Non-Markovian effect on the precision of parameter estimation

K. Berrada*
Al Imam Mohammad Ibn Saud Islamic University (IMSIU), College of Science, Department of Physics, Riyadh, Saudi Arabia
The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, Miramare-Trieste, Italy
(Received 22 July 2013; published 10 September 2013; corrected 26 September 2013)

We study the non-Markovian effect on the dynamics of the quantum Fisher information (QFI) by exactly solving a model consisting of a qubit system subjects to a zero-temperature reservoir for two different non-Markovian conditions: a high-Q cavity slightly off-resonant with the qubit transition frequency and a nonperfect photonic band gap, respectively. The phenomenon that the QFI, namely, the precision of estimation, changes dramatically with the environment structure and is affected by the environment memory effects. We find that revivals and retardation of QFI loss may occur by adjusting the cavity-qubit detuning, in the first case, while partial QFI trapping occurs in nonideal photonic band gaps and the decreasing gap width seriously destroys coherence, thus reducing the precision of estimation. These features make the qubit system in non-Markovian environments a good candidate for implementation of quantum optics schemes and information with high precision.

DOI: 10.1103/PhysRevA.88.035806 PACS number(s): 42.50.St, 03.67.−a, 03.65.Ta, 42.50.Dv

Parameter estimation is a significant pillar of different branches of science and technology, and new techniques developed for measurement of parameter sensitivity have often led to scientific breakthroughs and technological advancement. In the field of quantum estimation, the main task is to determine the value of an unknown parameter labeling the quantum system, and the primary goal is to enhance the precision of resolution. There is a great deal of work on phase estimation addressing the practical problems of state generation, loss, and decoherence [1–6]. Quantum Fisher information (QFI) lies at the heart of a parameter estimation theory that was originally introduced by Fisher [7]. QFI, which characterizes the sensitivity of the state with respect to changes in a parameter, is a key concept in parameter estimation theory. It provides, in particular, a boundary to distinguish the members of a family of probability distributions. When quantum systems are involved, especially for problems in which the quantity of interest is not directly accessible, the optimal measurement may be found using tools from quantum estimation theory. The quantum version of the Cramér-Rao inequality has been established and the lower bound is imposed by QFI [8]. Hence, the QFI becomes the key problem to be solved, an abstract quantity that measures the maximum information about a parameter ϕ that can be extracted from a given measurement procedure.

Every natural object is in contact with its environment, so its dynamics is that of an open system; thus the interaction between a composite quantum system and its environment and understanding the dynamics for different physical quantities have attracted more interest. This interaction results in the system experiencing “quantum noise,” which shows up in the system exhibiting fluctuations, decoherence, and possibly irreversible dissipative dynamics. Recently great attention has been paid to the development of a more general understanding of the dynamics of open quantum systems, in order to deal with the occurrence of memory effects [9–15]. In particular, different definitions of quantum non-Markovianity have been theoretically introduced [16,17] and, in some cases, experimentally investigated [18,19]. A particularly significant step forward in this context has been performed with the formulation of new theoretical tools able to characterize and quantify the deviations of related physical quantities of given dynamics from Markovianity [20,21]. However, a Markovian description for an open quantum systems is only an approximation to most realistic processes, which are non-Markovian. The non-Markovian effect, as an appealing feature, has led to the concept of non-Markovian quantum channels in quantum information processing [22,23]. Nevertheless, the physical reasons ruling whether an open quantum system exhibits non-Markovianity dynamics have yet to be fully clarified.

More recently, the dynamics of QFI under decoherence from a geometrical point of view is investigated. It has been shown that the collisional dephasing significantly diminishes the precision of the phase parameter with the Ramsey interferometry [24]. The QFI of the Greenberger-Horne-Zeilinger state with respect to SU(2) rotation under decoherence is studied. The authors observed the decay and sudden change of the QFI during the evolution [25]. The problem of the parameter estimation in a spin-j system surrounded by an environment which is modeled by a quantum Ising chain is investigated. It has been shown that the QFI decays with time almost monotonously when the environment reaches the critical point [26,27]. In this paper we continue the investigation of physical systems and physical effects that may lead to good precision of estimation. The main aim of this work is to examine the problem of parameter estimation in a qubit system under different non-Markovian conditions. In particular, we highlight the connection between the memory effects and open system information. We show that revivals and retardation of QFI loss may occur by adjusting the cavity-qubit detuning, in the first case, while partial QFI trapping occurs in nonideal photonic band gaps.

We first present a brief review of the QFI. A crucial goal of quantum estimation is to archive the best observable. For example, in order to estimate the true value of parameter θ provided that the system is in one state of the family $\{\rho_\theta\}$, an observable $\hat{\theta}$ is called to be the unbiased estimator, that is, the expectation of the estimator should satisfy $\mathrm{Tr}(\rho_\theta\hat{\theta}) = \theta$ and in

*Laboratoire de Physique Théorique, Faculté des Sciences, Université Mohammed V-Agdal, Avenue Ibn Battouta, Boîte Postale 1014, Agdal Rabat, Morocco.

general, the estimator $\hat{\theta}$ is not unique. We can quantify how accurately a state can measure an unknown parameter with the QFI associated with the quantum Cramér-Rao (QCR). QFI is defined as

$$F_Q = \mathrm{Tr}[\rho(\theta)L^2], \tag{1}$$

where $\rho(\theta)$ is the density matrix of the system, θ is the parameter to be measured, and L is the symmetric logarithmic derivation given by

$$\frac{\partial\rho(\theta)}{\partial\theta} = \frac{1}{2}[L\rho(\theta) + \rho(\theta)L]. \tag{2}$$

The QCR inequality has been formulated in which the bound is asymptotically archived by the maximum likelihood estimator as well as the classical theory,

$$\Delta\theta \geqslant (\Delta\theta_{\mathrm{QCR}}) = \frac{1}{\sqrt{\nu F_Q}}, \tag{3}$$

where $(\Delta\theta)^2$ is the mean square error in the parameter θ and ν is the number of repeated independent trials. The above inequality defines the principally smallest possible uncertainty in estimation of the value of phase.

We choose to compare the precision of the parameter estimation for different Markovian and non-Markovian dynamics using this widely accepted approach of QFI. The interferometric setup generally consists of four steps. The first is the preparation step, where the input state is chosen as a qubit optimal state $|\psi_{\mathrm{opt}}\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$ (here the qubit is assumed to be a two-level atom system), which maximizes the QFI of the output state and estimates the unknown parameter ϕ as precisely as possible [26]. Then a singlet-qubit phase gate is applied, given by

$$U(\phi) := |g\rangle\langle g| + e^{i\phi}|e\rangle\langle e|, \tag{4}$$

where $\theta = \phi$ in some inference parameter (see Fig. 1). The outcome state is called $|\psi_{\mathrm{out}}\rangle = U(\phi)|\psi_{\mathrm{opt}}\rangle$. After the phase gate operation and before the measurement is performed, the qubit is subjected to a reservoir. After the decoherence, the output mixed state $\rho_{\mathrm{out}}(\phi)$ is finally measured for the estimation of phase uncertainty.

For a pure state, QFI is given by $F_Q = 4[\langle\psi'|\psi'\rangle - |\langle\psi'|\psi_{\mathrm{out}}\rangle|^2]$ for $|\psi'\rangle = \partial|\psi_{\mathrm{out}}\rangle/\partial\theta$. If the outcome state is a mixed state, the QFI is given by

$$F_Q = \sum_{i,j}\frac{2}{\lambda_i + \lambda_j}|\langle\lambda_i|[\partial\rho_{\mathrm{out}}(\theta)/\partial\theta]|\lambda_j\rangle|^2, \tag{5}$$

where λ_i ($|\lambda_i\rangle$) are the eigenvalues (eigenvectors) of ρ_{out}.

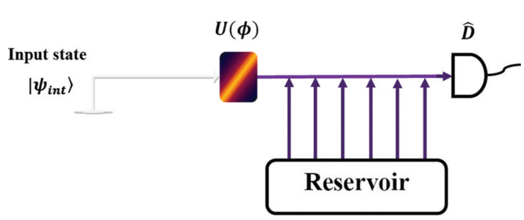

FIG. 1. (Color online) Interferometric phase estimation scheme in a unitary operation. After applying the gate operation, the system interacts with the reservoir. The precision of the estimation is impacted by characteristics of both the reservoir and the interaction.

We consider a qubit system S interacting with a reservoir R_S. The part "qubit S + reservoir R_S" is described by the Hamiltonian

$$H = \hbar\omega_0\hat{\sigma}_+\hat{\sigma}_- + \sum_k \hbar[\omega_k\hat{b}_k^\dagger\hat{b}_k + (g_k\hat{\sigma}_+\hat{b}_k + g_k^*\hat{\sigma}_-\hat{b}_k^\dagger)], \tag{6}$$

where ω_0 denotes the transition frequency of the two-level system (i.e., the qubit), with $\hat{\sigma}_\pm$ being the corresponding atomic raising and lowering operators. The index k labels different field modes of the reservoir frequency ω_k. $b_k^\dagger$ (b_k) is the creation (annihilation) operator of the reservoir field, with g_k being the coupling constant to the qubit. The Hamiltonian (6) may describe a large class of systems as, for example, a qubit formed by an exciton in a potential well environment. We assume that the qubit is initially in general a superposition with zero-temperature reservoir, $|\psi(0)\rangle = (C_0(0)|0\rangle_S + C_1(0)|1\rangle_S) \otimes |0\rangle_{R_S}$. The reduced density matrix of the qubit S under non-Markovian dynamics at any time can be obtained exactly by [23]

$$\rho_S(t) = \begin{pmatrix} \rho_{11}(0)|q(t)|^2 & \rho_{10}(0)q(t) \\ \rho_{01}(0)q^*(t) & \rho_{00}(0) + \rho_{11}(0)(1 - |q(t)|^2) \end{pmatrix}, \tag{7}$$

where $\rho_{ij} = C_i(0)C_j^*(0)$. The function $g(t)$ obeys the differential equation

$$\dot{q}(t) = \int_0^t d\tau f(t-\tau)q(\tau), \tag{8}$$

and the correlation function $f(t-\tau)$ is related to the spectral density $J(\omega)$ of the reservoir by

$$f(t-\tau) = \int d\omega J(\omega)\exp[i(\omega_0 - \omega)(t-\tau)]. \tag{9}$$

From the above equation, the exact form of $g(t)$ depends on the particular choice of the spectral density of the reservoir. The solution of the associated algebraic equation for $\dot{q}(t)$ can be obtained through Laplace transforms,

$$L[q(s)] = \frac{q(0)}{s + L[f(s)])}, \tag{10}$$

where $q(s)$ and $f(s)$ are the Laplace transforms of $q(t)$ and $f(t-\tau)$.

We shall now analyze the evolution of QFI for two different spectral densities: a single Lorentzian simulating a cavity with a mode nonresonant with the qubit transition frequency and a nonperfect photonic band gap at the qubit transition frequency. Let us begin with the first example by taking the spectral density $J(\omega)$ of the electromagnetic field inside a high-Q cavity supported by the detuning Δ of ω_0 and the center frequency of the cavity, resulting from the combination of the environment spectrum and the system-environment coupling. For this case, the Lorentzian spectral distribution is given by

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0 - \Delta - \omega)^2 + \lambda^2}, \tag{11}$$

where the parameter λ, defining the spectral width of the coupling, is connected to the reservoir correlation time τ_B by the relation $\tau_B \approx \lambda^{-1}$. The relaxation time scale τ_R over which the state of the system changes is then related to γ_0 by $\tau_R \approx \gamma_0^{-1}$. The Markovian and the non-Markovian regimes are distinguished by the relation of the parameters γ_0 and λ. In the Markovian regime there is $\gamma_0 < \lambda/2$ or $\tau_R > 2\tau_B$, and the non-Markovian regime corresponds to $\gamma_0 > \lambda/2$ or $\tau_R < 2\tau_B$. The reservoir correlation time is greater than the relaxation time and non-Markovian effects become relevant. For this reason, we are interested in this regime and we shall mainly limit our considerations to this case. Substituting $J(\omega)$ into Eq. (9), we get

$$f(t-\tau) = \frac{\gamma_0\lambda}{2}\exp[-(\lambda - i\Delta)(t-\tau)]. \tag{12}$$

Using this correlation function, the Laplace transform of $q(t)$ is

$$L[q(s)] = \frac{1}{\left(s + \frac{1}{2}\frac{\gamma_0\lambda}{s-(\lambda-i\Delta)}\right)}, \tag{13}$$

and the inverse Laplace transform finally gives

$$q(t) = e^{-\frac{1}{2}(\lambda - i\Delta)t}\left[\cosh\left(\frac{\Omega t}{2}\right) + \frac{\lambda - i\Delta}{\Omega}\sinh\left(\frac{\Omega t}{2}\right)\right], \tag{14}$$

where $\Omega = \sqrt{(\lambda - i\Delta)^2 - 2\gamma_0\lambda}$.

In Fig. 2, the dynamics of QFI for the initial optimal state can be compared with respect to Markovian and non-Markovian regimes for a different order of the spectral width of the coupling λ and detuning Δ. In Fig. 2(a), we plot the QFI variation as a function of the dimensionless quantity $\gamma_0 t$ for various values of Δ. In the non-Markovian regime ($\lambda = 0.05\gamma_0$), the dashed black line is for $\Delta = 0$, dash-dotted red line is for $\Delta/\gamma_0 = 0.2$, and the solid green line is for $\Delta/\gamma_0 = 0.8$. In the Markovian regime ($\lambda = 5\gamma_0$), the dotted blue line is for $\Delta = 0$. We start with the Markovian regime. In the resonant limit, $\Delta = 0$, one can see that the QFI exhibits a sudden drop to the minimum value. However, the increasing detuning number may retard the QFI loss during the time evolution. For a more detailed example, taking $\Delta = 0$, $\Delta = 0.8\gamma_0$, and $\Delta = 4\gamma_0$, the values of the QFI are equal to $Q_F = 0.014$, $Q_F = 0.020$, and $Q_F = 0.173$, respectively. The drop phenomenon of the QFI with time reflects that the parameter estimation of the open system becomes more inaccurate, because the weak coupling regime (i.e., the relation time is greater than the reservoir correlation time and $q(t)$ is essentially a Markovian exponential decay) destroys the quantum coherence and, consequently, the estimation based on the quantum coherence will be inaccurate. When the cavity bandwidth λ is smaller than the free-space atomic linewidth (non-Markovian regime for $\lambda = 0.05\gamma_0$), the resonant case shows that the QFI oscillates with time as the periodic function is suppressed to the zero value and raises the gain, exhibiting a sudden drop and revival of the information. Within this regime, $q(t)$ presents oscillations describing a quasicoherent exchange of energy between the qubit and the environment. The oscillation of QFI with time implies that the precision of estimation may rise again during some time period. In a sense, this phenomenon can be regarded as evidence of the enhancement of coherence in the open quantum system, which may be understood as the reversed flow of information from the environment back to the system. This indicates that in the weak-coupling-regime case, the environment remarkably suppresses the reversed flow of information. When the parameter Δ becomes larger than zero as shown in Fig. 2(a) for the cases $\Delta = 0.2$ and $\Delta = 0.8$, the QFI increases for each value of time and we find that revivals and retardation of QFI loss may occur by adjusting the cavity-qubit detuning. The phenomenon of the QFI revivals during that time is a clear manifestation of the environment memory effects. From this result, the enhancement of the QFI may occur by adjusting the cavity-qubit detuning. According to quantum estimation theory, the increasing of QFI means the optimal precision of estimation is increased. To get an intuitive understanding of the effects of detuning and non-Markovian characteristics on the QFI, we plot in Fig. 2(b) the QFI variation as a function of the dimensionless quantity Δ/γ_0 for various values of λ with $\gamma_0 t = 10$. The dotted blue line is for $\lambda = 3\gamma_0$, the dashed black line is for $\lambda = 0.1\gamma_0$, the dash-dotted red line is for $\lambda = 0.05\gamma_0$, and the solid green line is for $\lambda = 0.02\gamma_0$. We find that the QFI oscillates with detuning, exhibiting a periodic function of Δ/γ_0. Besides, we can observe that the amplitude of the QFI increases with decreasing λ/γ_0. It is clear that the spectral width of the environment significantly destroys the precision of the parameter estimation.

As a second example, we consider a non-Markovian environment with the spectral density

$$J(\omega) = \frac{1}{2\pi}\left[\frac{\gamma_1\lambda_1^2}{(\omega-\omega_0)^2+\lambda_1^2} + \frac{\gamma_2\lambda_2^2}{(\omega-\omega_0)^2+\lambda_2^2}\right], \tag{15}$$

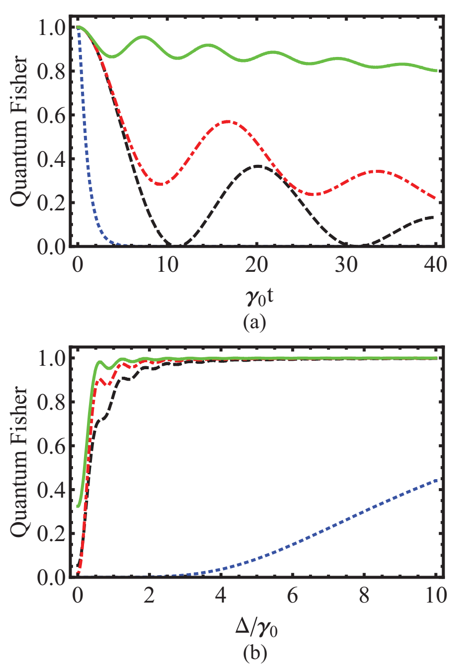

FIG. 2. (Color online) QFI as a function of the dimensionless quantities $\gamma_0 t$ and Δ/γ_0 for the initial state $|\psi\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$. (a) QFI in terms of $\gamma_0 t$ for different values of detuning. In the non-Markovian regime ($\lambda = 0.05\gamma_0$), $\Delta = 0$ (dashed line), $\Delta = 0.2\gamma_0$ (dash-dotted line), and $\Delta = 0.8\gamma_0$ (solid line). In the Markovian regime ($\lambda = 5\gamma_0$) and $\Delta = 0$ (dotted line). (b) QFI in terms of Δ/γ_0 for the different values of λ with $\gamma_0 t = 10$. The phenomenon of the slowing down of the QFI decay and of the QFI revivals are clear manifestation of the environment memory effects.

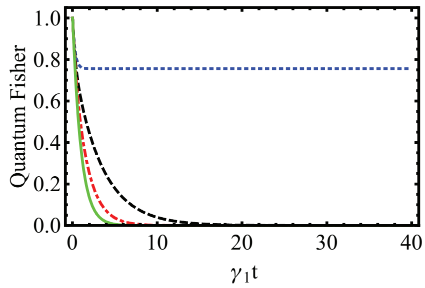

FIG. 3. (Color online) QFI as a function of the dimensionless quantities $\gamma_1 t$ starting from the initial optimal state $|\psi\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$ for the nonperfect PBG case, $\lambda_1 = 10\lambda_2 = 30\gamma_1$ for different values of γ_2: $\gamma_2 = \gamma_1$ (dotted line), $\gamma_2 = 2/3\gamma_1$ (dashed line), $\gamma_2 = 1/3\gamma_1$ (dash-dotted line), and $\gamma_2 = 0$ (solid line). QFI trapping is used and it decays with time almost monotonously when the width of the gap is increased.

which represents a Lorentzian with a dip used as a model to simulate the spontaneous decay of a qubit in a nonperfect photonic band gap (PBG). λ_1 defines the bandwidth of the flat background continuum, and λ_2 represents the width of the gap. γ_1 and γ_2 are the strength of the background and the gap, respectively. The positivity condition for the spectral density defined in Eq. (15) requires $\gamma_1\lambda_1^2 > \gamma_2\lambda_2^2$ at large values of ω and $\gamma_1 > \gamma_2$ at the center of resonance [23,28], combining these two inequalities in condition [28]. For the special case $\gamma_1 = \gamma_2$, the spectral density reduces exactly to zero at the center of the gap ($\omega = \omega_0$), leading to population trapping. In this considered case, the correlation function is given by

$$f(t-\tau) = \tfrac{1}{2}(\gamma_1\lambda_1 e^{-\lambda_1(t-\tau)} - \gamma_2\lambda_2 e^{-\lambda_2(t-\tau)}). \tag{16}$$

Using this function, the Laplace transform of $q(t)$ is

$$L[q(s)] = \frac{(\lambda_1 + s)(\lambda_2 + s)}{s^3 + s^2(\lambda_1+\lambda_2) + s(\lambda_1\lambda_2 + \Theta) + \lambda_1\lambda_2\Gamma}, \tag{17}$$

where $\Theta = (\gamma_1\lambda_1 - \gamma_2\lambda_2)/2$ and $\Gamma = (\gamma_1 - \gamma_2)/2$. The inverse Laplace transform finally gives

$$q(t) = \sum_i \frac{\mu_i^2 + \mu_i(\lambda_1+\lambda_2) + \lambda_1\lambda_2}{(\mu_i - \mu_j)(\mu_i - \mu_k)} e^{\mu_i t}, \tag{18}$$

where $i,j,k = 1,2,3$ are all different indexes, and μ_i are the three solutions of the third degree equation appearing in the denominator of Eq. (17).

We now investigate the QFI dynamics of the qubit in a nonperfect PBG as described by the spectral density of Eq. (15). In Fig. 3 we plot the QFI as a function of dimensionless quantity $\gamma_1 t$ for various values of γ_2. The blue dotted line is for $\gamma_2 = \gamma_1$, the black dashed line is for $\gamma_2 = 2/3\gamma_1$, the red dash-dotted line is for $\gamma_2 = 1/3\gamma_1$, and the green solid line is for $\gamma_2 = 0$. We find that the behavior of QFI versus time is affected by the strength γ_2, especially at the critical value $\gamma_2 = \gamma_1$. The spectral density goes to zero at the center of the gap and as a consequence, we obtain QFI trapping exhibiting asymptotic behavior with the value $F_Q = 0.746$. The critical reason is that the reservoir cannot take the information during the interaction with the open system in this limit. For other values of γ_2, QFI decays with time almost monotonously when the width of the gap is augmented. Interestingly, the plot evidences how by decreasing γ_2 the trapping of QFI is lost and the QFI decay always speeds up. In particular, for $\gamma_2 = 0$ the shape of the QFI is found to be similar to the simple Lorentzian case. From this result, the decreasing parameter γ_2 leads to the decay of QFI and destroys the coherence, thus reducing the optimal precision of estimation.

In conclusion, by using QFI we have investigated the problem of parameter estimation in a qubit system considering an exactly solvable model where the qubit is surrounded by a bosonic environment at zero temperature. We have examined two different spectral densities corresponding to two different environments: the first case considered is a Lorentzian spectrum representing a high-Q cavity out of resonance with the qubit transition frequency and the second, a nonperfect photonic band gap. We observed that revivals and retardation of QFI loss may occur by adjusting the cavity-qubit detuning, in the first case. On the other hand, the second spectral density has permitted the study of QFI dynamics when ideal conditions of the photonic band gap are not satisfied. In this case, the QFI trapping and the decreasing gap width seriously destroy the coherence and thus reduce the precision of estimation. An important future investigation will be the study of the effects of finite-temperature environments on the dynamics of QFI. In comparison with some recent work on the non-Markovian effect on the dissipation of the system in a microscopic way, our present work from a phenomenological viewpoint might be more practical to explain some experimental observations of the dissipation on the precision of the estimator parameter subject to a realistic environment, providing more hints for future investigation of this topic.

We acknowledge enlightening discussions with Fabio Benatti, Felipe Fanchini, and Adil Benmoussa.

[1] V. Giovannetti, S. Lloyd, and L. Maccone, Science 306, 1330 (2004).

[2] J. Dowling, Contemp. Phys. 49, 125 (2008).

[3] J. A. Jones, S. D. Karlen, J. Fitzsimons, A. Ardavan, S. C. Benjamin, G. A. D. Briggs, and J. J. L. Morton, Science 324, 1166 (2009).

[4] S. Simmons, J. A. Jones, S. D. Karlen, A. Ardavan, and J. J. L. Morton, Phys. Rev. A 82, 022330 (2010).

[5] B. L. Higgins, D. W. Berry, S. D. Bartlett, H. M. Wiseman, and G. J. Pryde, Nature (London) 450, 393 (2007).

[6] R. Demkowicz-Dobrzanski, U. Dorner, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, and I. A. Walmsley, Phys. Rev. A 80, 013825 (2009).

[7] R. A. Fisher, Proc. Cambridge Phil. Soc. 22, 700 (1929); reprinted in *Collected Papers of R. A. Fisher*, edited by

J. H. Bennett (Univ. of Adelaide Press, South Australia, 1972), pp. 15–40.

[8] H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, NJ, 1946).

[9] P. Haikka, J. Goold, S. McEndoo, F. Plastina, and S. Maniscalco, Phys. Rev. A **85**, 060101(R) (2012).

[10] E.-M. Laine, H.-P. Breuer, J. Piilo, C.-F. Li, and G.-C. Guo, Phys. Rev. Lett. **108**, 210402 (2012).

[11] L. Mazzola, C. A. Rodríguez-Rosario, K. Modi, and M. Paternostro, Phys. Rev. A **86**, 010102(R) (2012).

[12] C. A. Rodríguez-Rosario, K. Modi, L. Mazzola, and A. Aspuru-Guzik, Europhys. Lett. **99**, 20010 (2012).

[13] W. M. Zhang, P. Y. Lo, H. N. Xiong, T. MatisseWei-Yuan, and F. Nori, Phys. Rev. Lett. **109**, 170402 (2012).

[14] T. J. G. Apollaro, C. Di Franco, F. Plastina, and M. Paternostro, Phys. Rev. A **83**, 032103 (2011).

[15] M. Žnidarič, C. Pineda, and I. García-Mata, Phys. Rev. Lett. **107**, 080404 (2011).

[16] H.-P. Breuer, E.-M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).

[17] E.-M. Laine, J. Piilo, and H.-P. Breuer, Phys. Rev. A **81**, 062115 (2010).

[18] B. H. Liu, L. Li, Y.-F. Huang, C.-F. Li, G.-C. Guo, E.-M. Laine, H.-P. Breuer, and J. Piilo, Nat. Phys. **7**, 931 (2011).

[19] A. Smirne, L. Mazzola, M. Paternostro, and B. Vacchini, Phys. Rev. A **87**, 052129 (2013).

[20] X.-M. Lu, X. Wang, and C. P. Sun, Phys. Rev. A **82**, 042103 (2010).

[21] A. K. Rajagopal, A. R. Usha Devi, and R. W. Rendell, Phys. Rev. A **82**, 042107 (2010).

[22] D. Aharonov, A. Kitaev, and J. Preskill, Phys. Rev. Lett. **96**, 050504 (2006).

[23] B. Bellomo, R. LoFranco, and G. Compagno, Phys. Rev. Lett. **99**, 160502 (2007).

[24] W. Zhong, Z. Sun, J. Ma, X. Wang, and F. Nori, Phys. Rev. A **87**, 022337 (2013).

[25] J. Ma, Y.-X. Huang, X. Wang, and C. P. Sun, Phys. Rev. A **84**, 022302 (2011).

[26] Z. Sun, J. Ma, X.-M. Lu, and X. Wang, Phys. Rev. A **82**, 022306 (2010).

[27] X. Heng-Na and W. Xiaoguang, Physica A **390**, 4719 (2011).

[28] B. M. Garraway, Phys. Rev. A **55**, 4636 (1997).