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Bose-Fermi pairs in a mixture and the Luttinger theorem within a Nozières-Schmitt-Rink-like approach

T. Sogo,[1] P. Schuck,[1,2] and M. Urban[1]

[1]*Institut de Physique Nucléaire, CNRS-IN2P3 and Université Paris-Sud, F-91406 Orsay Cedex, France*

[2]*Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier, 25 Avenue des Martyrs, Boîte Postale 166, F-38042 Grenoble Cedex 9, France*

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Boson-fermion pair correlations in a mixture are considered at zero temperature in the T-matrix approximation. Special attention is paid to the Luttinger theorem. In a strict random-phase approximation variant of the Nozières-Schmitt-Rink approach, it is shown that this theorem is respected also in the homogeneous infinite matter case. We calculate the corresponding occupation numbers of fermions and bosons and the condensate depletion. We also show that in the limit of very small boson density, our results are in good agreement with the results found in the literature for the Fermi polaron in strongly imbalanced Fermi-Fermi mixtures.

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I. INTRODUCTION

Cold-atom physics is constantly progressing at a rapid pace. Fermi and Bose systems have been under consideration extensively. Bose-Fermi (BF) mixtures have been studied so far a little less. In early attempts to create a degenerate Fermi gas, bosonic ^{7}Li [1,2] or ^{23}Na atoms [3] were added to the fermionic ^{6}Li in order to allow for sympathetic cooling. The first BF mixture with an attractive BF interaction was that of ^{40}K and ^{87}Rb [4]. In present-day experiments with ^{6}Li, a small fraction of ^{7}Li atoms is kept to serve as a thermometer [5]. In Ref. [6] a mixture of ^{40}K, ^{41}K, and ^{6}Li was created with the main goal to produce a mass-imbalanced Fermi gas of ^{40}K and ^{6}Li, the boson ^{41}K acting again as a coolant. The possibility to produce a dipolar Fermi gas of polar fermionic molecules has triggered many experiments with different BF mixtures such as ^{87}Rb-^{40}K [7,8], ^{23}Na-^{40}K [9,10], and ^{23}Na-^{6}Li [11]. Experiments with BF mixtures are not restricted to alkaline atoms. For instance, also ^{84}Sr-^{87}Sr [12] mixtures were created.

From the theory perspective, BF mixtures are interesting in their own right, e.g., to study the interplay between different quantum statistics in various fields of physics. They also may serve to simulate definite physical systems. For example, BF mixtures have been considered as an analogy to what might happen in the quark-hadron phase transition [13] within the scenario that the first two quarks form a tightly bound diquark (boson), which then combines with a third quark (fermion) to form a nucleon. It, thus, is important to further develop the theory of BF correlations in BF mixtures. A particularly interesting question concerns the structure and behavior of BF pairs. In Ref. [14] we have shown that similar to the formation of Cooper pairs in two-component Fermi systems, also in BF mixtures stable BF pairs can exist with very weak attraction for which a bound state cannot be formed in free space.

In this work, we shall be concerned with bosons and fermions interacting via a broad Feshbach resonance. Under this condition, the system can be described by a Hamiltonian of bosons and (spinless) fermions interacting via an attractive (or repulsive) contact potential. There exist several Monte Carlo investigations in one-dimensional (1D) [15] and 3D [16,17] BF systems. However, also approximate many-body approaches have been applied. Among those several works using the so-called T-matrix approximation have appeared [14,18–22] and this shall also be our framework in this paper. The BF T matrix describes BF scattering states but also eventual formation of bound states. Bound states in the medium are especially interesting. The T matrix also can serve to study single-particle properties. In this respect, folding the T matrix with either a fermion or a boson propagator yields the boson or fermion self-energy of the Dyson equation.

The T-matrix approximation has become particularly popular since Nozières and Schmitt-Rink (NSR) showed that for attractive Fermi systems this approach interpolates between the weak-coupling (BCS) situation and the Bose-Einstein condensation (BEC) of strongly bound fermion pairs [23]. This approach has also been generalized to study the pairing properties of polarized Fermi systems where there exist more fermions with, e.g., spin $\uparrow$ than those with spin $\downarrow$ [24–27]. However, these studies have revealed that in this case the standard NSR approach may lead, in some regions of the parameter space, to pathological results. A special case of particular interest is that of an extremely imbalanced mixture, which can be treated by considering a single atom of the minority species, the so-called polaron limit. The case of Fermi polarons, i.e., a single fermion of spin $\downarrow$ in a system of fermions with spin $\uparrow$, has been intensively studied using a variational ansatz [28,29] and, equivalently, a T-matrix approach [30,31], as well as using a diagrammatic Monte Carlo technique [32–34]. All these results can be directly applied to BF mixtures with a very small number of bosons, because if one considers only a single impurity it does not matter whether it is a fermion or a boson.

In the present paper, we will pay special attention to the Luttinger theorem [35]. This theorem states that the volume of the Fermi sphere is not changed by interactions, or in other words, that the reduction of the occupation numbers $\rho_{k<k_F}$ inside the Fermi sphere is exactly compensated by the nonvanishing occupation numbers $\rho_{k>k_F}$ outside the Fermi sphere. It is highly nontrivial to respect this theorem within a nonperturbative approximation scheme. Here, we will use a variant of the NSR approach adapted to BF systems. A particularity of the NSR approach is that it treats the self-energy in the single-particle Dyson equation only to first

order. This, for instance, means that the NSR approach, if suitably adapted, is strictly equivalent to the random-phase approximation (RPA), here in the so-called particle-particle (pp) channel, which sums pp and hole-hole (hh) ladders simultaneously [36]. The fact that pp-RPA satisfies, among other things, the analog of the Luttinger theorem in a system with a discrete level structure such as atomic nuclei has been known for many years [36,37]. It has also been demonstrated for a BF system on a lattice [38]. It will be one of the results of the present work to show this explicitly in a continuum case for an attractively interacting infinite BF system.

The paper is organized as follows. In Sec. II we discuss the BF scattering in a BF mixture within the pp-RPA framework. In Sec. III we discuss the correlation effects on the ground-state properties. Finally, in Sec. IV we summarize and conclude.

II. BOSON-FERMION T MATRIX WITHIN PARTICLE-PARTICLE RPA

The starting point of our study is the following BF Hamiltonian:

$$H = \int d^3r \Big\{ -\psi^\dagger(\mathbf{r})\frac{\nabla^2}{2m_F}\psi(\mathbf{r}) - \varphi^\dagger(\mathbf{r})\frac{\nabla^2}{2m_B}\varphi(\mathbf{r}) + g\psi^\dagger(\mathbf{r})[\sqrt{n_0}+\varphi^\dagger(\mathbf{r})][\sqrt{n_0}+\varphi(\mathbf{r})]\psi(\mathbf{r})\Big\}, \quad (1)$$

where ψ and φ are the fermion and boson field operators, $m_{F,B}$ are the fermion and boson masses, and g is the coupling constant. The field operator φ has been shifted by a c-number $\sqrt{n_0}$, where n_0 denotes the density of condensed bosons [39]. The field operators ψ and φ can be written in terms of fermion and boson annihilation operators $c_\mathbf{k}$ and $b_\mathbf{k}$ as

$$\psi(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} c_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (2)$$

$$\varphi(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} b_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (3)$$

Analogously, $\psi^\dagger$ and $\varphi^\dagger$ can be written in terms of fermion and boson creation operators $c_\mathbf{k}^\dagger$ and $b_\mathbf{k}^\dagger$.

The Hamiltonian (1) is suitable for the case of a broad Feshbach resonance in the BF interaction [21]. We neglect the boson-boson (BB) interaction. A repulsive BB interaction would essentially result in a mean-field shift that can be absorbed in a redefinition of the boson chemical potential and does not change the results very much [18,19]. Since we assume that the fermions are present in only one spin state ("spinless fermions"), there cannot be an s-wave fermion-fermion (FF) interaction and higher partial waves are usually negligible in ultracold trapped atoms.

As mentioned before, we want to apply a suitably adapted NSR approach to the present BF problem. We will work at zero temperature and with chronological Green's functions (GFs). In standard notation [39] we have for the single-particle fermion and noncondensed boson GFs

$$G_F(\mathbf{k},t-t') = -i\langle 0|T c_\mathbf{k}(t) c_\mathbf{k}^\dagger(t')|0\rangle, \quad (4)$$

$$G_B(\mathbf{k},t-t') = -i\langle 0|T b_\mathbf{k}(t) b_\mathbf{k}^\dagger(t')|0\rangle, \quad (5)$$

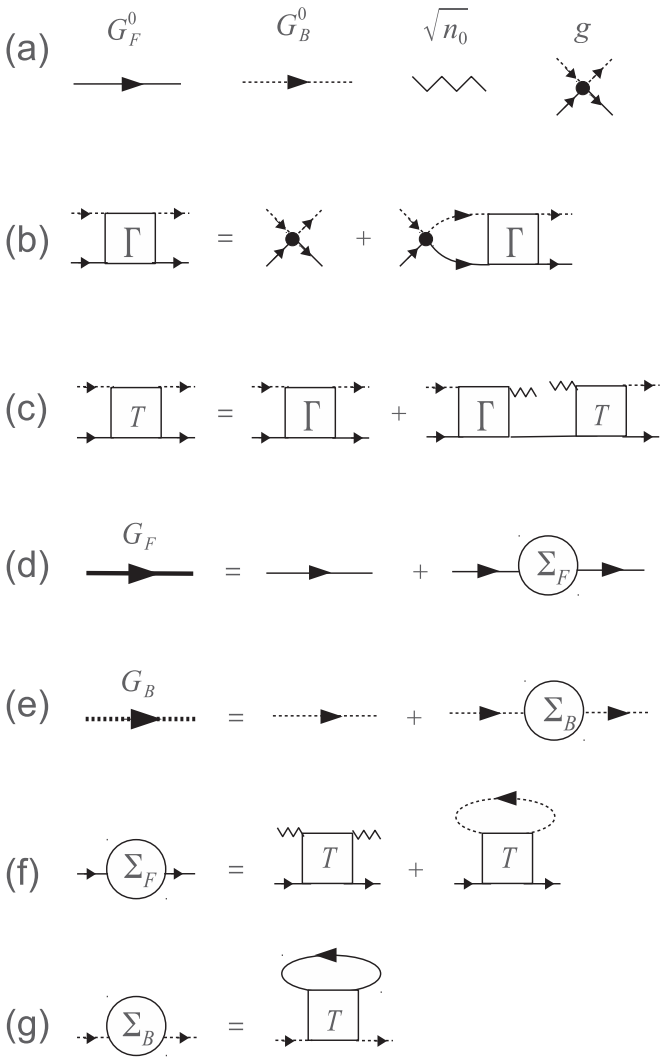

FIG. 1. Feynman diagrams corresponding to the formulas in the text.

where T means time ordering. The corresponding free boson and fermion propagators in frequency space are given by

$$G_B^0(\mathbf{k},\omega) = \frac{1}{\omega - \varepsilon_B(k) + i\eta}, \quad (6)$$

and

$$G_F^0(\mathbf{k},\omega) = \frac{\theta(k-k_F)}{\omega - \varepsilon_F(k) + i\eta} + \frac{\theta(k_F-k)}{\omega - \varepsilon_F(k) - i\eta}, \quad (7)$$

where $\varepsilon_{B,F}(k) = k^2/(2m_{B,F})$ are the noninteracting boson and fermion energies and k_F is the Fermi momentum, related to the fermion density n_F by $n_F = k_F^3/(6\pi^2)$.

We use these free GFs to construct the BF T matrix in ladder approximation. The result can be written as [14,18]

$$T(\mathbf{k},\omega) = \frac{1}{\Gamma^{-1}(\mathbf{k},\omega) - n_0 G_F^0(\mathbf{k},\omega)}. \quad (8)$$

The regularized BF scattering matrix Γ with no boson in the condensate is a standard expression, which can be found in the

literature [18,19,21]

$$\Gamma(\mathbf{k},\omega) = \frac{1}{\frac{m_r}{2\pi a} - J(\mathbf{k},\omega)}, \tag{9}$$

where $m_r = m_F m_B/(m_F + m_B)$ is the reduced mass, a is the BF scattering length, and J denotes the uncorrelated BF propagator that is given by

$$J(\mathbf{k},\omega) = \int \frac{d^3k'}{(2\pi)^3}\left[\frac{1-\theta\left(k_F - \left|\frac{m_F}{M}\mathbf{k}+\mathbf{k}'\right|\right)}{\omega - \frac{k^2}{2M} - \frac{k'^2}{2m_r} + i\eta} + \frac{2m_r}{k'^2}\right], \tag{10}$$

with $M = m_B + m_F$. The Feynman diagrams corresponding to the Γ and T matrices are shown in Figs. 1(b) and 1(c).

For the following calculations it is important to study the analytical properties of the T matrix. The continuum threshold, i.e., the energy above which the T-matrix has an imaginary part, lies at

$$\omega_{th}(k) = \begin{cases} \frac{(k-k_F)^2}{2m_B} + E_F & \left(k \leqslant \frac{M}{m_F}k_F\right) \\ \frac{k^2}{2M} + E_F & \left(k > \frac{M}{m_F}k_F\right) \end{cases}, \tag{11}$$

where $E_F = k_F^2/(2m_F)$. For not too high momenta k, $\Gamma(k,\omega)$ has a pole at $\Omega_\Gamma(k)$ below this threshold. As a consequence, the T matrix has one or two poles below threshold:

$$\begin{aligned} T(\mathbf{k},\omega) &= \frac{\omega - \varepsilon_F(k)}{(\omega-\varepsilon_F(k))\Gamma^{-1}(\mathbf{k},\omega) - n_0} \\ &= \frac{(\omega-\varepsilon_F(k))S_1(k)\theta(k_F-k)}{\omega - \Omega_1(k) - i\eta} \\ &\quad + \frac{(\omega-\varepsilon_F(k))S_1(k)\theta(k-k_F)}{\omega - \Omega_1(k) + i\eta} \\ &\quad + \frac{(\omega-\varepsilon_F(k))S_2(k)}{\omega - \Omega_2(k) + i\eta} + T_c(\mathbf{k},\omega), \end{aligned} \tag{12}$$

where $T_c(\mathbf{k},\omega)$ is the continuum part and $(\Omega_i - \varepsilon_F)S_i$ is the residue of the pole at $\omega = \Omega_i$ (if there is only one pole, we set $S_2 = 0$).

In Fig. 2 we show the dispersion of the two poles of T for some cases. The poles are physically of very different nature. Ω_2 is a collective pole created by BF scattering with the boson always out of the condensate. Ω_1 stems from the elastic scattering of the fermion off the Bose condensate and, thus, it is essentially given by the free fermion dispersion. This is also the reason why Ω_1 lies in the upper half of the complex plane for $k < k_F$, as it is in the case of the free fermion GF (7). Of course, in T both branches interact and depending on the system parameters they can be more or less repelled from one another. An interesting aspect, already revealed in Ref. [14], is that the Ω_2 branch corresponds to a stable BF pair that exists even for very weak attraction so that there is no bound state in free space. This phenomenon is similar to the existence of the Cooper pole in a pure two-species Fermi gas, since the stability of the BF pair in weak coupling is due to the fact that there is still a sharp Fermi edge in the problem.

At this point it is worth discussing a subtle point of the theory related to a possible crossing or inversion of the two branches shown as the dashed lines in Fig. 2. In an unpolarized spin-1/2 Fermi system, it is known from the Thouless criterion that once the T matrix has a pole at $\omega = 2\mu$ (where μ is the fermion chemical potential), an instability towards the superfluid (superconducting) state appears, and that for lower temperatures and in particular for zero temperature the ground state of the system has to be changed from the Hartree-Fock (HF) to the BCS one [36]. In our BF case, one would think that there should be also some criterion that tells us when our description of a single BF pair in an uncorrelated ground state becomes invalid and the ground state has to be changed into a state consisting of many interacting BF pairs. We are not aware that such a criterion has been given in the literature. However, we will see in Sec. III that as soon as Ω_Γ drops below ε_F for $k < k_F$, the correlation energy does no longer vanish in the limit $n_0 \to 0$, as it should. We therefore suspect that in this case our theory is not valid any more and we discard in the present work cases in the parameter space where this happens (e.g., lower panels of Fig. 2).

Contrary to the case of spin-1/2 fermions, where the new ground state of Bose condensed Cooper pairs can be described within BCS theory, it is unclear how this new ground state of correlated BF pairs should look. In any case, as it was pointed out in Ref. [41], it is obvious that since the BF pairs are fermions, this cannot be treated as in BCS theory as suggested in Ref. [40]. This problem shall be a very interesting subject for future studies.

III. SINGLE-PARTICLE GREEN'S FUNCTIONS AND CORRELATED GROUND STATE

In order to obtain the occupation numbers in the correlated ground state, let us get back to the single-particle GFs. As in the standard NSR approach [23], where the particle number is obtained from a GF in which the self-energy in the Dyson equation is treated only to lowest order [diagrams Figs. 1(d) and 1(e)], we will write for the boson and fermion GFs

$$G_{B,F}(\mathbf{k},\omega) = G^0_{B,F}(\mathbf{k},\omega) + G^{02}_{B,F}(\mathbf{k},\omega)\Sigma_{B,F}(\mathbf{k},\omega). \tag{13}$$

The self-energies in ladder approximation are defined by

$$\begin{aligned} \Sigma_F(\mathbf{k},\omega) &= n_0 T(\mathbf{k},\omega) + i\int\frac{d^3K}{(2\pi)^3}\int\frac{d\omega'}{2\pi}e^{i\omega'\eta} \\ &\quad \times T(\mathbf{K},\omega')G^0_B(\mathbf{K}-\mathbf{k},\omega'-\omega), \end{aligned} \tag{14}$$

and

$$\begin{aligned} \Sigma_B(\mathbf{k},\omega) &= -i\int\frac{d^3K}{(2\pi)^3}\int\frac{d\omega'}{2\pi}e^{i\omega'\eta} \\ &\quad \times T(\mathbf{K},\omega')G^0_F(\mathbf{K}-\mathbf{k},\omega'-\omega), \end{aligned} \tag{15}$$

see Feynman diagrams in Figs. 1(f) and 1(g).

By truncating the Dyson equation already at first order in Σ in Eq. (13), we treat the correlation effects only to leading order. To be consistent, we should therefore not include the condensate depletion into the calculation of the T matrix. In other words, for the condensate density n_0 that enters the calculation of T and $\Sigma_{B,F}$, we put

$$n_0 = n_B, \tag{16}$$

n_B being the total boson density, since in an uncorrelated system at zero temperature all bosons are condensed. Although one might be tempted to use the "final" condensate density as a better approximation for n_0, one should remember that

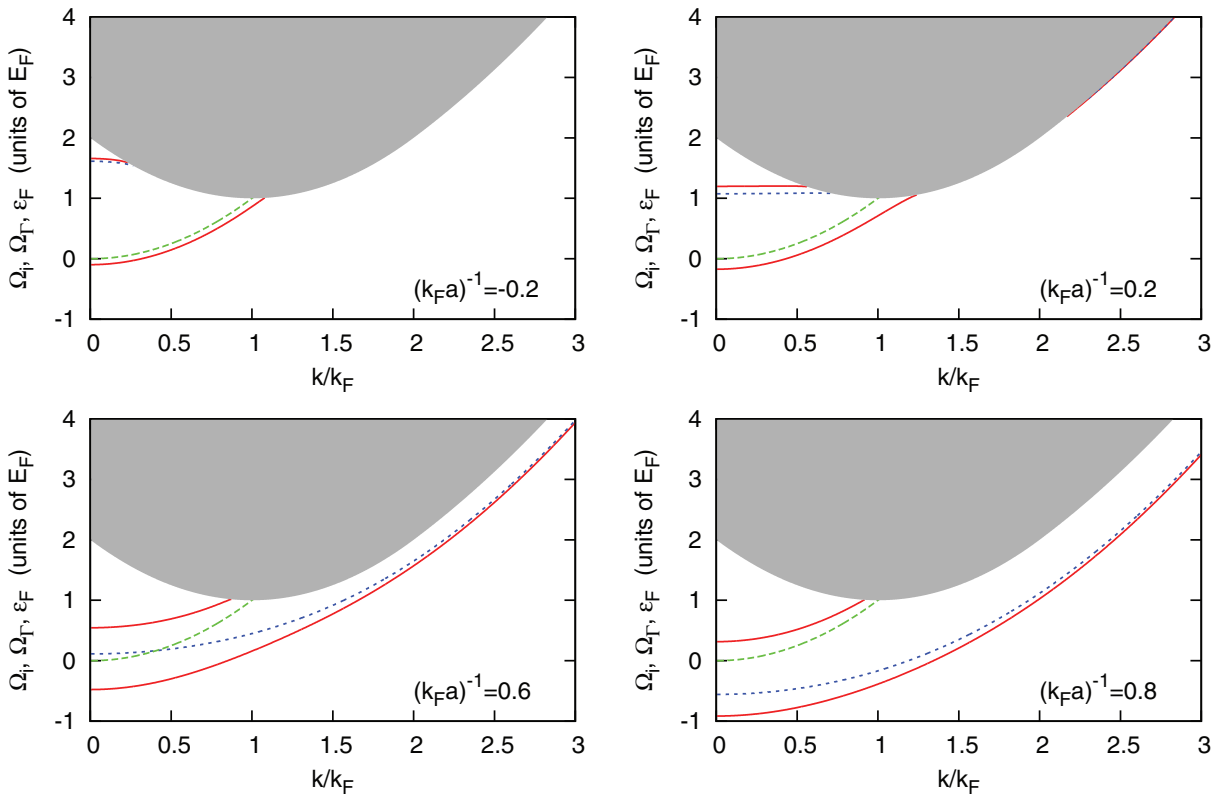

FIG. 2. (Color online) Dispersion relation of the poles of the T matrix for $m_B = m_F$ and $n_B = n_F$ for various values of $(k_F a)^{-1}$. The solid red lines represent poles in T matrix. The short-dashed blue line represents the pole in Γ. The long-dashed green line is the noninteracting fermion single-particle energy $\varepsilon_F(k) = k^2/(2m_F)$. The gray area corresponds to the continuum where Γ and T have a nonzero imaginary part.

standard RPA [36,39] is always built on top of the uncorrelated ground state, and only in this way one can be sure that it respects all theorems (see discussion below).

Notice that the first term of Σ_F contains one-particle reducible diagrams (i.e., diagrams that can be separated by cutting a single fermion line), because in the T matrix the boson can disappear in the condensate. Nevertheless, this term has to be retained within RPA, and as long as Σ_F is kept only to first order in Eq. (13) this is not a problem.

Using the above equations, the fermion and boson occupation numbers can be calculated from

$$\rho_{B,F}(k) = \pm i \int \frac{d\omega}{2\pi} e^{i\eta\omega} G_{B,F}(\mathbf{k},\omega), \tag{17}$$

the upper (lower) sign being valid for bosons (fermions). Inserting the explicit expressions for the self-energies, one obtains

$$\rho_F(k) = \theta(k_F - k)\Gamma^{-1}(k,\Omega_1(k))S_1(k) + \theta(k - k_F) \times \int \frac{d^3K}{(2\pi)^3} \frac{[\Omega_1(K) - \varepsilon_F(K)]S_1(K)\theta(k_F - K)}{[\Omega_1(K) - \varepsilon_F(k) - \varepsilon_B(\mathbf{K} - \mathbf{k})]^2}, \tag{18}$$

$$\rho_B(k) = \int \frac{d^3K}{(2\pi)^3} S_1(K)[\Omega_1(K) - \varepsilon_F(K)] \times \frac{\theta(k_F - K)\theta(|\mathbf{K} - \mathbf{k}| - k_F)}{[\Omega_1(K) - \varepsilon_F(\mathbf{K} - \mathbf{k}) - \varepsilon_B(k)]^2}. \tag{19}$$

The results for the occupation numbers are presented in Fig. 3 for various system parameters. Note that, as a consequence of the perturbative treatment of the self-energy in Eq. (13), the Z factor determining the jump of ρ_F at the Fermi surface can become negative, or the number of bosons out of the condensate can become larger than the total number of bosons. We discard such cases and restrict ourselves to parameters in which the correlations are not too strong.

The Luttinger theorem states that in the fermion distribution the momentum integral over what is missing with respect to the free case below k_F is exactly compensated by the part above k_F, i.e.,

$$\int_{k<k_F} \frac{d^3k}{(2\pi)^3}[1 - \rho_F(k)] = \int_{k>k_F} \frac{d^3k}{(2\pi)^3}\rho_F(k). \tag{20}$$

From general properties of RPA theory (see below) one expects that the Luttinger theorem should be exactly fulfilled in our scheme, although from the final expression (18) for the occupation numbers this is hard to see. In our numerical calculations, Eq. (20) is fulfilled to a relative accuracy of better than 10^{-3}. This is the advantage of treating the self-energy perturbatively in Eq. (13). If we had resummed the Dyson equation to all orders, as in [19,21,22], the Luttinger theorem would most likely have been violated. For instance, in Fig. 8(a) of Ref. [21] it seems that the number of fermions above the

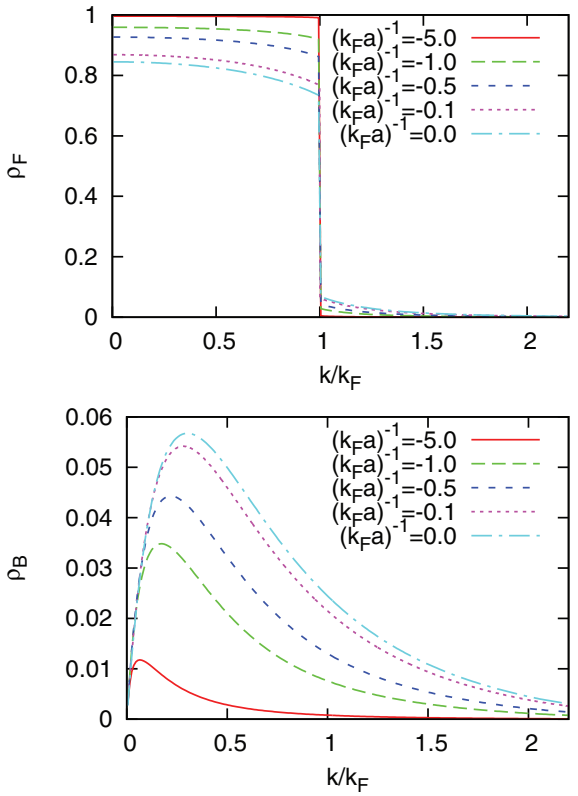

FIG. 3. (Color online) Occupation numbers of fermions and bosons at $m_B = m_F$ and $n_B = n_F$ for various values of $(k_F a)^{-1}$.

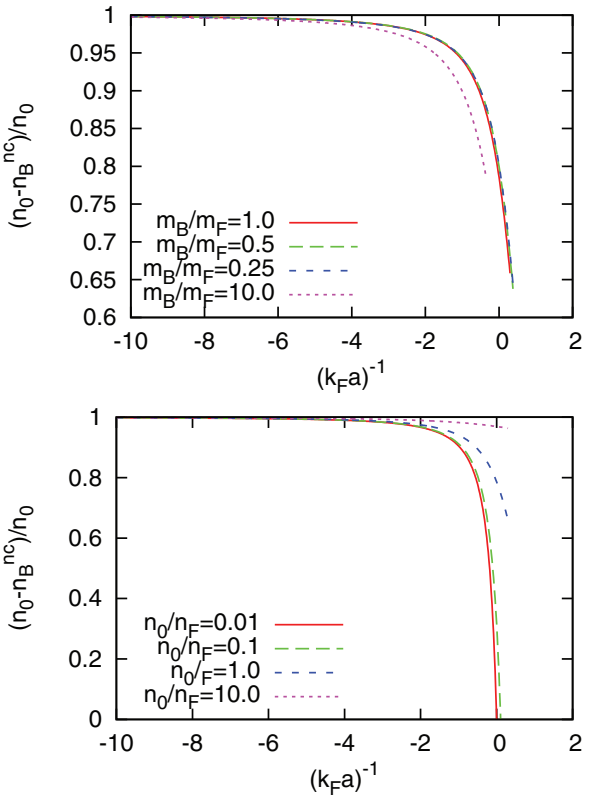

FIG. 4. (Color online) Condensate depletion as a function of the interaction strength: (a) for various mass ratios m_B/m_F and $n_B = n_F$; (b) for various density ratios n_B/n_F and $m_B = m_F$.

Fermi surface[1] is larger than the number of fermions missing below.

In addition to the Luttinger theorem (20) for the fermions, our formulation satisfies the following relation:

$$n_B^{nc} = \int_{k>0} \frac{d^3k}{(2\pi)^3}\rho_B(k) = \int_{k>k_F} \frac{d^3k}{(2\pi)^3}\rho_F(k), \qquad (21)$$

where n_B^{nc} denotes the density of noncondensed bosons. The relation has a very intuitive interpretation: each time a boson is scattered out of the condensate, also a fermion is scattered out of the Fermi sea. Therefore the total number of fermions above k_F must be equal to the number of bosons out of the condensate. The condensate depletion as a function of the interaction strength is shown in Fig. 4 for different mass and density ratios. As mentioned before, we stop the calculation as soon as $n_0 - n_B^{nc}$ or the Z factor of the fermions becomes negative or the branch Ω_Γ drops below ε_F for $k < k_F$.

Let us discuss the approximation scheme that is set up in the foregoing equations. One recognizes the similarity with the NSR approach for the treatment of interacting two-component Fermi gases close to the transition temperature to the superfluid state [23]. The main difference is that the NSR formalism is transcribed here to an interacting BF system at zero temperature.

The T matrix, which, in the case of a pure Fermi system, simultaneously sums the particle-particle (pp) and hole-hole (hh) ladders, is sometimes also called the pp-RPA [36]. It is well known that RPA theory has appreciable properties as the fulfillment of conservation laws and sum rules. (The latter statements are, strictly speaking, only valid if the RPA is solved in the HF basis [36]. However, in our case the HF shifts are unimportant because they disappear in the regularization procedure when the coupling constant g tends to zero while the cutoff tends to infinity, keeping the scattering length a constant [18,19,21].)

It is, in principle, straightforward to prove that the Luttinger theorem is fulfilled in strict application of RPA. The proof is straightforward and well known in the case of particle-hole (ph) RPA in a system with discrete single-particle states as it is often considered in, e.g., nuclear or atomic and chemical physics [36,42,43], i.e., for finite Fermi systems.[2] In the case of pp-RPA, things are less well known but corresponding expressions can also be found in the literature [37]. In the BF case, the fulfillment of the Luttinger theorem, i.e., the fact that the occupation numbers of levels above the Fermi surface exactly cancel the reduction of the occupation numbers of levels below the Fermi surface, has also been demonstrated for finite size cases with the Hubbard model [38]. We intend to show it with RPA in continuum cases where things are, of

[1]Note that in Ref. [21] the Fermi surface is not at $k = k_F$ because k_F has a different meaning in that paper.

[2]In Refs. [37,42] explicit expressions for the correlated parts of the single-particle occupation numbers are given. From these expressions, it becomes so obvious that particle number is conserved that this property is most of the time not even stated in the literature.

course, a little more tricky, for instance from the numerical point of view.

It is, however, very important to notice a subtle difference between this strict RPA approach and the NSR scheme. The latter is generally formulated in finite-temperature formalism, and the zero-temperature case is obtained as a limiting procedure as, e.g., in [21]. However, the two formalisms do not become equivalent in this limit (see, e.g., chapter 3.3 of Ref. [44]), even if the self energy is only treated to first order and not summed as in Ref. [21]. This point is a topic for future research.

In our scheme, the particle numbers $n_{B,F}$ are fixed from the beginning and they are not modified by the inclusion of correlations (because the Luttinger theorem is satisfied). Therefore we cannot determine the chemical potentials in the way this is usually done in the NSR scheme by inverting the $n(\mu)$ relation obtained by integrating Eq. (17) over k. But of course, also in our scheme the correlations change the equation of state, i.e., the relation between n and μ. Therefore, we have free chemical potentials, $\mu_F^0 = E_F = k_F^2/(2m_F)$ and $\mu_B^0 = 0$, and modified ones $\mu_{F,B}$ that include the correlation effects. But here the corrections to the chemical potentials are calculated perturbatively to first order in the correlations. They are obtained from the correlated ground-state energy density, i.e.,

$$\mu_{F,B} = \frac{\partial \mathcal{E}}{\partial n_{F,B}}. \tag{22}$$

The energy density $\mathcal{E}$ is calculated within RPA in the usual way from the coupling constant integration [39]

$$\mathcal{E} - \mathcal{E}_0 = -i \int_0^1 \frac{d\lambda}{\lambda} \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} e^{i\omega\eta} \Sigma_F^\lambda(k,\omega) G_F^0(k,\omega), \tag{23}$$

where Σ_F^λ is the self-energy calculated with coupling constant $g\lambda$ instead of g. Considering a finite value of the coupling constant g and a cutoff and taking the cutoff to infinity only in the end of the calculation, one obtains the following simple formula for the ground-state energy:

$$\mathcal{E} - \mathcal{E}_0 = \int_{k<k_F} \frac{d^3k}{(2\pi)^3} [\Omega_1(k) - \varepsilon_F(k)]. \tag{24}$$

This expression for the energy density agrees with that given in Ref. [14], besides the fact that here the extra term of the fermion-hole boson-condensate matrix element is missing, since it has been absorbed by the regularization procedure. From this formula it is clear that as long as Ω_Γ lies above ε_F for $k < k_F$, the branch Ω_1 approaches ε_F in the limit $n_0 \to 0$ and the correlation energy tends to zero, which is not true if Ω_Γ drops below ε_F for $k < k_F$.

The boson and fermion chemical potentials calculated in this way are shown in Fig. 5. As expected, we see that the chemical potentials are lowered by the correlations if the boson density n_0 or the interaction strength $|a|$ ($a < 0$) increases.

One can show analytically that the boson chemical potential satisfies

$$\mu_B = \Sigma_B(0,0), \tag{25}$$

which agrees to leading order in Σ_B with the usual condition for Bose condensed systems.

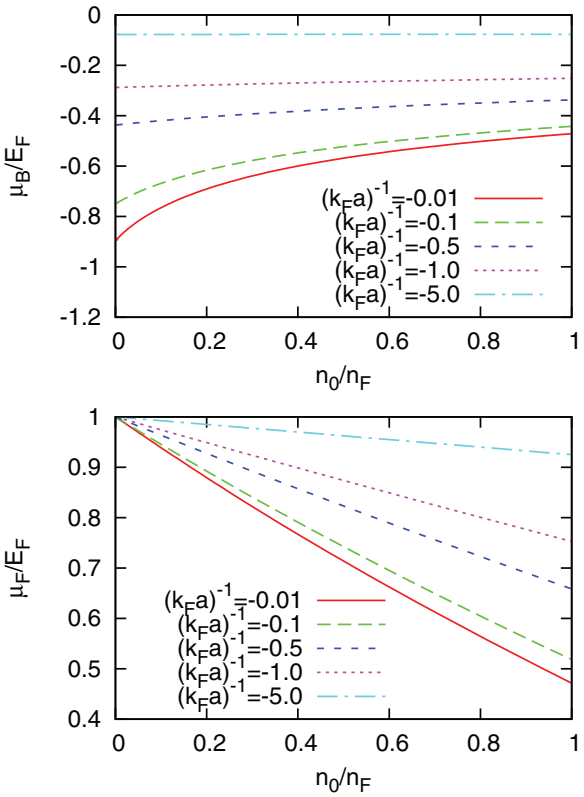

FIG. 5. (Color online) Fermion and boson chemical potentials as a function of the boson density n_0 for various values of the scattering length and $m_B = m_F$.

Let us now consider the case with almost vanishing boson number, i.e., the polaron limit, where it is immaterial whether the impurity is a boson or a fermion of another species (or opposite spin). Boson chemical potentials in this limit are displayed in Fig. 6 as functions of the interaction strength for various mass ratios m_B/m_F. We compare our results with those of Combescot *et al.* [30]. We see that the agreement is quite good for negative and not too large scattering lengths. For values of $(k_F a)^{-1}$ close to -1 the agreement deteriorates. This is not surprising, since we treat the self-energy only to

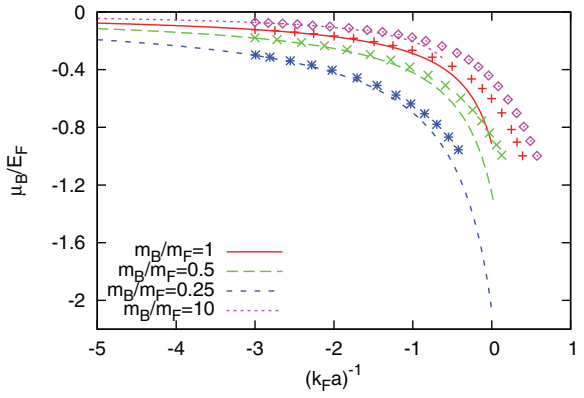

FIG. 6. (Color online) Boson chemical potential as a function of $(k_F a)^{-1}$ at $n_0/n_F = 0.001$ for various mass ratios. The symbols $+$, $\times$, $*$, and $\diamond$ are polaron chemical potentials extracted from Fig. 1 of Ref. [30] for $m_B/m_F = 1$, 0.5, 0.25, and ∞.

first order whereas in the polaron approach the whole series is summed.

It would therefore be desirable to sum up the self-energy to all orders. However, with the present form of the self-energy, this would cause other problems, such as the violation of the Luttinger theorem (20). We think that these issues should be settled before definite conclusions can be drawn from a nonperturbative approach.

IV. SUMMARY, DISCUSSION, AND OUTLOOK

In this work we used a T matrix approach to describe BF pair correlations in a BF mixture. The approach is very similar to the usual NSR theory for fermions [23]. However, there are subtle differences because we work within the zero-temperature formalism. Our approach is a strict application of what has been known as pp-RPA in nuclear physics [36,37]. As expected, this approach respects the Luttinger theorem. This is explicitly verified numerically to high precision in calculating the correlated fermion and boson occupation numbers. We also studied for the bosons the condensate depletion and found that the number of bosons scattered out of the condensate is exactly equal to the number of fermions scattered above the Fermi surface. In studies of spin-1/2 Fermi gases, it is often supposed that the Luttinger theorem is satisfied [see, e.g., Eq. (6) of Ref. [45]] but it is rarely checked whether the approximations that are used preserve this property. The problems found in studies of polarized Fermi systems [24–27] might also be related to this problem.

As in the original NSR approach, we keep the self-energies only to first order in the Dyson equation. Besides the nice properties mentioned before, this has of course also some drawbacks. For instance, the Z factor of the fermion GF (i.e., the jump of the occupation numbers at k_F) may become negative if the correlations are too strong. A possible way to avoid this overestimation of the correlation effects, without violating the Luttinger theorem, would be to use in the T matrix the self-consistently determined correlated occupation numbers instead of the uncorrelated ones. In nuclear physics this approximation is known as "renormalized RPA", see, e.g., Refs. [46,47].

We also investigated the polaron limit and found that the boson chemical potential agrees well with the results by Combescot *et al.* [30] in the weak-coupling region. Close to unitarity the results start to diverge, which is again a consequence of our perturbative treatment of the self-energy.

If one goes in the molecular regime beyond the polaron limit, one expects the system to have a completely different ground state, namely a Fermi sea of composite molecules. Actually this transition might already happen before the molecular limit, since there is, as in the Cooper pair problem, always a stable BF branch in the in-medium T matrix, even if in free space there is no bound state. How this transition happens is still unclear [41] and needs further investigation.

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