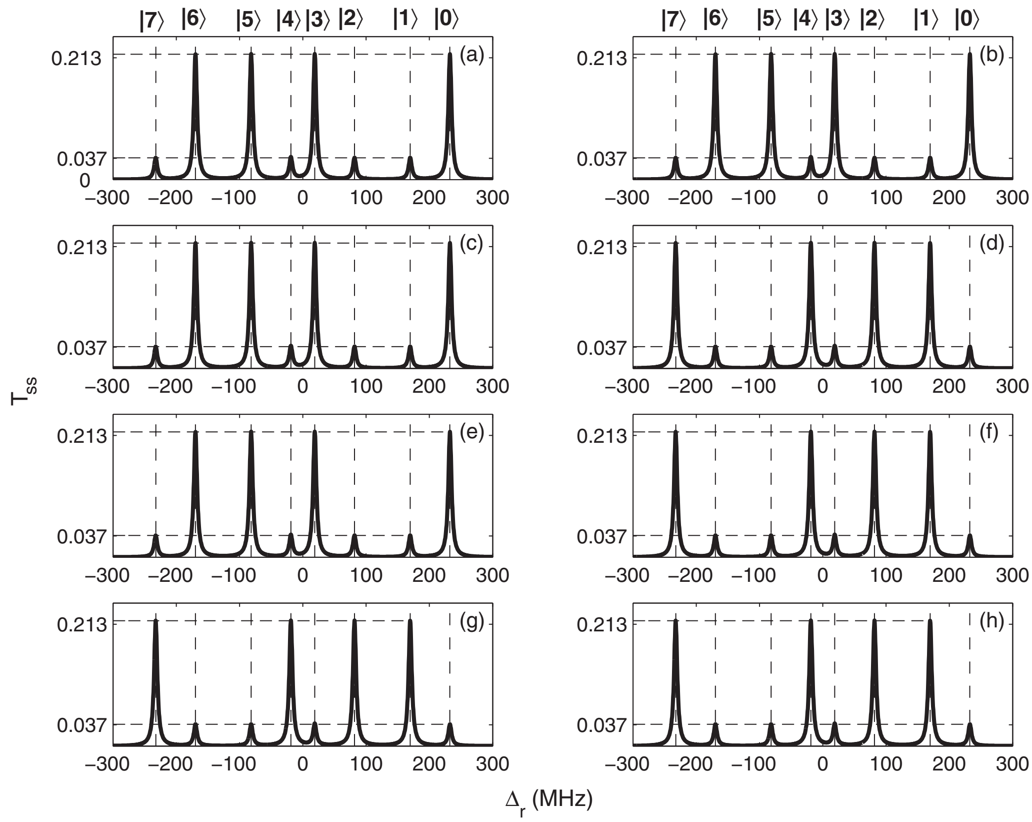

FIG. 2. Steady-state transmission spectra of the driven cavity T_{ss} vs the detuning $\Delta_r = \omega_d - \omega_r$ for the encoded state $|\Psi\rangle$ with one set of local variables chosen as $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\} = \{\frac{\pi}{4},\frac{\pi}{2},0,\frac{3\pi}{4},\pi,\frac{\pi}{2}\}$. Panels (a)–(h) correspond to the local variables $\{\theta_1,\theta_2,\theta_3\}$, $\{\theta_1,\theta_2,\theta_3'\}$, $\{\theta_1,\theta_2',\theta_3\}$, $\{\theta_1,\theta_2',\theta_3'\}$, $\{\theta_1',\theta_2,\theta_3\}$, $\{\theta_1',\theta_2,\theta_3'\}$, $\{\theta_1',\theta_2',\theta_3\}$, and $\{\theta_1',\theta_2',\theta_3'\}$. Here, the parameters are chosen as $(\Gamma_1,\Gamma_2,\Gamma_3,\kappa) = 2\pi \times (20,12,5,1)$ MHz. $|\mathbf{0}\rangle = |000\rangle$, $|\mathbf{1}\rangle = |001\rangle$, $|\mathbf{2}\rangle = |010\rangle$, $|\mathbf{3}\rangle = |011\rangle$, $|\mathbf{4}\rangle = |100\rangle$, $|\mathbf{5}\rangle = |101\rangle$, $|\mathbf{6}\rangle = |110\rangle$, and $|\mathbf{7}\rangle = |111\rangle$.

As a consequence, the SI [13] is robustly violated. Also, with the required correlation functions $E(\theta_1,\theta_2,\theta_3')$, $E(\theta_1,\theta_2',\theta_3)$, $E(\theta_1',\theta_2,\theta_3)$, and $E(\theta_1',\theta_2',\theta_3')$ in Eq. (18) for the MI test [12], we can calculate the M function (1) as

$$M_1 = 2.816 > 2. \tag{20}$$

This indicates that the MI [12] is also violated.

Similar to the case for the GHZ states discussed above, the SI can also be tested for the W state and biseparable three-qubit entangled state $|\chi\rangle_{12}|\xi\rangle_3$ as follows.

B. SI test with tripartite W states

With tripartite W states [26],

$$|\phi\rangle_{123} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)_{123}, \tag{21}$$

we can encode the local variables $\theta_j (j = 1,2,3)$ by performing a single-qubit operation [30] on each TLA,

$$V_j(\theta_j) = R_y(\theta_j/2) = \begin{pmatrix} \cos\frac{\theta_j}{2} & \sin\frac{\theta_j}{2} \\ -\sin\frac{\theta_j}{2} & \cos\frac{\theta_j}{2} \end{pmatrix}. \tag{22}$$

Afterwards, the original W state $|\phi\rangle_{123}$ is changed into

$$|\Phi\rangle_{123} = V_1(\theta_1)V_2(\theta_2)V_3(\theta_3)|\phi\rangle_{123}. \tag{23}$$

Again, the correlation function for the present case can be theoretically calculated as

$$\begin{aligned} E(\theta_1,\theta_2,\theta_3) &= \langle\Phi|P|\Phi\rangle \\ &= -\tfrac{2}{3}\cos(\theta_1+\theta_2+\theta_3) - \tfrac{1}{3}\cos\theta_1\cos\theta_2\cos\theta_3, \end{aligned} \tag{24}$$

with $P = \sigma_{z1}\otimes\sigma_{z2}\otimes\sigma_{z3}$. Thus, for one set of local variables $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\}$, the S function in Eq. (2) can be calculated as

$$\begin{aligned} S_2 = \tfrac{1}{3}|&2\cos(\theta_1+\theta_2+\theta_3) + \cos\theta_1\cos\theta_2\cos\theta_3 \\ &+2\cos(\theta_1+\theta_2+\theta_3') + \cos\theta_1\cos\theta_2\cos\theta_3' \\ &+2\cos(\theta_1+\theta_2'+\theta_3) + \cos\theta_1\cos\theta_2'\cos\theta_3 \\ &-2\cos(\theta_1+\theta_2'+\theta_3') - \cos\theta_1\cos\theta_2'\cos\theta_3' \\ &+2\cos(\theta_1'+\theta_2+\theta_3) + \cos\theta_1'\cos\theta_2\cos\theta_3 \\ &-2\cos(\theta_1'+\theta_2+\theta_3') - \cos\theta_1'\cos\theta_2\cos\theta_3' \\ &-2\cos(\theta_1'+\theta_2'+\theta_3) - \cos\theta_1'\cos\theta_2'\cos\theta_3 \\ &-2\cos(\theta_1'+\theta_2'+\theta_3') - \cos\theta_1'\cos\theta_2'\cos\theta_3'|. \end{aligned} \tag{25}$$

Therefore, for the local variables selected as $\theta_1 = \theta_2 = \theta_3 = 35.264^\circ$ and $\theta_1' = \theta_2' = \theta_3' = 144.736^\circ$, we have

$$S_2 = 4.354 > 4. \tag{26}$$

This indicates that the SI [13] is violated.

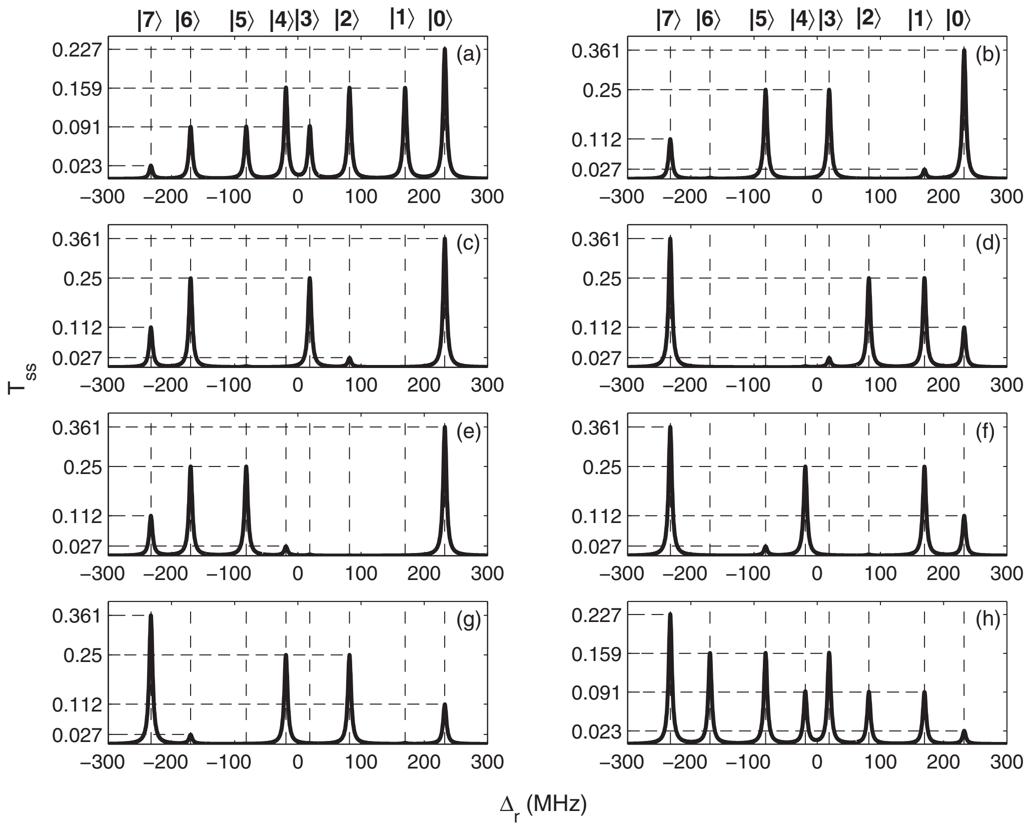

FIG. 3. Steady-state transmission spectra of the driven cavity T_{ss} vs the detuning $\Delta_r = \omega_d - \omega_r$ for the encoded state $|\Phi\rangle$ with the local variables chosen as $\theta_1 = \theta_2 = \theta_3 = 35.264^\circ$ and $\theta_1' = \theta_2' = \theta_3' = 144.736^\circ$. Panels (a)–(h) correspond to the local variables $\{\theta_1,\theta_2,\theta_3\}$, $\{\theta_1,\theta_2,\theta_3'\}$, $\{\theta_1,\theta_2',\theta_3\}$, $\{\theta_1,\theta_2',\theta_3'\}$, $\{\theta_1',\theta_2,\theta_3\}$, $\{\theta_1',\theta_2,\theta_3'\}$, $\{\theta_1',\theta_2',\theta_3\}$, and $\{\theta_1',\theta_2',\theta_3'\}$. The parameters are the same as those in Fig. 2.

For the experimental test, all the correlation functions in the SI can be read out from steady-state transmission spectra of the driven cavity. For the local variables selected as $\theta_1 = \theta_2 = \theta_3 = 35.264^\circ$ and $\theta_1' = \theta_2' = \theta_3' = 144.736^\circ$, with the same parameters as those in Fig. 2, the steady-state transmission spectra T_{ss} of the driven cavity (as a function of the detuning $\Delta_r = \omega_d - \omega_r$) for the encoded state $|\Phi\rangle$ is shown in Fig. 3. Panels (a)–(h) correspond to local parameters $\{\theta_1,\theta_2,\theta_3\}$, $\{\theta_1,\theta_2,\theta_3'\}$, $\{\theta_1,\theta_2',\theta_3\}$, $\{\theta_1,\theta_2',\theta_3'\}$, $\{\theta_1',\theta_2,\theta_3\}$, $\{\theta_1',\theta_2,\theta_3'\}$, $\{\theta_1',\theta_2',\theta_3\}$, and $\{\theta_1',\theta_2',\theta_3'\}$. According to Eq. (16) and from Figs. 3(a)–3(h), the correlation functions in SI are evaluated as

$$\begin{aligned} E(\theta_1,\theta_2,\theta_3) &= 0,\\ E(\theta_1,\theta_2,\theta_3') &= 0.722,\\ E(\theta_1,\theta_2',\theta_3) &= 0.722,\\ E(\theta_1,\theta_2',\theta_3') &= -0.722,\\ E(\theta_1',\theta_2,\theta_3) &= 0.722,\\ E(\theta_1',\theta_2,\theta_3') &= -0.722,\\ E(\theta_1',\theta_2',\theta_3) &= -0.722,\\ E(\theta_1',\theta_2',\theta_3') &= 0. \end{aligned} \tag{27}$$

Inserting these correlation functions in (27) into Eq. (2), we get

$$S_2' = 4.332 > 4. \tag{28}$$

This indicates that the SI [13] is robustly violated. Additionally, with the desired correlation functions in Eq. (27) for the MI test [12], the M function (1) is evaluated as

$$M_2 = 2.166 > 2. \tag{29}$$

Obviously, the MI [12] is violated as well.

C. SI test with biseparable three-qubit entangled states

Above, we have shown that both the SI and MI are violated by the genuine tripartite entangled GHZ and W states. Now, we show in this subsection that the MI, rather than the SI, can still be violated by biseparable three-qubit entangled states.

Without loss of generality, let us consider the following biseparable three-qubit entangled state:

$$|\varphi\rangle_{123} = |\chi\rangle_{12}|\xi\rangle_3 = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{12}|0\rangle_3. \tag{30}$$

The local variables $\theta_j (j = 1,2,3)$ can be encoded into the state $|\varphi\rangle$ by performing the single-qubit operation $V_j(\theta_j)$ (22) on each TLA. Then, the state $|\varphi\rangle_{123}$ is transformed into

$$|\tilde{\varphi}\rangle_{123} = V_1(\theta_1)V_2(\theta_2)V_3(\theta_3)|\varphi\rangle_{123}. \tag{31}$$

Consequently, the correlation function can be theoretically calculated as

$$E(\theta_1,\theta_2,\theta_3) = \langle\tilde{\varphi}|P|\tilde{\varphi}\rangle = \cos(\theta_1 - \theta_2)\cos\theta_3, \tag{32}$$

with $P = \sigma_{z1} \otimes \sigma_{z2} \otimes \sigma_{z3}$. Thus, for one set of local variables $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\}$, the S function in Eq. (2) can be

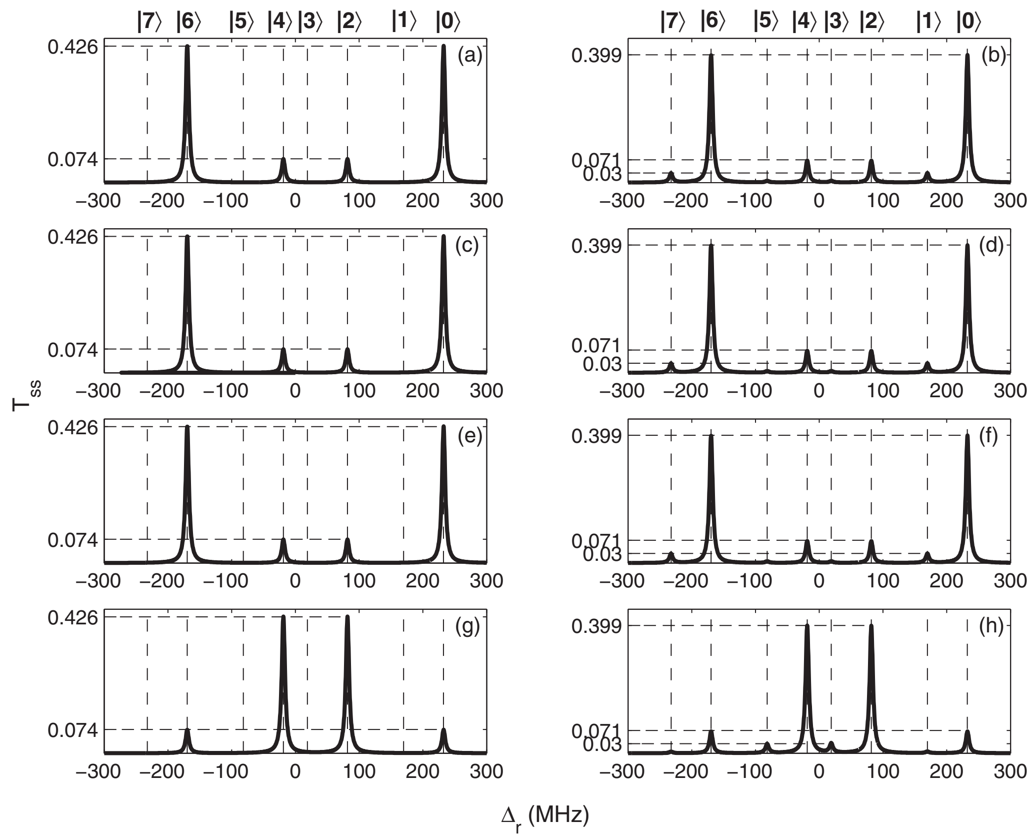

FIG. 4. Steady-state transmission spectra of the driven cavity T_{ss} versus the detuning $\Delta_r = \omega_d - \omega_r$ for the encoded state $|\tilde{\varphi}\rangle$ with the local variables chosen as $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\} = \{\frac{\pi}{4},0,0,\frac{7\pi}{4},\frac{\pi}{2},\frac{\pi}{6}\}$. Panels (a)–(h) correspond to the local variables $\{\theta_1,\theta_2,\theta_3\}$, $\{\theta_1,\theta_2,\theta_3'\}$, $\{\theta_1,\theta_2',\theta_3\}$, $\{\theta_1,\theta_2',\theta_3'\}$, $\{\theta_1',\theta_2,\theta_3\}$, $\{\theta_1',\theta_2,\theta_3'\}$, $\{\theta_1',\theta_2',\theta_3\}$, and $\{\theta_1',\theta_2',\theta_3'\}$. Here, the parameters are the same as those in Fig. 2.

calculated as

$$\begin{aligned} S_3 = |&\cos(\theta_1-\theta_2)\cos\theta_3 + \cos(\theta_1-\theta_2)\cos\theta_3' \\ &+ \cos(\theta_1-\theta_2')\cos\theta_3 - \cos(\theta_1-\theta_2')\cos\theta_3' \\ &+ \cos(\theta_1'-\theta_2)\cos\theta_3 - \cos(\theta_1'-\theta_2)\cos\theta_3' \\ &- \cos(\theta_1'-\theta_2')\cos\theta_3 - \cos(\theta_1'-\theta_2')\cos\theta_3'|. \end{aligned} \quad (33)$$

For the local variables chosen as $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\} = \{\frac{\pi}{4},0,0,\frac{7\pi}{4},\frac{\pi}{2},\frac{\pi}{6}\}$, we have

$$S_3 = 2.828 < 4. \quad (34)$$

This indicates that the SI [13] is not violated. The numerical experiments to verify such an argument are similar to those done previously. Experimentally, for the local variables chosen as $\{\theta_1,\theta_2,\theta_3,\theta_1',\theta_2',\theta_3'\} = \{\frac{\pi}{4},0,0,\frac{7\pi}{4},\frac{\pi}{2},\frac{\pi}{6}\}$ and the parameters selected as the same as those in Fig. 2, the steady-state transmission spectra of the driven cavity as a function of the detuning $\Delta_r = \omega_d - \omega_r$ for the encoded state $|\tilde{\varphi}\rangle$ is plotted in Fig. 4. Panels (a)–(h) correspond to the local variables $\{\theta_1,\theta_2,\theta_3\}$, $\{\theta_1,\theta_2,\theta_3'\}$, $\{\theta_1,\theta_2',\theta_3\}$, $\{\theta_1,\theta_2',\theta_3'\}$, $\{\theta_1',\theta_2,\theta_3\}$, $\{\theta_1',\theta_2,\theta_3'\}$, $\{\theta_1',\theta_2',\theta_3\}$, and $\{\theta_1',\theta_2',\theta_3'\}$. According to Eq. (16) and from Figs. 4(a)–4(h), the correlation functions in the SI are evaluated as

$$\begin{aligned} E(\theta_1,\theta_2,\theta_3) &= 0.704, \\ E(\theta_1,\theta_2,\theta_3') &= 0.596, \\ E(\theta_1,\theta_2',\theta_3) &= 0.704, \\ E(\theta_1,\theta_2',\theta_3') &= 0.596, \\ E(\theta_1',\theta_2,\theta_3) &= 0.704, \\ E(\theta_1',\theta_2,\theta_3') &= 0.596, \\ E(\theta_1',\theta_2',\theta_3) &= -0.704, \\ E(\theta_1',\theta_2',\theta_3') &= -0.596. \end{aligned} \quad (35)$$

Inserting these correlation functions in (35) into Eq. (2), we get

$$S_3' = 2.816 < 4. \quad (36)$$

Obviously, the SI [13] is not violated. However, with the available correlation functions in (35) for the MI test [12], the M function (1) is calculated as

$$M_3 = 2.6 > 2. \quad (37)$$

This means that the MI [12] is still violated. Therefore, the MI cannot be served as a proper criterion for the existence of genuine tripartite entanglement.

IV. DISCUSSIONS AND CONCLUSIONS

We would like to emphasize that the deviations of S function between ideal predictions and the results of numerical experiments in (19) and (28) are really negligible, e.g., $\delta S_1 = S_1 - S_1' = 0.024$ for the case of GHZ states, and $\delta S_2 = S_2 - S_2' = 0.022$ for the case of W states. This implies that the influence of the dissipation of the cavity (which leads to various finite widths of the transmission spectra) is practically unimportant. Therefore, the present proposal for testing the SI could be feasible with a dissipative cavity.

Presently, there are many existing schemes to generate three-qubit GHZ and W states in a driven cavity system, which are available for quantum nonlocality test. Taking a kind of driven cavity system (circuit QED [31]) as an example, GHZ and W states can be readily generated in circuit QED by several methods, such as a sequence of quantum gates [18], measurement-based generation [32,33], and one-step dynamical evolution [25,34–38]. References [18,32,33] have really shown that the GHZ and W states can be generated with different transition frequencies of three atoms (superconducting qubits). Certainly, these states can be easily generated with the same transition frequencies of three atoms [25,34–38] by adjusting the applied external biased fluxes. Moreover, it can also be found that both GHZ and W states of three different atoms can be prepared with three different external driving [18] or only one external driving [25,32–34] on the cavity. Alternatively, without the external driving on the cavity, GHZ and W states can be effectively generated with local classical drivings on the atoms [35–38]. Therefore, there are many feasible and flexible methods to generate GHZ and W states in circuit QED, which are available for quantum nonlocality tests.

In conclusion, we have proposed a feasible method to test the SI with three TLAs dispersively coupled to a driven cavity. The joint measurements of three TLAs were realized by probing the steady-state transmission spectra of the driven cavity. With these kinds of joint measurements, the local-variable-dependent probabilities of various basis states superposed in the local-variable-encoded states can be directly readout. Consequently, various correlation functions for different local variables can be easily calculated. As a consequence, both the SI and MI can be efficiently tested for three kinds of tripartite entangled states: GHZ states, W states, and biseparable three-qubit entangled state $|\chi\rangle_{12}|\xi\rangle_3$. It is shown that the MI can be violated by these three kinds of states, while the SI can only be violated by GHZ and W states, not by a biseparable three-qubit entangled state $|\chi\rangle_{12}|\xi\rangle_3$. Therefore, it is verified that the violation of SI can be used as a robust criterion to confirm the existence of genuine tripartite quantum nonlocality, rather than the usual MI. We believe that our proposal can be generalized to test genuine multipartite quantum nonlocality characterized by multiqubit SI [19] in a straightforward way.

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