Stable long-distance propagation and on-off switching of colliding soliton sequences with dissipative interaction

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We study propagation and on-off switching of two colliding soliton sequences in the presence of second-order dispersion, Kerr nonlinearity, linear loss, cubic gain, and quintic loss. Employing a Lotka-Volterra (LV) model for dynamics of soliton amplitudes along with simulations with two perturbed coupled nonlinear Schrödinger (NLS) equations, we show that stable long-distance propagation can be achieved for a wide range of the gain-loss coefficients, including values that are outside of the perturbative regime. Furthermore, we demonstrate robust on-off and off-on switching of one of the sequences by an abrupt change in the ratio of cubic gain and quintic loss coefficients, and extend the results to pulse sequences with periodically alternating phases. Our study significantly strengthens the recently found relation between collision dynamics of sequences of NLS solitons and population dynamics in LV models, and indicates that the relation might be further extended to solitary waves of the cubic-quintic Ginzburg-Landau equation.

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I. INTRODUCTION

The cubic nonlinear Schrödinger (NLS) equation, which describes wave propagation in the presence of second-order dispersion and cubic (Kerr) nonlinearity, is one of the most widely researched nonlinear wave models in physics. It was successfully employed to describe water wave dynamics [1], nonlinear waves in plasma [2], Bose-Einstein condensates (BECs) [3], and pulse propagation in optical waveguides [4]. The fundamental NLS solitons are the most ubiquitous solutions of the cubic NLS equation due to their stability and to the fact that a generic wave pattern typically evolves into a sequence of fundamental solitons in the presence of anomalous dispersion and Kerr nonlinearity. In the absence of additional physical processes (perturbations), the fundamental solitons propagate without any change in their amplitude, group velocity, and shape. However, the presence of perturbations usually breaks this ideal picture, by inducing changes in the solitons' amplitude, group velocity, and shape. Dissipative perturbations due to linear and nonlinear gain or loss are of particular interest, since they are very common in soliton systems. In optical waveguides, for example, nonlinear loss or gain arise due to multiphoton absorption or emission, respectively [5]. Moreover, nonlinear loss and gain play an important role in many phenomena described by the complex Ginzburg-Landau (GL) equation [6], such as convection and pattern formation in fluids and mode-locked lasers.

Despite its success, the single NLS equation is limited to describing a scalar physical field and is also unsuitable for handling generic broadband nonlinear wave systems. If the physical field is a vector, or if the spectra of the waves in a broadband wave system are concentrated about multiple widely separated wavelengths, the single NLS equation should be replaced by a system of coupled-NLS equations. Indeed, in recent years, coupled-NLS models have been employed in studies of a wide range of phenomena in fluid dynamics [7,8], nonlinear optics [4,9], multiphase BECs [10], and plasma physics [11]. Coupled-NLS models are particularly useful in describing broadband wave systems, in which the

waves are organized in sequences (trains) moving with very different group velocities, such as in crossing seas [8], or in broadband optical waveguide transmission [4]. Due to the large group-velocity differences in these systems, collisions between pulses from different sequences are very frequent, and thus play a major role in the dynamics.

In the current paper, we study the dynamics of two colliding sequences of fundamental NLS solitons in the presence of dissipative perturbations due to linear loss, cubic gain, and quintic loss (i.e., a GL gain-loss profile). In the absence of perturbations, the solitons' amplitudes and group velocities do not change in the collisions. However, the presence of dissipative interaction due to cubic gain and quintic loss induces additional amplitude shifts during the collisions, whose magnitude depends on the initial amplitudes of the two colliding solitons. Consequently, amplitude dynamics of solitons in the two sequences become nonlinearly coupled, and important questions arise regarding the nature of this dynamics. In three recent studies we showed that amplitude dynamics of N sequences of colliding solitons in the presence of dissipative perturbations might be described by Lotka-Volterra (LV) models for N species, where the exact form of the LV model depends on the nature of the perturbations [12–14]. This potential relation between coupled-NLS equations and LV systems is of great interest, due to the central role of these seemingly unrelated models in the natural sciences. While the importance of coupled-NLS equations was explained in the previous paragraph, LV models are widely used in environmental sciences to describe population dynamics [15–17], in chemistry, to study dynamics of chemical reactions [18], in economics, to explain interaction between different technologies [19], and in neural networks, to compute neuron firing rates [20]. Due to the importance of coupled-NLS and LV models, it is essential to further enhance and extend the understanding of the relation between them. Moreover, a better understanding of this relation can be used for stabilizing the propagation, for controlling and tuning of soliton amplitudes and group velocities, and even for broadband switching.

In order to give further motivation for the current study and to clarify the importance of its results, we provide a brief critical description of our three earlier studies of the problem. In Ref. [12], we studied the propagation of colliding soliton sequences in the presence of delayed Raman response. We showed that amplitude dynamics of N soliton sequences is described by an N-dimensional predator-prey model, but the results were not checked by numerical simulations with the corresponding coupled-NLS model. Furthermore, the equilibrium states of the predator-prey model were found to be centers, i.e., the equilibria were not asymptotically stable. In order to resolve the problem of asymptotic stability in *the LV model*, we turned to investigate collision dynamics of soliton sequences in the presence of weak linear gain and cubic loss [13]. We showed that in this case dynamics of soliton amplitudes might be approximately described by a LV model for competing species with quadratic interaction terms. Stability analysis of the equilibrium states of the model yielded conditions on the physical parameters for stable propagation of two sequences with equal soliton amplitudes. Numerical simulations with the full coupled-NLS model confirmed stability at short-to-intermediate distances, but uncovered an instability at longer distances [13]. The latter finding was attributed to weak (second-order) radiative instability due to the impact of linear gain on the two sequences. In order to overcome this destabilizing effect, we turned to study a different setup, where the soliton sequences propagate in the presence of weak linear loss, cubic gain, and quintic loss [14]. We showed that, in this setup, the LV model for amplitude dynamics contains quadratic and quartic interaction terms. The presence of linear loss was expected to suppress the weak radiative instability and by this to enable stable long-distance propagation. However, numerical solution of the corresponding perturbed coupled-NLS model still showed instability at intermediate-to-long propagation distances [14]. Thus the important problem of further stabilization of the propagation remained unresolved. Moreover, due to the instability at intermediate distances, all the previous studies were unable to explore the possibility of achieving stable on-off and off-on switching, i.e., the turning on and off of transmission of one sequence by a fast change of one or more of the physical parameters. In addition, the earlier studies were limited in scope to small values of the dissipative coefficients and to uniform initial distribution of soliton amplitudes and phases in each sequence.

In the current paper we address these important deficiencies of the earlier studies. Considering propagation of two colliding soliton sequences in the presence of a GL gain-loss profile, we observe stable long-distance transmission over a wide range of the gain-loss coefficients, including values that are outside of the perturbative regime. The stable transmission is enabled by the presence of linear loss, a sufficiently large initial intersoliton separation, and the absence of significant initial position shift between the two soliton sequences. Moreover, we show that robust on-off and off-on switching of one soliton sequence can be realized by an abrupt change in the ratio of the cubic gain and quintic loss coefficients. In addition, we extend the results to soliton sequences with periodically alternating phases. Our findings significantly enhance the surprising relation between propagation of colliding NLS soliton sequences and population dynamics in LV models. The

results also indicate that the relation might be further extended to sequences of solitons of the cubic-quintic GL equation.

The rest of the paper is organized as follows. In Sec. II, we present the coupled-NLS model for the propagation along with the reduced LV model for amplitude dynamics. We also discuss the predictions of the latter model for stability of propagation. In Sec. III, we present the results of numerical simulations with the coupled-NLS model for long distance transmission, as well as for on-off and off-on switching. In addition, we compare the simulation results with the predictions of the LV model. Our conclusions are presented in Sec. IV.

II. COUPLED-NLS AND LOTKA-VOLTERRA MODELS

We consider propagation of sequences of soliton pulses in the presence of second-order dispersion, Kerr nonlinearity, and a GL gain-loss profile, consisting of linear loss or gain, cubic gain, and quintic loss. We denote the physical field of the jth sequence by ψ_j. In the context of propagation of light through optical waveguides, for example, ψ_j is proportional to the envelope of the electric field of the jth sequence. In the absence of gain and loss, the propagation of the jth soliton sequence is described by the cubic NLS equation,

$$i\partial_z \psi_j + \partial_t^2 \psi_j + 2|\psi_j|^2 \psi_j = 0, \qquad (1)$$

where we adopt the waveguide optics notation, in which z is propagation distance and t is time. The fundamental soliton solution of the NLS equation with group velocity $2\beta_j$ is $\psi_{sj}(t,z) = \eta_j \exp(i\chi_j)\mathrm{sech}(x_j)$, where $x_j = \eta_j(t - y_j - 2\beta_j z)$, $\chi_j = \alpha_j + \beta_j(t - y_j) + (\eta_j^2 - \beta_j^2)z$, and η_j, y_j, and α_j are the soliton amplitude, position, and phase, respectively.

We focus attention on the dynamics of two sequences of fundamental NLS solitons propagating with group velocities $2\beta_j$, where $j = 1,2$. Assuming a large group velocity difference $|\beta_1 - \beta_2| \gg 1$, the solitons undergo a large number of fast intersequence collisions. We now take into account the effects of linear gain-loss, cubic gain, and quintic loss as well as intersequence interaction due to Kerr nonlinearity. Thus the propagation is described by the following system of perturbed coupled-NLS equations:

$$
\begin{aligned}
i\partial_z \psi_j &+ \partial_t^2 \psi_j + 2|\psi_j|^2 \psi_j + 4|\psi_k|^2 \psi_j \\
&= i g_j \psi_j/2 + i\epsilon_3 |\psi_j|^2 \psi_j + 2i\epsilon_3 |\psi_k|^2 \psi_j \\
&\quad - i\epsilon_5 |\psi_j|^4 \psi_j - 3i\epsilon_5 |\psi_k|^4 \psi_j - 6i\epsilon_5 |\psi_k|^2 |\psi_j|^2 \psi_j, \quad (2)
\end{aligned}
$$

where $j = 1,2$, $k = 1,2$, g_j is the linear gain-loss coefficient for the jth sequence, and ϵ_3 and ϵ_5 are the cubic gain and quintic loss coefficients, respectively. The term $4|\psi_k|^2 \psi_j$ in Eq. (2) describes intersequence interaction due to Kerr nonlinearity, while $i g_j \psi_j/2$, $i\epsilon_3 |\psi_j|^2 \psi_j$, and $-i\epsilon_5 |\psi_j|^4 \psi_j$ correspond to intrasequence effects due to linear gain-loss, cubic gain, and quintic loss, respectively. In addition, the term $2i\epsilon_3 |\psi_k|^2 \psi_j$ represents intersequence effects due to cubic gain, while $-3i\epsilon_5 |\psi_k|^4 \psi_j$ and $-6i\epsilon_5 |\psi_k|^2 |\psi_j|^2 \psi_j$ describe dissipative intersequence interaction due to quintic loss.

In Ref. [14], we showed that, under certain assumptions, amplitude dynamics of solitons in the two sequences can be approximately described by a LV model for two species with quadratic and quartic interaction terms. The derivation of the LV model was based on the following assumptions.

(1) The temporal separation T between adjacent solitons in each sequence is a large constant: $T \gg 1$. In addition, the amplitudes are equal for all solitons from the same sequence, but are not necessarily equal for solitons from different sequences. (2) The pulses circulate in a closed loop, e.g., in an optical waveguide ring. (3) As $T \gg 1$, the pulses in each sequence are temporally well separated. As a result, intrasequence interaction is exponentially small and is neglected. (4) High-order effects due to collision-induced frequency shift and emission of radiation are also neglected.

Since the soliton sequences are periodic, the amplitudes of all pulses in a given sequence undergo the same dynamic evolution. Taking into account collision-induced and single-pulse amplitude changes, we obtain the following equation for the rate of change of the amplitude of the solitons in the jth sequence η_j [14]:

$$\frac{d\eta_j}{dz} = \eta_j \left[g_j + \frac{4}{3}\epsilon_3 \eta_j^2 - \frac{16}{15}\epsilon_5 \eta_j^4 + \frac{8}{T}\epsilon_3 \eta_k \right.$$
$$\left. - \frac{8}{T}\epsilon_5 \eta_k \left(2\eta_j^2 + \eta_k^2 \right) \right], \tag{3}$$

where $j = 1,2$. Note that Eq. (3) can be described as a LV model for two species with quadratic cooperation terms and quartic competition terms.

The results described in the previous paragraph indicate that there is a relation between collision-induced dynamics of soliton sequences in nonlinear waveguides with a GL gain-loss profile and dynamics of population size of species with nonlinear competition and cooperation. Alternatively, one may speak about a connection between the perturbed coupled-NLS model (2) and the LV model (3). This relation along with knowledge about properties of LV models can be used to develop ways for controlling the dynamics of the colliding solitons. A straightforward way for achieving this goal is by tuning of the linear gain-loss coefficients g_j. In optical waveguide systems, for example, this can be realized by adjusting the linear amplifier gain.

As a particular example, we consider the important case where the values of the g_j coefficients are chosen such that the LV model (3) has a steady state (η, η) with equal amplitudes for both sequences. This case is of special interest, since in optical waveguide systems it is usually easier to control the transmission when the amplitudes of all pulses are equal. Requiring that (η, η) is a steady state, we obtain $g_j = 4\epsilon_5 \eta(-\kappa \eta/3 + 4\eta^3/15 - 2\kappa/T + 6\eta^2/T)$, where $\kappa = \epsilon_3/\epsilon_5$ and $\epsilon_5 \neq 0$. Substituting this relation into Eq. (3), we arrive at the following LV model for amplitude dynamics:

$$\frac{d\eta_j}{dz} = \epsilon_5 \eta_j \left\{ \frac{4\kappa}{3}\left(\eta_j^2 - \eta^2\right) - \frac{16}{15}\left(\eta_j^4 - \eta^4\right) + \frac{8\kappa}{T}(\eta_k - \eta) \right.$$
$$\left. - \frac{8}{T}\left[\eta_k \left(2\eta_j^2 + \eta_k^2\right) - 3\eta^3\right] \right\}. \tag{4}$$

We note that (η, η) and $(0,0)$ are equilibrium points of Eq. (4) for any positive values of η, κ, and T.

Let us discuss the predictions of the LV model (4) regarding stability of propagation of the two soliton sequences. For concreteness, we choose $\eta = 1$, and require that the equilibrium points at $(1,1)$ and $(0,0)$ are stable nodes (sinks). The stability requirement on $(0,0)$, which is important for

suppression of weak instability due to radiation emission, leads to $g_j < 0$ for $j = 1,2$, i.e., the solitons are subject to net linear loss. The above stability requirements yield the following conditions on T and κ [14]: $(4T + 90)/(5T + 30) < \kappa < (8T + 135)/(5T + 15)$ for $3 < T < 60/17$, and $(4T + 90)/(5T + 30) < \kappa < (8T - 15)/(5T - 15)$ for $T \geqslant 60/17$ [21].

III. NUMERICAL SIMULATIONS WITH THE COUPLED-NLS MODEL

As explained above, the LV model (4) provides an approximate description for collision-induced dynamics of circulating soliton sequences. The propagation is fully described by the coupled-NLS system (2) with periodic boundary conditions. A key question about the collision-induced dynamics concerns the possibility to suppress the instability observed in numerical simulations in Ref. [14]. Here we address this important question. More specifically, we show that the instability can be effectively mitigated by using a larger temporal separation value T, compared with the one used in [14], and by eliminating the initial intersequence position shift, which was introduced in [14] to avoid incomplete collisions. Thus, the numerical simulations setup in the current paper consists of the coupled-NLS model (2) with periodic boundary conditions and an initial condition in the form of two periodic sequences of $2J + 1$ *overlapping* solitons with amplitudes $\eta_j(0)$ and zero phase:

$$\psi_j(t,0) = \sum_{k=-J}^{J} \frac{\eta_j(0)\exp[i\beta_j(t - kT)]}{\cosh[\eta_j(0)(t - kT)]}, \tag{5}$$

where $j = 1,2$. The temporal separation T is taken as $T = 20$, compared with $T = 10$ that was used in Ref. [14]. Four different values of ϵ_5 are used: $\epsilon_5 = 0.01$, $\epsilon_5 = 0.06$, $\epsilon_5 = 0.1$, and $\epsilon_5 = 0.5$, compared with $\epsilon_5 = 0.01$ that was considered in Ref. [14]. Note that a value of $\epsilon_5 = 0.5$ is outside of the parameter range, where the perturbative approach leading to the LV model (3) is expected to be valid. The other physical parameters in the simulations are taken as $\kappa = 1.5$, $\eta = 1$, $\beta_1 = 0$, $\beta_2 = 40$, and $J = 2$.

The z dependence of η_1 obtained by numerical solution of Eq. (2) with initial amplitudes $\eta_1(0) = 1.25$ and $\eta_2(0) = 0.7$ is shown in Fig. 1(a) for $\epsilon_5 = 0.01$, $\epsilon_5 = 0.06$, and $\epsilon_5 = 0.1$, and in Fig. 1(b) for $\epsilon_5 = 0.5$. The predictions of the LV model (4) are also presented. The agreement between the coupled-NLS simulations and the predictions of the LV model are excellent. Moreover, in all four cases $\eta_1(z)$ tends to the equilibrium value of $\eta = 1$ and the approach to $\eta = 1$ is faster as ϵ_5 increases. Similar results are obtained for $\eta_2(z)$ and for other initial amplitude values in the vicinity of the equilibrium point $(1,1)$. Note that the distances over which stable propagation is observed are larger by a factor of 6.67 compared with distances in Ref. [13], and by a factor of 2 or more compared with the distances in Ref. [14]. The fact that the predictions of the LV model hold even for ϵ_5 values as large as 0.5, i.e., outside of the perturbative regime, is quite surprising. Furthermore, as shown in the inset of Fig. 1(b), the shape of the two soliton trains is retained during the propagation despite the magnitude of ϵ_5 and the large number of collisions.
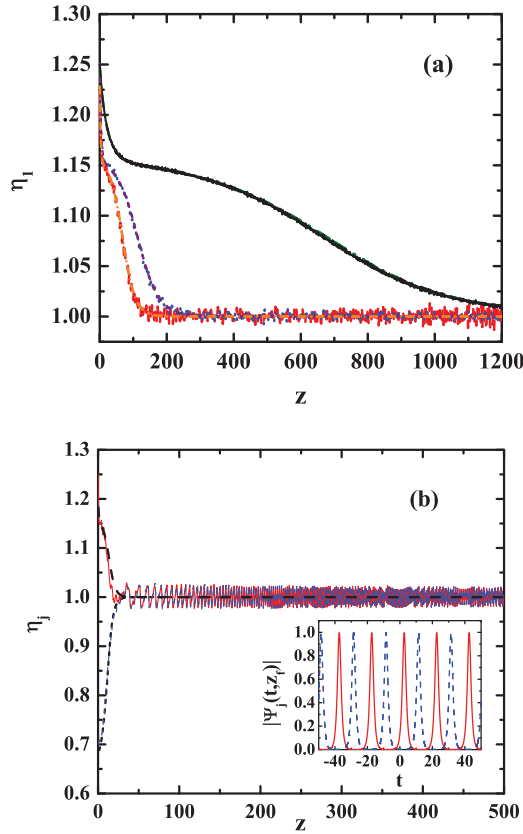
FIG. 1. (Color online) (a) z dependence of η_1 for various ϵ_5 values and $T = 20, \kappa = 1.5, \eta_1(0) = 1.25,$ and $\eta_2(0) = 0.7$. The solid black, dotted blue, and dashed red lines correspond to $\eta_1(z)$ values obtained by numerical solution of the coupled-NLS model (2) with $\epsilon_5 = 0.01$, $\epsilon_5 = 0.06$, and $\epsilon_5 = 0.1$, respectively. The dashed-dotted green, short dashed purple, and short dashed-dotted orange lines represent $\eta_1(z)$ values predicted by the LV model (4) with $\epsilon_5 = 0.01$, $\epsilon_5 = 0.06$, and $\epsilon_5 = 0.1$. (b) The z dependence of η_1 and η_2 for $\epsilon_5 = 0.5$ and the same values of T, κ, $\eta_1(0)$, and $\eta_2(0)$ as in (a). The solid red and dashed blue lines represent $\eta_1(z)$ and $\eta_2(z)$ as obtained by numerical solution of Eq. (2), while the dashed and short dashed black lines correspond to $\eta_1(z)$ and $\eta_2(z)$ values as obtained by the LV model (4). The inset shows the final pulse patterns $|\psi_1(t,z_f)|$ (solid red) and $|\psi_2(t,z_f)|$ (dashed blue) obtained in the coupled-NLS simulation.

The observation of long-distance propagation of the two soliton sequences opens the way for studying broadband on-off and off-on switching, i.e., the turning on and off of one of the propagating pulse sequences. The two switching scenarios are based on bifurcations of the steady state $(1,1)$ of the LV model. More specifically, in on-off switching, the value of the parameter κ is abruptly increased at the switching distance z_s from $\kappa_i < \kappa_c$ to $\kappa_f > \kappa_c$, where $\kappa_c = (8T − 15)/(5T − 15)$, such that the steady state $(1,1)$ becomes unstable, while another steady state $(\eta_s,0)$ is stable. As a result, soliton amplitudes in sequences 2 and 1 tend to 1 for $z < z_s$, but tend to 0 and η_s, respectively, for $z > z_s$. This means that transmission of sequence 2 is effectively turned off at z_s. Off-on switching is realized in a similar manner, by decreasing the value of κ at the switching distance z_s from $\kappa_i > \kappa_c$ to $\kappa_f < \kappa_c$ such that the steady state $(1,1)$ becomes stable. Consequently, for $z < z_s$, the amplitudes of solitons in sequences 1 and 2 tend to η_s and
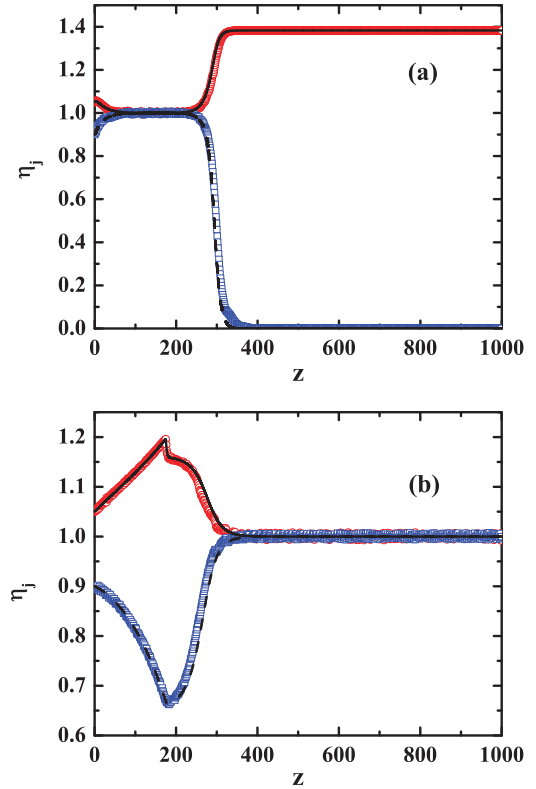

FIG. 2. (Color online) z dependence of soliton amplitudes in on-off switching (a), and in off-on switching (b) with $z_s = 175$ and parameter values as described in the text. The red circles and blue squares represent $\eta_1(z)$ and $\eta_2(z)$ values obtained with the coupled-NLS model (2), while the solid and dashed black lines correspond to $\eta_1(z)$ and $\eta_2(z)$ values obtained with the LV model (4).

0, respectively, but for $z > z_s$, both amplitudes tend to 1. Thus, in this case, transmission of sequence 2 is turned on at z_s.

In order to check if these switching scenarios can be realized with sequences of colliding solitons, we numerically solve Eq. (2) with initial pulse patterns of the form (5), where $T = 20$, $\eta_1(0) = 1.05$, and $\eta_2(0) = 0.9$. Since $T = 20$, the critical κ value is $\kappa_c = 29/17$. As an example, the switching distance is taken as $z_s = 175$. In on-off switching, $\epsilon_5 = 0.1$, and κ is increased from 1.5 to 2.0 at z_s. In off-on switching, we use $\epsilon_5 = 0.01$ and $\kappa = 2.0$ for $z \leqslant 175$, and $\epsilon_5 = 0.1$ and $\kappa = 1.5$ for $z > 175$. The results of the simulations for $\eta_j(z)$ are shown in Figs. 2(a) and 2(b) for on-off and off-on switching, respectively. Also shown is the prediction of the LV model (4). The overall agreement between the coupled-NLS simulations and the predictions of the LV model is very good for both switching scenarios. The only exception is for small values of $\eta_2(z)$ in on-off switching, where the LV model underestimates the value obtained with the coupled-NLS model. This is due to the fact that, in this regime, the linear loss term is comparable to or larger than the Kerr nonlinearity term, leading to the breakdown of the perturbative description. Despite this fact, the amplitudes of the solitons in sequence 2 do tend to zero, and thus the on-off switching is fully realized. Based on the good agreement between coupled-NLS simulations and predictions of the LV model, we conclude that on-off and off-on switching can be realized for the two sequences of colliding solitons. It should be noted that stable off-on switching can be achieved
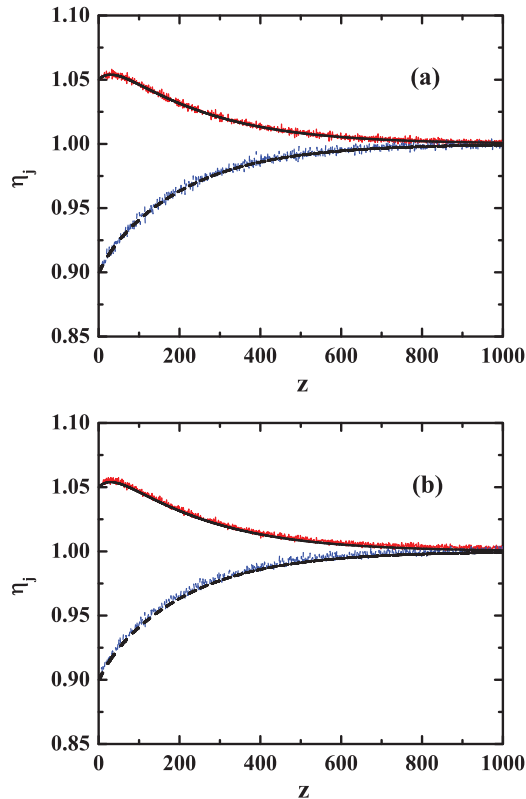
FIG. 3. (Color online) z dependence of pulse amplitudes for soliton sequences with alternating initial phases and parameter values $T = 20, \kappa = 1.5, \epsilon_5 = 0.01, \eta_1(0) = 1.05$, and $\eta_2(0) = 0.9$. (a) η_j vs z for $\alpha_{jk}(0) = 0$ and $\alpha_{jk+1}(0) = \pi$. (b) η_j vs z for $\alpha_{jk}(0) = 0$ and $\alpha_{jk+1}(0) = \pi/2$. The solid red and dashed blue lines represent $\eta_1(z)$ and $\eta_2(z)$ values obtained with the coupled-NLS model (2), while the solid and dashed black lines correspond to $\eta_1(z)$ and $\eta_2(z)$ values obtained with the LV model (4).

only when $\eta_2(z_s)$ is larger than some threshold value η_{th}. For the parameter values used in our simulations, $\eta_{\text{th}} = 0.65$. Thus, in order to implement off-on switching with the current setup of two soliton sequences, the value of the decision level distinguishing between 0 and 1 states should be set larger than η_{th}.

All the propagation setups discussed so far are limited, in the sense that all amplitudes and phases within each soliton sequence are equal. It is therefore important to extend the results to more general setups. Possible extension can be achieved by launching soliton sequences with periodically alternating phases. Indeed, the LV model (4) is independent of the soliton phases and, as a result, amplitude dynamics for sequences with periodically alternating phases is expected to be the same as the dynamics for soliton sequences with a uniform phase pattern. On the other hand, it is known

that intrasequence soliton interaction strongly depends on the relative phase difference between the solitons. The interplay between this interaction and other effects, such as loss or radiation emission might induce instabilities, which would lead to the breakdown of the LV model description. For this reason, it is important to check whether the predictions of the LV model (4) do apply for soliton sequences with periodically alternating phases. For this purpose, we numerically solve Eq. (2) with pulse sequences consisting of two solitons each with initial amplitudes $\eta_1(0) = 1.05$ (sequence 1) and $\eta_2(0) = 0.9$ (sequence 2). Two sets of initial phases are used: $\alpha_{j1}(0) = 0$ and $\alpha_{j2}(0) = \pi$ [set (a)] and $\alpha_{j1}(0) = 0$ and $\alpha_{j2}(0) = \pi/2$ [set (b)]. The other physical parameter values are taken as $T = 20, \kappa = 1.5$, and $\epsilon_5 = 0.01$. The results of the numerical simulations for $\eta_j(z)$ are shown in Fig. 3 along with the predictions of the LV model. As can be seen, for both sets of initial phases the agreement between full-scale simulations and the predictions of the LV model is excellent over the entire propagation distance. Furthermore, the soliton amplitudes tend to the equilibrium value $\eta = 1$, i.e., the propagation is indeed linearly stable. Thus our numerical simulations demonstrate that the predictions of the LV model (4) can be applied to generic setups with periodic phase patterns.

IV. CONCLUSIONS

In summary, we investigated long-distance propagation as well as on-off and off-on switching of two colliding soliton sequences in the presence of second-order dispersion, Kerr nonlinearity, and a GL gain-loss profile. Using coupled-NLS simulations and a reduced LV model, we showed that stable long-distance propagation is possible for a wide range of the gain-loss coefficients, including values that are outside of the perturbative regime. The stable propagation is enabled by the presence of linear loss, the choice of a sufficiently large initial interpulse separation within each sequence, and the absence of significant initial time delay between the two sequences. Furthermore, we found that robust on-off and off-on switching of one of the propagating sequences can be achieved by an abrupt change in the ratio of cubic gain and quintic loss coefficients. Additionally, we demonstrated that the results can be extended to pulse sequences with periodically alternating phases. Our study significantly enhances the recently found relation between collision-induced dynamics of sequences of NLS solitons and evolution of population sizes in LV models. Moreover, it indicates that the relation might be further extended to sequences of solitary waves of the cubic-quintic GL equation, at least in some regions of parameter space. Finally, our results might find useful applications in nonlinear waveguides with saturable absorption for stable energy equalization and broadband switching.

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[21] Note that the value $(8T + 135)/(5T + 15)$ for the right boundary of the first κ interval is a correction to the value given in Eq. (26) in Ref. [14].