PHYSICAL REVIEW A **88**, 033817 (2013)

Quantum-state transfer between tripod atoms over a dark fiber

G. Bevilacqua[1] and F. Renzoni[2]

[1]*Department of Information Engineering and Mathematical Science, University of Siena, Via Roma 56, 53100 Siena, Italy*

[2]*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*

(Received 21 May 2013; published 9 September 2013)

In this work we introduce a model for quantum-state transfer between tripod atoms over a dark fiber. Two tripod atoms are confined in separate cavities linked by an optical fiber. The cavities and the fiber sustain two optical modes of opposite circular polarization. For each atom, the two ground states encode the quantum state to be transferred and are coupled to a common excited state by the cavity modes of opposite polarization. The remaining transition for each atom is used to control the transfer process. We demonstrate that by using laser pulses the dynamics of the system can be confined within a degenerate dark state subspace, with the different dark states interacting via nonadiabatic couplings. We solve analytically the dynamics in the dark state subspace, and determine the conditions on the pulse shape for the implementation of the quantum transfer. We identify a possible pulse shape which satisfies the required conditions, and demonstrate the quantum-state transfer via numerical simulations.

DOI: 10.1103/PhysRevA.88.033817 PACS number(s): 42.50.Gy, 42.50.Ex, 03.67.Hk, 42.81.Qb

I. INTRODUCTION

Quantum interference in atomic systems driven by laser fields gives rise to a wealth of new quantum-optical phenomena, and to new mechanisms for the control of quantum systems. In a three-level atomic system interacting with two laser fields in the Λ configuration, destructive quantum interference results in a coherent superposition of ground states—the so called dark state—not interacting with the laser radiation [1–3]. For time-dependent laser fields, the dark state is time dependent. By properly tailoring the laser pulses, the transfer of the system from one ground state to the other ground state can be obtained by adiabatic following of the dark state, a process named stimulated Raman adiabatic passage (STIRAP [4]).

STIRAP was extended to a variety of multilevel systems [5,6], and found important applications in quantum computation [7–11] and quantum communication [12,13]. In Ref. [13], a scheme for quantum-state transfer via STIRAP between two atoms placed in distinct cavities linked via a fiber was introduced. An important feature of the scheme is that the fiber is not populated at any time during the transfer process, as a result of quantum interference. This suppresses the important channel of decoherence represented by photon losses in the fiber.

In this work we introduce a model for quantum-state transfer between tripod atoms over a dark fiber. With respect to the work of Ref. [13], our scheme involves a simpler level atomic scheme and a smaller number of applied laser fields. In our scheme, two tripod atoms are confined in distinct cavities linked by an optical fiber. The cavities and the fiber sustain two optical modes of opposite circular polarization. For each atom, the two ground states encode the quantum state to be transferred and are coupled to a common excited state by the cavity modes of opposite polarization. The remaining transition for each atom is used to control the transfer process. We demonstrate that by using laser pulses the dynamics of the system can be confined within a degenerate dark state subspace, with the different dark states interacting via nonadiabatic couplings. We solve analytically the dynamics in the dark state subspace, and determine the conditions on the pulse shape for the implementation of the quantum transfer. We identify a possible pulse shape which satisfies the required conditions, and demonstrate the quantum-state transfer via numerical simulations.

This paper is organized as follows. In Sec. II we introduce the system under consideration, and state the aim of the process to be implemented. In Sec. III we define the Hamiltonian of the system, and identify the relevant dark state subspace. By solving the dynamics in the dark state subspace, we determine the condition on the laser pulses for quantum-state transfer. In Sec. IV we validate the choice of the identified pulse sequence by numerical simulations of the system evolution. We also study numerically the dependence of the transfer fidelity on the cavity-fiber coupling and on other experimentally relevant parameters. Finally, in Sec. V conclusions are drawn.

II. STATEMENT OF THE PROBLEM

We consider two optical cavities, containing one atom each and linked via an optical fiber. The atomic level scheme and the cavities' configuration are shown in Fig. 1. The atoms are tripod atoms, with three ground states $|-\rangle$, $|+\rangle$, and $|a\rangle$ coupled to a common excited state $|e\rangle$. Each cavity can sustain two modes with opposite circular polarization. For each atom, the two ground states $|\pm\rangle$ are coupled to the excited state via the cavity fields with opposite polarizations, with the same coupling constant g for the two polarizations. The transition between the ground state $|a\rangle$ and the excited state is driven via an applied laser field, with Rabi frequencies Ω_1 and Ω_2 for the first and second atoms, respectively.

We assume that each laser field, applied perpendicularly to the cavity axis, does not directly pump the cavity or the fiber. The proposed scheme can be realized with alkali-metal atoms. For example, the $F_g = 1 \rightarrow F_e = 0$ transition of the ^{87}Rb D_1 line can be used for this purpose, with the qubit states $|\pm\rangle$ encoded in the $|F_g = 1, m_F = \pm 1\rangle$ sublevels and the auxiliary state $|a\rangle$ encoded in $|F_g = 1, m_F = 0\rangle$.

We choose as basis of the system the states $|t_1 n_L n_R\rangle|t_2 m_L m_R\rangle|p_L p_R\rangle$, where the first (second) ket refers

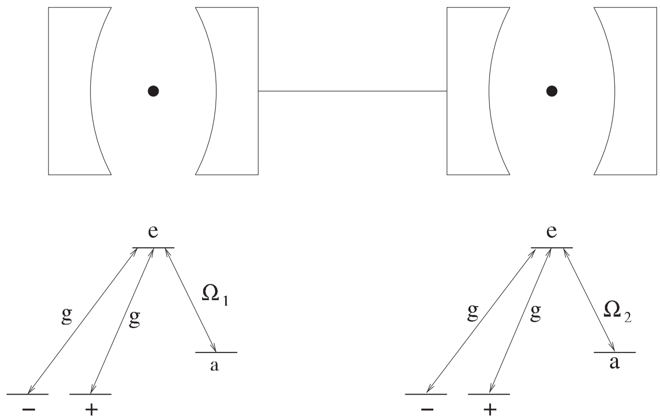

FIG. 1. Optical cavity setup, and atom-light interaction scheme for the proposed implementation of quantum-state transfer.

to the first (second) atom + cavity system, and the third ket to the fiber. The atom + cavity system is defined by the internal states t_1 and t_2 of the first and second atoms ($|-\rangle$, $|+\rangle$, and $|a\rangle$) and the number of photons in the cavity modes with left (n_L, m_L) and right (n_R, m_R) polarization. The state of the fiber is defined by the number of photons with (p_L) and right (p_R) polarization.

Our aim is to identify a pulse sequence for the two applied laser fields so that the superposition

$$|\psi_{\rm in}\rangle = (\alpha|-00\rangle + \beta|+00\rangle)|a00\rangle|00\rangle \tag{1}$$

will be mapped onto

$$|\psi_{\rm fin}\rangle = |a00\rangle(\alpha|-00\rangle + \beta|+00\rangle)|00\rangle. \tag{2}$$

This corresponds to quantum-state transfer over the fiber. Additionally, we require the system to evolve within a dark state subspace, so that the atomic excited states are not occupied at any time during the evolution, and the fiber is not occupied by any photon.

III. THEORETICAL ANALYSIS

A. Hamiltonian and dark state subspace

The Hamiltonian for the system is

$$\begin{aligned} H = {} & \Omega_1|e\rangle_{11}\langle a| + g|e\rangle_{11}\langle -|a_- + g|e\rangle_{11}\langle +|a_+ + \text{H.c.} \\ & + \Omega_2|e\rangle_{22}\langle a| + g|e\rangle_{22}\langle -|b_- + g|e\rangle_{22}\langle +|b_+ + \text{H.c.} \\ & + \nu[c_+(a_+^+ + b_+^+) + c_-(a_-^+ + b_-^+) + c_+^+(a_+ + b_+) \\ & + c_-^+(a_- + b_-)]. \end{aligned} \tag{3}$$

Here $a_\pm$ ($b_\pm$) is the annihilation operator for a photon of $\sigma_\pm$ polarization of the first (second) cavity mode. The operator $c_\pm$ describes the annihilation of a fiber photon of $\sigma_\pm$ polarization and ν is the cavity-fiber coupling strength, assumed to be the same for the two modes. The above Hamiltonian is valid in the short-fiber limit $2l\Gamma/(2\pi c) \leqslant 1$, where l is the length of the fiber, Γ the decay rate of the cavity into the fiber, and c the speed of light.

We consider the low-excitation regime, with one photon at most in the cavity-fiber system when the atoms are in the ground states. By repeatedly applying the Hamiltonian to the initial state $|a00\rangle|-00\rangle|00\rangle$, it is easy to show that the relevant Hilbert space has dimension 32 and it spanned by the states: $|a00\rangle|\pm00\rangle|00\rangle$, $|\pm00\rangle|\pm01\rangle|00\rangle$, $|\pm00\rangle|\pm10\rangle|00\rangle$, $|e00\rangle|\pm00\rangle|00\rangle$, $|\pm00\rangle|\pm00\rangle|10\rangle$, $|\pm00\rangle|\pm00\rangle|01\rangle$, and the permutations between the first and second atoms. The system has a dark space of dimensionality height. The explicit form of our choice for the basis vectors $|\psi_D^{(j)}\rangle$ ($j = 1$–8) spanning the dark subspace is reported in the Appendix.

B. Dynamics in the dark state subspace

We now consider a pulse sequence, with the two pulses Ω_1 and Ω_2 delayed but overlapping. The dark states relevant for the quantum-state transfer process are those which show a dependence on the Rabi frequencies, i.e. $|\psi_D^{(j)}\rangle$ with $j = 5$–8. For $t \to \pm\infty$ the condition $\Omega_{1,2} \to 0$ implies that asymptotically these states have the form

$$\left|\psi_D^{(5)}(t \to -\infty)\right\rangle = |-00\rangle|a00\rangle|00\rangle, \tag{4}$$

$$\left|\psi_D^{(6)}(t \to +\infty)\right\rangle = |a00\rangle|-00\rangle|00\rangle, \tag{5}$$

$$\left|\psi_D^{(7)}(t \to -\infty)\right\rangle = |+00\rangle|a00\rangle|00\rangle, \tag{6}$$

$$\left|\psi_D^{(8)}(t \to +\infty)\right\rangle = |a00\rangle|+00\rangle|00\rangle. \tag{7}$$

Thus the transformation

$$\begin{aligned} \left|\psi_D^{(5)}(t \to -\infty)\right\rangle &= |-00\rangle|a00\rangle|00\rangle \to \left|\psi_D^{(6)}(t \to +\infty)\right\rangle \\ &= |a00\rangle|-00\rangle|00\rangle \end{aligned} \tag{8}$$

$$\begin{aligned} \left|\psi_D^{(7)}(t \to -\infty)\right\rangle &= |+00\rangle|a00\rangle|00\rangle \to \left|\psi_D^{(8)}(t \to +\infty)\right\rangle \\ &= |a00\rangle|+00\rangle|00\rangle \end{aligned} \tag{9}$$

is required to implement the wanted quantum-state transfer.

Further analysis requires solving the dynamics of the system. The Schrödinger equation in the rotating wave approximation is

$$\frac{d}{dt}|\psi\rangle = -iH(t)|\psi\rangle. \tag{10}$$

We now consider the time dependent basis $\{\psi_D^{(1)}, \ldots, \psi_D^{(8)}, \psi^{(9)}, \ldots, \psi^{(32)}\}$ constituted by the height orthonormal dark states, completed by the 24 orthonormal states, as obtained by the standard Gram-Schmidt orthonormalization procedure. In this basis, the Schrödinger equation becomes

$$\frac{d}{dt}|\tilde{\psi}\rangle = -i\tilde{H}(t)|\tilde{\psi}\rangle, \tag{11}$$

where

$$|\tilde{\psi}\rangle = U^{-1}|\psi\rangle \tag{12}$$

with U the operator whose column are the new basis vectors

$$U = \left[\psi_D^{(1)}, \ldots, \psi_D^{(8)}, \psi^{(9)}, \ldots, \psi^{(32)}\right]. \tag{13}$$

The transformed Hamiltonian $\tilde{H}$ is

$$\tilde{H} = U^{-1} H U + i \dot{U}^{-1} U. \tag{14}$$

We assume the adiabatic limit—an assumption which will be justified *a posteriori* —and we restrict our analysis to the dark state subspace [5,6]. The Hamiltonian restricted to such a subspace consists of the nonadiabatic couplings between dark states and has the form

$$\tilde{H}_{NA} = i\chi \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}, \tag{15}$$

where

$$\chi = \frac{2\Omega_1 \dot{\Omega}_2 \sqrt{3} g^2}{\sqrt{3g^4 + 4\Omega_2^2 g^2 + 4\Omega_1^2 g^2 + 4\Omega_1^2 \Omega_2^2} \left(3g^2 + 4\Omega_2^2\right)}. \tag{16}$$

The nonadiabatic dynamics in the dark state subspace can be studied with the help of the time-evolution operator

$$U_A = \exp\left(-i \int_{-\infty}^{t} \tilde{H}_{NA}(t') dt'\right), \tag{17}$$

which after substituting the expression (15) for $\tilde{H}_{NA}$ reads

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(A) & \sin(A) & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin(A) & \cos(A) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos(A) & \sin(A) \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sin(A) & \cos(A) \end{pmatrix}, \tag{18}$$

where

$$A = \int_{-\infty}^{t} \chi(t') dt'. \tag{19}$$

From the expression for the time-evolution operator it follows that, in order to implement the transformations (8) and (9) required for the quantum-state transfer over the dark fiber, the condition

$$A = -\frac{\pi}{2} \tag{20}$$

has to be satisfied.

To evaluate the integral (19), first notice that the main contributions come from the regions where $\Omega_2(t)$ has a steep variation; then, if in one of those regions, say $t_a < t < t_b$, $\Omega_1(t)$ is nearly constant, the partial contribution to A is

$$A(t_a,t_b) = \int_{t_a}^{t_b} \chi(t') dt' = \operatorname{sgn}(\Omega_1) \arctan\left[\frac{2\sqrt{3}\, \Omega_2(t)\, |\Omega_1|}{\sqrt{3\{3g^4 + 4g^2[\Omega_1^2 + \Omega_2(t)^2] + 4\Omega_1^2 \Omega_2(t)^2\}}}\right]\Bigg|_{t=t_a}^{t=t_b}, \tag{21}$$

which assumes the following simpler form in the limit $|\Omega_1|, |\Omega_2(t_a)|, |\Omega_2(t_b)| \gg g$:

$$A(t_a,t_b) \approx \operatorname{sgn}(\Omega_1)\Big[\operatorname{sgn}[\Omega_2(t_b)] - \operatorname{sgn}[\Omega_2(t_a)]\Big]\left[\frac{\pi}{6} + O\left(g^2/\Omega_1^2\right)\right]. \tag{22}$$

The total A is just a matter of counting the steep edges of the $\Omega_2(t)$ pulse. With this in mind a possible choice for pulses satisfying condition (20) is given by delayed, but overlapping, pulses with steep leading and trailing edges, as for example

$$\Omega_1(t) = \Omega_{1,0} f[(t - t_0 + \tau)/w] \tag{23a}$$

$$\Omega_2(t) = \Omega_{2,0} f[(t - t_0 - \tau)/w] \tag{23b}$$

with

$$f(t) = -\exp[-(t+2)^8] + \exp[-t^8]. \tag{24}$$

For such pulses the leading and trailing edges of $\Omega_2(t)$ provide the appropriate contribution to obtain $A \simeq (\pi/2)\operatorname{sgn}(\Omega_{1,0}\Omega_{2,0}\tau)$, for $\tau \neq 0$. Instead, for $\tau = 0$ the integration produces exactly $A = 0$.

IV. NUMERICAL ANALYSIS

We numerically solved the Schrödinger equation for a pulse sequence of the form defined by Eqs. (23) and (24). We assumed that the laser pulses are overlapping, and examined all possible situations: pulse Ω_1 preceding pulse Ω_2, pulse Ω_2 preceding pulse Ω_1, and no delay between pulses. For a given nonzero delay, the relative sign between Rabi frequencies is adjusted so that the condition for quantum-state transfer, Eq. (20), is satisfied. For an initial preparation of the system in the state $|-00\rangle|a00\rangle|00\rangle$ we verified, as shown in Fig. 2, that the dynamics remains confined in the dark state subspace and, for a nonzero delay between the pulses, the system evolves from the initial state to the wanted final state $|a00\rangle|-00\rangle|00\rangle$. Furthermore we verified that, due to quantum interference, the atomic excited states are not populated during the evolution

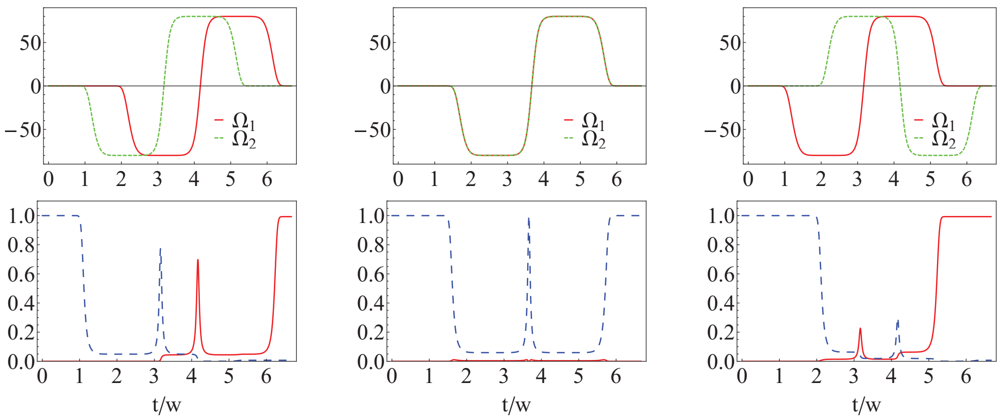

FIG. 2. (Color online) Result of numerical simulations for the process implementing quantum-state transfer over a dark fiber. For each column, the top panel represents the pulse sequence, and the lower panel the time evolution of the populations of the states most relevant to the transfer process: the dashed line corresponds to the population of the $|-00\rangle|a00\rangle|00\rangle$ state, while the solid line is the population of the $|a00\rangle|-00\rangle|00\rangle$ state. The parameter of the calculations are: left column: $\tau = -150$, $\nu = 20$, $\Omega_{1,0} = \Omega_{2,0} = 80$; center column: $\tau = 0$, $\Omega_{1,0} = \Omega_{2,0} = 80$; right column $\tau = 150$, $\Omega_{1,0} = -\Omega_{2,0} = 80$. Furthermore, $g = 21$, $w = 300$, and $\nu = 20$ for all simulation data sets.

of the system. The same applies to the fiber connecting the two cavities: quantum interference allows the quantum-state transfer over the fiber, without the fiber ever being populated by photons.

We verified numerically that the same pulse sequence produces an essentially perfect quantum-state transfer also for the symmetric case of initial preparation in the $|+00\rangle|a00\rangle|00\rangle$ state, as well as for arbitrary initial superposition $(\alpha|+00\rangle + \beta|-00\rangle)|a00\rangle|00\rangle$. This confirms that the used pulse sequence leads to quantum-state transfer over the dark fiber by implementing the correct evolution in the dark state subspace.

We note that the scheme works both for $\tau > 0$ and $\tau < 0$, provided that the signs of the Rabi frequencies are adjusted to satisfy condition (20). However for exactly superimposed pulses ($\tau = 0$, Fig. 2, center column) the dynamics is time reversible and the system evolves within the dark state subspace simply to return to the initial state.

We now address the issue of the assumption of the adiabaticity, and investigate the robustness of the scheme with respect to the variation of system parameters. We numerically calculated the eigenvalues of the Hamiltonian in the complete 32×32 Hilbert space. In Fig. 3 a typical time evolution of the eigenvalues is shown, for a pulse sequence of the form of Eq. (23). Throughout the time evolution there is a clear separation between the nonzero eigenvalues and the ones identically equal to zero, the latter ones corresponding to the dark subspace. We have verified that by decreasing ν or g the clear separation between the two classes of eigenvalues vanishes, and the condition of adiabaticity is broken. We can thus conclude that, provided that g and ν are large enough, the adiabatic approximation applies.

Provided that we are in the range of applicability of the adiabatic approximation, the proposed scheme is robust with respect to the variation of several system parameters. This is shown in Fig. 4(a) where the fidelity of quantum transfer

$$F = |\langle\psi_t|\psi(+\infty)\rangle|^2 \tag{25}$$

into the wanted target state $|\psi_t\rangle$ is plotted as a function of the fiber-cavity coupling strength ν for different values of the pulse duration w. The transfer process is robust over a wide range of variations in the cavity-fiber coupling constant and in the pulse duration. In particular, these data confirm the previous result that ν has to be large enough to guarantee adiabaticity. Beyond that value, the process is robust against any variation of ν.

We also studied the robustness of the transfer process with respect to variations of the delay time τ between the pulses. Our results, shown in Fig. 4(b), show that the process is indeed robust against variations in the delay time between pulses.

Finally, we investigated how the pulse shape affects the quantum-state transfer process. We considered a more general form of pulses replacing the function $f(t)$ as defined by Eq. (24) with

$$f(t) = \exp\{-[(t+2)^2]^\gamma\} + \exp\{-[t^2]^\gamma\}. \tag{26}$$

We notice that for $\gamma = 4$ we recover the original form of $f(t)$. For smaller values of γ the pulses show less steep leading and trailing edges, thus producing values of the parameter A, as defined by Eq. (19), further differing from the $A = -\pi/2$ value leading to an ideal transfer process.

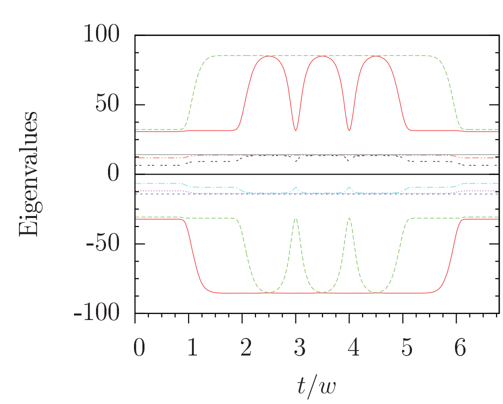

FIG. 3. (Color online) Dependence on time of the eigenvalues of the 32×32 Hamiltonian of the system. The relevant parameters are $w = 300$, $\tau = 150$, $g = 21$, $t_0 = 1400$, $\Omega_{1,0} = -\Omega_{2,0} = 80$, and $\nu = 10$.

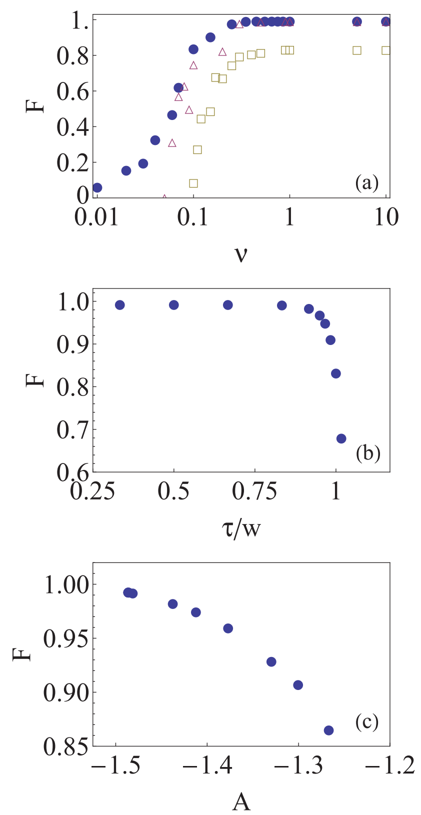

FIG. 4. (Color online) Result of numerical simulations for the quantum-state transfer over a dark fiber of the superposition $(\alpha|+00\rangle + \beta|-00\rangle)|a00\rangle|00\rangle$, with $\alpha = 1/\sqrt{3}$, $\beta = -\sqrt{2/3}$. (a) The fidelity of transfer is plotted as a function of ν. The different data sets differ for the value of w: filled circles correspond to $w = 300$, open triangles to $w = 200$, and open squares to $w = 150$. The time delay between pulses is $\tau = 150$ for all the data sets. (b) The fidelity is plotted as a function of the delay time τ between the pulses, for a cavity-fiber coupling $\nu = 1$, and $w = 300$. (c) The fidelity is calculated for pulses of different shape, as defined by Eq. (26), and plotted as a function of the parameter A, given by Eq. (19). For the data in panel (c), $\nu = 1$, $\tau = 150$, and $w = 300$. The parameters common to all calculations are $g = 21$, $t_0 = 1400$, and $\Omega_{1,0} = -\Omega_{2,0} = 80$.

We studied the dependence of the transfer process on the pulse shape by considering values of γ in the range [3/5 : 8] and studying the fidelity as a function of A. Our results are shown in Fig. 4(c). Fidelities larger than 0.99 are obtained for $\gamma \geqslant 3/2$ (i.e., $|A| > 1.48$). The fidelity smoothly decreases for pulses with less steep leading and trailing edges. For example, for Gaussian pulses ($\gamma = 1$) a fidelity of 0.98 is obtained.

V. CONCLUSIONS

In this work we introduced a model for quantum-state transfer between tripod atoms in cavities over a dark fiber. Two tripod atoms are confined in distinct cavities linked by an optical fibers. The cavities and the fiber sustain two optical modes of opposite circular polarization. For each atom, the two ground states encode the quantum state to be transferred and are coupled to a common excited state by the cavity modes of opposite polarization. The remaining transition for each atom is used to control the transfer process. We demonstrate that by using laser pulses the dynamics of the system can be confined within a degenerate dark state subspace, with the different dark states interacting via nonadiabatic couplings. We solve analytically the dynamics in the dark state subspace, and determine the conditions on the pulse shape for the implementation of the quantum trasfer. We identify a possible pulse shape which satisfies the required conditions, and demonstrate the quantum-state transfer via numerical simulations. We also demonstrated numerically the robustness of the scheme with respect to several system parameters.

ACKNOWLEDGMENTS

This work was supported by the Royal Society and EPSRC (Grant No. EP/H049231/1).

APPENDIX: THE DARK STATES

The basis vectors $|\psi_D^{(j)}\rangle$ ($j = 1$–8) spanning the dark subspace are as follows:

$$\left|\psi_D^{(1)}\right\rangle = \frac{-|-00\rangle|+10\rangle - |+00\rangle|-10\rangle + |+00\rangle|+01\rangle + |+10\rangle|-00\rangle + |-10\rangle|+00\rangle - |+01\rangle|+00\rangle}{\sqrt{6}}|00\rangle, \tag{A1}$$

$$\left|\psi_D^{(2)}\right\rangle = \frac{|-00\rangle|-10\rangle - |-00\rangle|+01\rangle - |+00\rangle|-01\rangle - |-10\rangle|-00\rangle + |+01\rangle|-00\rangle + |-01\rangle|+00\rangle}{\sqrt{6}}|00\rangle, \tag{A2}$$

$$\left|\psi_D^{(3)}\right\rangle = \frac{-|+00\rangle|+10\rangle + |+10\rangle|+00\rangle}{\sqrt{2}}|00\rangle, \tag{A3}$$

$$\left|\psi_D^{(4)}\right\rangle = \frac{|-00\rangle|-01\rangle - |-01\rangle|-00\rangle}{\sqrt{2}}|00\rangle, \tag{A4}$$

$$\left|\psi_D^{(5)}\right\rangle = \frac{\Omega_2}{\sqrt{3\left(3g^2 + 4\Omega_2^2\right)}}[(3g/\Omega_2)|-00\rangle|a00\rangle - |-00\rangle|-10\rangle - 2|-00\rangle|+01\rangle$$
$$+ |+00\rangle|-01\rangle + |-10\rangle|-00\rangle - |+01\rangle|-00\rangle + 2|-01\rangle|+00\rangle]|00\rangle, \tag{A5}$$

$$
\begin{aligned}
|\psi_D^{(6)}\rangle = & \frac{1}{\sqrt{3g^4+4\Omega_2^2 g^2+4\Omega_1^2 g^2+4\Omega_1^2\Omega_2^2}}\left\{ g\sqrt{3g^2+4\Omega_2^2}|a00\rangle|{-00}\rangle+\frac{\Omega_1}{\sqrt{3g^2+4\Omega_2^2}}\big[-2g\Omega_2|{-00}\rangle|a00\rangle+\right.\\
&+g^2(|{-01}\rangle|{+00}\rangle-|{-00}\rangle|{+01}\rangle)+2\big(g^2+\Omega_2^2\big)(|{+00}\rangle|{-01}\rangle-|{+01}\rangle|{-00}\rangle)\\
&\left.+\big(g^2+2\Omega_2^2\big)(|{-00}\rangle|{-10}\rangle-|{-10}\rangle|{-00}\rangle)\big]\right\}|00\rangle,
\end{aligned}
\tag{A6}
$$

$$
\begin{aligned}
|\psi_D^{(7)}\rangle = & \frac{\Omega_2}{\sqrt{3(3g^2+4\Omega_2^2)}}[(3g/\Omega_2)|{+00}\rangle|a00\rangle+|{-00}\rangle|{+10}\rangle-2|{+00}\rangle|{-10}\rangle,\\
&-|{+00}\rangle|{+01}\rangle+2|{+10}\rangle|{-00}\rangle-|{-10}\rangle|{+00}\rangle+|{+01}\rangle|{+00}\rangle]|00\rangle,
\end{aligned}
\tag{A7}
$$

$$
\begin{aligned}
|\psi_D^{(8)}\rangle = & \frac{1}{\sqrt{3g^4+4\Omega_2^2 g^2+4\Omega_1^2 g^2+4\Omega_1^2\Omega_2^2}}\left\{ g\sqrt{3g^2+4\Omega_2^2}|a00\rangle|{+00}\rangle+\frac{\Omega_1}{\sqrt{3g^2+4\Omega_2^2}}\big[-2g\Omega_2|{+00}\rangle|a00\rangle\right.\\
&+g^2(|{+10}\rangle|{-00}\rangle-|{+00}\rangle|{-10}\rangle)+2\big(g^2+\Omega_2^2\big)(|{-00}\rangle|{+10}\rangle-|{-10}\rangle|{+00}\rangle)\\
&\left.+\big(g^2+2\Omega_2^2\big)(|{+00}\rangle|{+01}\rangle-|{+01}\rangle|{+00}\rangle)\big]\right\}|00\rangle.
\end{aligned}
\tag{A8}
$$

[1] G. Alzetta, A. Gozzini, L. Moi, and G. Orriols, Nuovo Cimento B **36**, 5 (1976).

[2] E. Arimondo and G. Orriols, Lett. Nuovo Cimento **17**, 333 (1976).

[3] For a review see E. Arimondo, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1996), Vol. 35, pp. 257–354.

[4] K. Bergmann, H. Theuer, and B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).

[5] Z. Kis and S. Stenholm, Phys. Rev. A **64**, 063406 (2001).

[6] Z. Kis and S. Stenholm, J. Mod. Opt. **49**, 111 (2002).

[7] Z. Kis and F. Renzoni, Phys. Rev. A **65**, 032318 (2002).

[8] H. Goto and K. Ichimura, Phys. Rev. A **70**, 012305 (2004).

[9] N. Sangouard, X. Lacour, S. Guérin, and H. R. Jauslin, Phys. Rev. A **72**, 062309 (2005).

[10] J.-M. Chen, L.-M. Liang, C.-Z. Li, and Z.-J. Deng, Opt. Commun. **282**, 4020 (2009).

[11] X. Lacour, N. Sangouard, S. Guérin, and H. R. Jauslin, Phys. Rev. A **73**, 042321 (2006).

[12] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**, 3221 (1997).

[13] Y. L. Zhou, Y. M. Wang, L. M. Liang, and C. Z. Li, Phys. Rev. A **79**, 044304 (2009).